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On the Delay-Throughput Tradeoff in Distributed Wireless Networks

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Abstract

The delay-throughput of a single-hop wireless network with n randomly distributed links is analyzed. We consider a general shadow-fading model, described by parameters (α, ϖ), where α denotes the probability of shadowing and ϖ represents the average cross-link gains. The analysis relies on the distributed *on-off power allocation strategy* (i.e., links with a direct channel gain above a certain threshold transmit at full power and the rest remain silent) for the deterministic and stochastic packet arrival processes. In the first part of the paper, we analyze the effective throughput maximization of the network. It is proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. Then, we present the delay characteristics of the underlying network in terms of a packet dropping probability. We derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, we study the trade-off between the effective throughput of the network and delay-bounds for different packet arrival processes. In particular, we determine how much degradation will be enforced in the throughput by introducing other constraints.

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Index Terms

Throughput maximization, delay-throughput tradeoff, dropping probability, Poisson arrival process.

I. Introduction

As the demand for higher data rates increases, effective resource allocation emerges as the primary issue in wireless networks in order to satisfy Quality of Service (QoS) requirements. Central to the study of resource allocation schemes, the distributed power control algorithms for maximizing the network throughput have attracted significant research attention [1]–[6]. Also, achieving a low transmission delay is an important QoS requirement in buffer-limited networks [7]. In particular, for backlogged users[1] with real-time services (e.g., interactive games, live sport videos, etc), too much delay results in dropping some packets. Therefore, the main challenge in wireless networks with real-time services is to utilize an efficient power allocation scheme such that the delay is minimized, while achieving a high throughput.

The throughput maximization problem in cellular and multihop wireless networks has been extensively studied in [8]–[12]. In these works, delay analysis is not considered. However, it is shown that the high throughput is achieved at the cost of a large delay [13]. This problem has motivated the researchers to study the relation between the delay characteristics and the throughput in wireless networks [14]–[17]. In particular, in most recent literature [13], [18]–[25], the tradeoffs between delay and throughput have been investigated as a key measure of the network's performance. The first studies on achieving a high throughput along with a low delay in ad hoc wireless networks are framed in [16] and [17]. This line of work is further expanded in [13], [19] and [20] by using different mobility models. El Gamal *et al.* [13] analyze the optimal delay-throughput scaling for some wireless network topologies. For a static random network with n nodes, they prove that the optimal tradeoff between throughput T_n and delay D_n is given by $D_n = \Theta(nT_n)$. Reference [13] also shows that the same result is achieved in random mobile networks, when

[1]For each user, there is always a packet available to be transmitted.

$T_n = O(1/\sqrt{n \log n})$. Neely and Modiano [20] consider the delay-throughput tradeoff for mobile ad hoc networks under the assumption of redundant packets transmission through multiple paths. Sharif and Hassibi [21] analyze the delay characteristics and the throughput in a broadcast channel. They propose an algorithm to reduce the delay without too much degradation in the throughput. This line of work is further extended in [22] by demonstrating that it is possible to achieve the maximum throughput and short-term fairness simultaneously in a large-scale broadcast network.

In [26], we addressed the throughput maximization of a distributed single-hop wireless network with K links, where the links are partitioned into a fixed number (M) of clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. We proposed a distributed and non-iterative power allocation strategy, where the objective for each user is to maximize its best estimate (based on its local information, i.e., direct channel gain) of the average sum-rate of the network. Under the Rayleigh fading channel model possibly with shadowing effect, it is proved that when the number of links is large, the optimum power allocation strategy for each user is the threshold-based on-off power scheme (i.e., the links with a direct channel gain above a certain threshold τ_n transmit at full power and the rest remain silent). It is also demonstrated that the maximum average sum-rate of the network for every value of $1 \leq M \leq K$ is achieved at $M = 1$ and it scales as $\Theta(\log K)$. Also, the optimum threshold level that achieves the maximum average sum-rate of the network is obtained as $\tau_n = \log n - 2 \log \log n + O(1)$, where $n = \frac{K}{M}$ is the number of links in each cluster. However, the delay related issues were not addressed in [26].

In this work, we follow the distributed single-hop wireless network model proposed in [26] with $M = 1$ (which is the case with the maximum throughput) and address the delay-throughput tradeoff of the network. In the first part, we define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we obtain the *full buffer probability* of a link for the deterministic and stochastic packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process. It is proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process.

In the second part, we present the delay characteristics of the underlying network in terms of a packet dropping probability, and for deterministic and stochastic packet arrival processes. These are quite different from the delay analysis with the ON/OFF Bernoulli scheme in [27]. Primarily, we utilize a distributed approach using local information, i.e., direct channel gains, while [27] relies on a central controller which studies the channel conditions of all the links and decides accordingly. We use a homogeneous network with *quasi-static block fading* without path loss. This differs from the geometric models proposed in [13], [19] and [20], which are based on the distance between the source and the destination (i.e., power decay-versus-distance law).

It is shown that increasing the number of links gives rise to increasing the network throughput, at the cost of increasing the delay. This will cause the higher packet droppings in the network with a limited buffer size. We derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, we study the tradeoff between the effective throughput of the network and other performance measures, i.e., dropping probability and delay-bounds for different arrival processes. In particular, we determine how much degradation will be enforced in the throughput by introducing other constraints, and how much this degradation depends on the arrival process.

The rest of the paper is organized as follows. In Section II, the network model and objectives are described. The throughput maximization of the underlying network is presented in Section III. The delay characteristics in terms of the dropping probability are analyzed in Section IV. Section V establishes the delay-throughput tradeoff for the network. In Section VI, the simulation results are presented. Finally, in Section VII, an overview of the results and conclusions are presented.

Notations: For any functions $f(n)$ and $g(n)$ [28]:

- $f(n) = O(g(n))$ means that $\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| < \infty$.

- $f(n) = o(g(n))$ means that $\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| = 0$.
- $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.
- $f(n) = \Omega(g(n))$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0$.
- $f(n) = \Theta(g(n))$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where $0 < c < \infty$.
- $f(n) \sim g(n)$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$.
- $f(n) \approx g(n)$ means that $f(n)$ is approximately equal to $g(n)$, i.e., if we replace $f(n)$ by $g(n)$ in the equations, the results still hold.

Throughout the paper, we use $\log(.)$ as the natural logarithm function and $\mathbb{N}_n$ for representing the set $\{1, 2, \cdots, n\}$. Also, $\mathbb{E}[.]$ represents the expectation operator, and $\mathbb{P}\{.\}$ denotes the probability of the given event.

II. Network Model and Problem Description

A. Network Model

In this work, we consider a distributed single-hop wireless network, in which n pairs of nodes[2], indexed by $\{1, ..., n\}$, are located within the network area (Fig. 1). We assume the number of links, n, is known information for the users. All the nodes in the network are assumed to have a single antenna. Also, it is assumed that all the transmissions occur over the same bandwidth. In addition, we assume that each receiver knows its direct channel gain with the corresponding transmitter, as well as the interference power imposed by other users. However, each transmitter is assumed to be only aware of the direct channel gain to its corresponding receiver. The power of Additive White Gaussian Noise (AWGN) at each receiver is assumed to be N_0.

We assume that the time axis is divided into slots with the duration of one transmission block, which is defined as the unit of time. The channel model is assumed to be flat Rayleigh fading with the shadowing effect. The channel gain[3] between transmitter j and receiver i at

[2]The term "pair" is used to describe the transmitter and the related receiver, while the term "user" is used only for the transmitter.

[3]In this paper, *channel gain* is defined as the square magnitude of the *channel coefficient*.

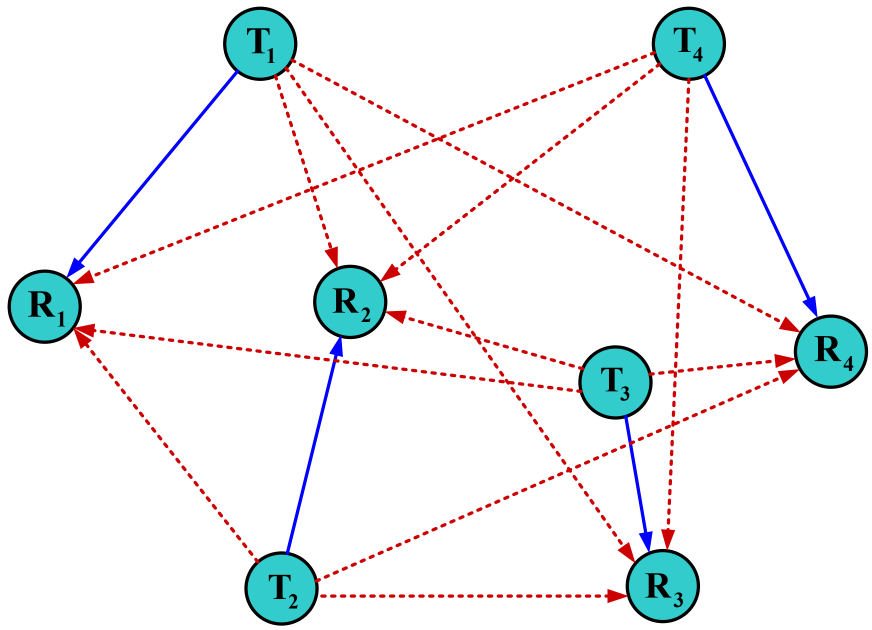


Fig. 1. A distributed single-hop wireless network with $n = 4$.

time slot t is represented by the random variable $\mathcal{L}_{ji}^{(t)}$ [4]. For $j = i$, the *direct channel gain* is defined as $\mathcal{L}_{ji}^{(t)} \triangleq h_{ii}^{(t)}$, where $h_{ii}^{(t)}$ is exponentially distributed with unit mean (and unit variance). For $j \neq i$, the *cross channel gains* are defined based on a shadowing model as follows[5]:

$$\mathcal{L}_{ji}^{(t)} \triangleq \begin{cases} \beta_{ji}^{(t)} h_{ji}^{(t)}, & \text{with probability} \quad \alpha \\ 0, & \text{with probability} \quad 1-\alpha, \end{cases} \tag{1}$$

where $h_{ji}^{(t)}$s have the same distribution as $h_{ii}^{(t)}$s, $0 \leq \alpha \leq 1$ is a fixed parameter, and the random variable $\beta_{ji}^{(t)}$, referred to as the *shadowing factor*, is independent of $h_{ji}^{(t)}$ and satisfies the following conditions:

- $\beta_{min} \leq \beta_{ji}^{(t)} \leq \beta_{max}$, where $\beta_{min} > 0$ and β_{max} is finite,
- $\mathbb{E}\big[\beta_{ji}^{(t)}\big] \triangleq \varpi \leq 1$.

All the channels in the network are assumed to be quasi-static block fading, i.e., the channel gains remain constant during one block and change independently from block to block. In

[4]In the sequel, we use the superscript (t) for some events to show that the events occur in time slot t.

[5]For more details, the reader is referred to [29] and [30] and references therein.

other words, $\mathcal{L}_{ji}^{(t)}$ is independent of $\mathcal{L}_{ji}^{(t')}$ for $t \neq t'$. This model is also used in [17], [21] and [22].

Assuming that the transmitted signals are Gaussian, the interference term seen by receiver $i \in \mathbb{N}_n$ at time slot t will be Gaussian with power

$$I_i^{(t)} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathcal{L}_{ji}^{(t)} p_j^{(t)}, \tag{2}$$

where $p_j^{(t)}$ is the transmission power of user j at time slot t. Under these assumptions, the achievable data rate of each link $i \in \mathbb{N}_n$ is expressed as

$$R_i^{(t)} = \mathbb{E}_{I_i^{(t)}} \left[\log \left(1 + \frac{h_{ii}^{(t)} p_i^{(t)}}{I_i^{(t)} + N_0} \right) \right] \quad \text{nats/channel use}, \tag{3}$$

assuming no constraint on the decoding delay, i.e., decoding can be performed over an arbitrarily large number of blocks.

B. On-Off Power Allocation Strategy

We consider a homogeneous network in the sense that all the links have the same configuration and use the same protocol. Thus, the transmission strategy for all users are agreed in advance. We assume a limited buffer network, where each link has a buffer size equal to one packet. Also, the transmission blocks of the users are assumed to be synchronous with each other with the same duration. In this work, we assume that all the links utilize the threshold-based on-off power allocation strategy proposed in [26]. In this reference, it is shown that the on-off power allocation scheme is asymptotically (in terms of the number of links) optimum in terms of the sum-rate throughput, assuming the availability of direct channel gains at the transmitters. Unlike most of the works in the literature that assume backlogged users, here we assume a practical model for the packet arrivals in which the buffer of each link is not necessarily full (of packet) all the time. Based on this observation, we adopt the on-off power allocation scheme during each time slot t as follows:

1- Based on the direct channel gain, the transmission policy is[6]

$$p_i^{(t)} = \begin{cases} 1, & \text{if } \ h_{ii}^{(t)} > \tau_n \text{ and the buffer of link } i \text{ is full at time slot } t \\ 0, & \text{Otherwise}, \end{cases} \tag{4}$$

where τ_n is a prespecified threshold level that is a function of n and also depends on the channel model and packet arrival process.

2- Knowing its corresponding direct channel gain, each active user i transmits with full power and the rate (3).

C. Packet Arrival Process

One of the most important parameters in the network analysis is the model for the packet arrival process. The packet arrival process is a random process which is described by either the arrival time of the packets or the interarrival time between the subsequent packets. These quantities may be modeled by the deterministic or stochastic processes (Fig. 2). In this paper, we consider the following packet arrival processes:

- *Poisson Arrival Process (PAP):* In this process, the number of arrived packets in any interval of unit length is assumed to have a Poisson distribution with the parameter $\frac{1}{\lambda}$. This process is a commonly used model for random and mutually independent packet arrivals in queueing theory [31].
- *Bernoulli Arrival Process (BAP):* In this process, in any given time slot, the probability that a packet arrives is $\rho \triangleq \frac{1}{\lambda}$ [7]. Moreover, the arrival of the packets in different slots occurs independently. This model has been used in many works in the literature such as [20] and [32].
- *Constant Arrival Process (CAP):* In this process, packets arrive continuously with a constant rate of $\frac{1}{\lambda}$ packets per unit length (Fig. 2-b) [33].

It is assumed that the packet arrival process for all links is the same. Let us denote $t_{A_k}^{(i)}$ as the time instant of the k^{th} packet arrival into the buffer of link i. It is observed from Fig.

[6] In fact, if there is no packet in the buffer, it does not make sense for the user to be active, even if its channel is good.
[7] We choose the parameter ρ as $\frac{1}{\lambda}$ to be consistent with other packet arrival processes.

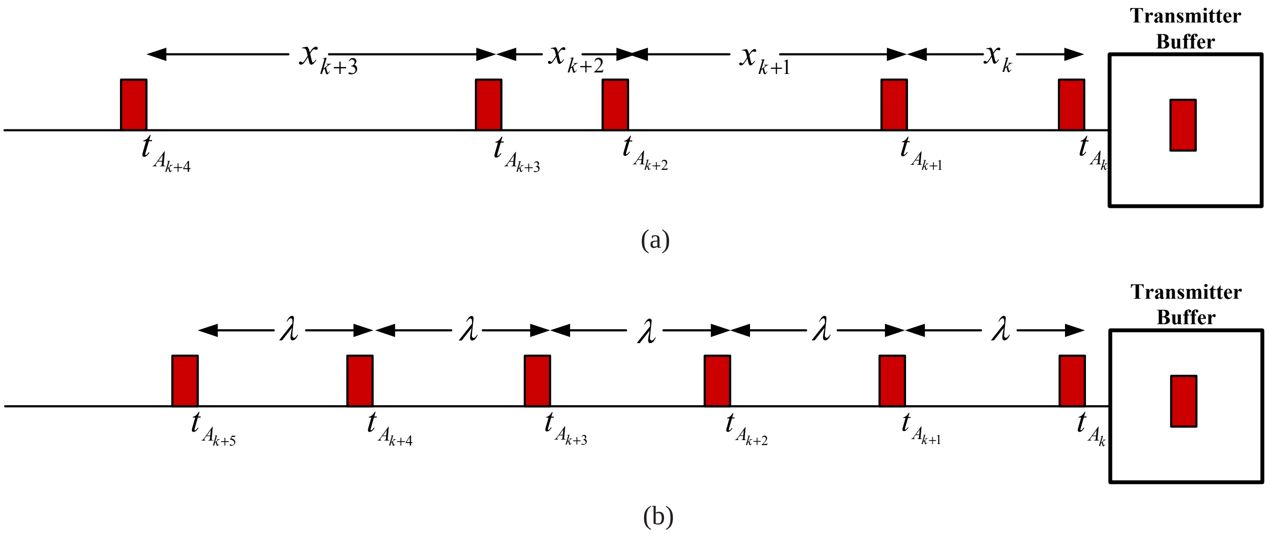


Fig. 2. A schematic figure for a) stochastic packet arrival process, b) constant packet arrival process.

2-a that $t_{A_k}^{(i)} = \sum_{j=1}^{k-1} x_j^{(i)} + t_0^{(i)}$ where $t_0^{(i)}$ is the starting time for link i, and the random variable $x_j^{(i)}$ is the interarrival time defined as

$$x_j^{(i)} \triangleq t_{A_{j+1}}^{(i)} - t_{A_j}^{(i)}, \tag{5}$$

with $\mathbb{E}[x_j^{(i)}] = \lambda$. For the CAP, $x_j^{(i)} = \lambda$ and $t_{A_k}^{(i)} = (k-1)\lambda + t_0^{(i)}$[8], while for the PAP, $x_j^{(i)}$'s are independent samples of an exponential random variable x with the probability density function (pdf)

$$f_X(x) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}x}, \qquad x > 0. \tag{6}$$

Also for the BAP, $x_j^{(i)}$'s are independent samples of a geometric random variable X with the probability mass function (pmf)

$$p_X(m) \triangleq \mathbb{P}\{X = m\} = (1-\rho)^{m-1}\rho, \qquad m = 1, 2, ..., \tag{7}$$

with $\rho \triangleq \frac{1}{\lambda}$.

We represent $t_{D_k}^{(i)}$ as the time instant at which either the k^{th} arriving packet departs the buffer of link i for the transmission or drops from the buffer. In such configuration, we have the following definition:

[8]For analysis simplicity, we assume that λ is an integer number.

Definition 1 ***(Delay):*** *The random variable* $\mathscr{D}_k^{(i)} \triangleq t_{D_k}^{(i)} - t_{A_k}^{(i)}$ *for each link* i *is defined as the delay between the departure and the arrival time of each packet* k*, expressed in terms of the number of time slots.*

Due to the finite buffer size and the on-off power allocation strategy, the existing buffered packet may be dropped. The dropping happens when one packet arrives before the previous arrived packet has any chance to be served. Therefore, the event that the dropping of packet k occurs in link $i \in \mathbb{N}_n$ is defined as

$$\begin{aligned} \mathscr{B}_i &\equiv \left\{ \mathscr{D}_k^{(i)} \geq t_{A_{k+1}}^{(i)} - t_{A_k}^{(i)} \right\} && (8) \\ &\equiv \left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \right\}. && (9) \end{aligned}$$

The packet dropping probability in each link $i \in \mathbb{N}_n$, denoted by $\mathbb{P}\{\mathscr{B}_i\}$, can be obtained as

$$\begin{aligned} \mathbb{P}\{\mathscr{B}_i\} &= \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \right\} && (10) \\ &= \int_0^\infty \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \middle| x_k^{(i)} = x \right\} f_X(x) dx, \quad \text{for PAP}, && (11) \\ &= \sum_{m=1}^\infty \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \middle| x_k^{(i)} = m \right\} p_X(m), \quad \text{for BAP}, && (12) \\ &= \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq \lambda \right\}, \quad \text{for CAP}. && (13) \end{aligned}$$

where $f_X(x)$ and $p_X(m)$ are defined as (6) and (7), respectively. In Section IV, we will obtain $\mathbb{P}\{\mathscr{B}_i\}$ for different packet arrival processes.

D. Objectives

Part I: Throughput Maximization: The main objective of the first part of this paper is to maximize the throughput of the underlying network. To address this problem, we first define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we obtain the *full buffer probability* of a link for the deterministic and stochastic packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process.

Part II: Delay Characteristics: The main objective of the second part is to analyze the delay characteristics of the underlying network in terms of the number of links (n) and λ. For this purpose, we first formulate the packet dropping probabilities based on the aforementioned packet arrival processes. Then, we derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Part III: Delay-Throughput Tradeoff: The main goal of the third part is to study the tradeoff between the effective throughput of the network and other performance measures, i.e., the dropping probability and the delay-bound (λ) for different packet arrival processes. In particular, we are interested to determine how much degradation will be enforced in the throughput by introducing the other constraints, and how much this degradation depends on the packet arrival process.

III. Throughput Maximization

In this section, we aim to derive the maximum throughput of the network with a large number (n) of links, based on using the distributed on-off power allocation strategy. The throughput of the network is defined as the average sum-rate of all links. However, to capture the effect of the packet arrival process, we define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we first obtain the *full buffer probability* of each link $i \in \mathbb{N}_n$ for different packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process.

A. Effective Throughput

In this section, we present a new performance metric in the network, called *effective throughput*, which is a function of the threshold level τ_n and λ. Let us start with the following definition.

Definition 2 ***(Effective Throughput):*** *Under the on-off power allocation strategy, the effective throughput of each link i, $i \in \mathbb{N}_n$, is defined (on a per-block basis) as*

$$\mathfrak{T}_i \triangleq \lim_{L\to\infty} \frac{1}{L} \sum_{t=1}^{L} R_i^{(t)} \mathcal{I}_i^{(t)}, \tag{14}$$

where $R_i^{(t)}$ is defined as (3) and $\mathcal{I}_i^{(t)}$ is an indicator variable which is equal to 1, if user i transmits at time slot t, and 0 otherwise. Furthermore, the effective throughput of the network is defined as

$$\mathfrak{T}_{\text{eff}} \triangleq \sum_{i=1}^{n} \mathfrak{T}_i. \tag{15}$$

The quantity $\mathfrak{T}_i$ represents the average amount of information conveyed through link i in a long period of time. This metric is suitable for real-time applications, where the packets have a certain amount of information and certain arrival rates. It should be noted that $\mathcal{I}_i^{(t)} = 1$ is equivalent to the case in which the buffer is full and the channel gain $h_{ii}^{(t)}$ is greater than the threshold level τ_n at time slot t. Defining the full buffer event as follows

$$\mathscr{C}_i^{(t)} \equiv \{\text{Buffer of link } i \text{ is full at time slot } t\}, \tag{16}$$

we have

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{I}_i^{(t)} = 1\right\} &= \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n,\ \mathscr{C}_i^{(t)}\right\} && (17)\\
&\overset{(a)}{=} \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\} \mathbb{P}\left\{\mathscr{C}_i^{(t)}\right\} && (18)\\
&= q_n \Delta_n, && (19)
\end{aligned}$$

where $q_n \triangleq \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\}$, and $\Delta_n \triangleq \mathbb{P}\left\{\mathscr{C}_i^{(t)}\right\}$ is the *full buffer probability*. In the above equations, (a) follows from the fact that the full buffer event depends on the packet arrival process as well as the direct channel gains $h_{ii}^{(t^{'})}$, for $t^{'} < t$, which is independent of the channel gain $h_{ii}^{(t)}$ (due to the block fading channel model). Thus,

$$\mathcal{I}_i^{(t)} = \begin{cases} 1, & \text{with probability } q_n \Delta_n, \\ 0, & \text{with probability } 1 - q_n \Delta_n. \end{cases} \tag{20}$$

It is observed that $\mathcal{I}_i^{(t)}$ is a Bernoulli random variable with parameter $q_n\Delta_n$. In fact, $q_n\Delta_n$ is the probability of the link activation which is a function of n. In the sequel, we derive Δ_n for the aforementioned packet arrival processes.

B. Full Buffer Probability

Let us denote $t_a^{(i)}$ as the time instant the last packet has arrived in the buffer of link i before or at the same time t. The event $\mathscr{C}_i^{(t)}$ implicitly indicates that during $\mathscr{X}_i^{(t)} \triangleq t - t_a^{(i)}$ time slots, the channel gain of link i is less than the threshold level τ_n. Clearly, $\mathscr{X}_i^{(t)}$ is a random variable which varies from zero to infinity for the stochastic packet arrival processes and is finite for the CAP[9]. Under the on-off power allocation scheme and using the block fading model property, the full buffer probability can be obtained as[10]

$$\Delta_n = \mathbb{E}\left[(1-q_n)^{\mathscr{X}_i^{(t)}}\right], \tag{21}$$

where the expectation is computed with respect to $\mathscr{X}_i^{(t)}$, and $q_n \triangleq \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\} = e^{-\tau_n}$.

Lemma 1 *Let us denote the full buffer probability of an arbitrary link $i \in \mathbb{N}_n$, for the Poisson, Bernoulli and constant arrival processes as Δ_n^{PAP}, Δ_n^{BAP} and Δ_n^{CAP}, respectively. Then,*

$$\Delta_n^{PAP} = \frac{1}{1+\lambda\log(1-q_n)^{-1}}, \tag{22}$$

$$\Delta_n^{BAP} = \frac{1}{1+(\lambda-1)q_n}, \tag{23}$$

$$\Delta_n^{CAP} = \frac{1-(1-q_n)^{\lambda}}{\lambda q_n}. \tag{24}$$

Proof: For the PAP, since $\mathscr{X}_i^{(t)}$ is an exponential random variable, (21) can be simplified as

$$\Delta_n^{PAP} = \int_0^{\infty} \frac{1}{\lambda}(1-q_n)^x e^{-\frac{1}{\lambda}x} dx \tag{25}$$

$$= \frac{1}{1+\lambda\log(1-q_n)^{-1}}. \tag{26}$$

[9] Note that, here we assume that if a packet arrives at time t and the channel gain is greater than τ_n at this time, the packet will be transmitted.

[10] As we will show in Lemma 1, Δ_n is independent of index i.

Also for the BAP, $\mathscr{X}_i^{(t)}$ is a geometric random variable with parameter $\rho = \frac{1}{\lambda}$. Thus, (21) can be simplified as

$$\Delta_n^{BAP} = \sum_{m=0}^{\infty}(1-q_n)^m \rho(1-\rho)^m \tag{27}$$

$$\stackrel{(a)}{=} \frac{1}{1+(\lambda-1)q_n}, \tag{28}$$

where (a) follows from the following geometric series:

$$\sum_{m=0}^{\infty} x^m = \frac{1}{1-x}, \quad |x| < 1. \tag{29}$$

For the CAP, the full buffer probability in (21) can be written as

$$\Delta_n^{CAP} \stackrel{(a)}{=} \sum_{m=0}^{\lambda-1}(1-q_n)^m \mathbb{P}\{\mathscr{X}_i^{(t)} = m\} \tag{30}$$

$$\stackrel{(b)}{=} \sum_{m=0}^{\lambda-1}(1-q_n)^m \frac{1}{\lambda} \tag{31}$$

$$\stackrel{(c)}{=} \frac{1-(1-q_n)^{\lambda}}{\lambda q_n}, \tag{32}$$

where (a) follows from Fig. 2-b, in which $\mathscr{X}_i^{(t)}$ varies from zero to $\lambda - 1$ and (b) follows from the fact that for the deterministic process, $\mathscr{X}_i^{(t)}$ has a uniform distribution. In other words, for every value of $m \in [0, \lambda - 1]$, $\mathbb{P}\{\mathscr{X}_i^{(t)} = m\} = \frac{1}{\lambda}$. Also, (c) comes from the following geometric series:

$$\sum_{m=0}^{s} x^m = \frac{1-x^{s+1}}{1-x}. \tag{33}$$

■

Having derived the full buffer probability, we obtain the effective throughput of the network in the following section.

C. Effective Throughput of the Network

Rewriting (14), the effective throughput of link i can be obtained as

$$\begin{aligned}
\mathfrak{T}_i &= \lim_{L\to\infty}\frac{1}{L}\sum_{t=1}^{L} R_i^{(t)}\mathcal{I}_i^{(t)} && (34)\\
&\overset{(a)}{=} \mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\right] && (35)\\
&= \mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\middle|\mathcal{I}_i^{(t)}=1\right]\mathbb{P}\left\{\mathcal{I}_i^{(t)}=1\right\}+\mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\middle|\mathcal{I}_i^{(t)}=0\right]\mathbb{P}\left\{\mathcal{I}_i^{(t)}=0\right\} && (36)\\
&\overset{(b)}{=} q_n\Delta_n\mathbb{E}\left[R_i^{(t)}\middle|h_{ii}^{(t)}>\tau_n,\mathscr{C}_i^{(t)}\right] && (37)\\
&\overset{(c)}{=} q_n\Delta_n\mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{I_i^{(t)}+N_0}\right)\middle|h_{ii}^{(t)}>\tau_n\right], && (38)
\end{aligned}$$

where the expectation is computed with respect to $h_{ii}^{(t)}$ and the interference term $I_i^{(t)}$. In the above equations, (a) follows from the ergodicity of the channels (due to the block fading model), which implies that the average over time is equal to average over realization. (b) results from (17)-(19) and $\mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\middle|\mathcal{I}_i^{(t)}=0\right]=0$. Finally, (c) results from the fact that the term $\log\left(1+\frac{h_{ii}^{(t)}}{I_i^{(t)}+N_0}\right)$ is independent of $\mathscr{C}_i^{(t)}$.

In order to derive the effective throughput, we need to find the statistical behavior of $I_i^{(t)}$ which is performed in the following lemmas:

Lemma 2 *Under the on-off power scheme, we have*

$$\mathbb{E}\left[I_i^{(t)}\right]=(n-1)\hat{\alpha}q_n\Delta_n, \tag{39}$$

$$\mathit{Var}\left[I_i^{(t)}\right]\le(n-1)(2\alpha\kappa q_n\Delta_n), \tag{40}$$

where $\hat{\alpha}\triangleq\alpha\varpi$ *and* $\kappa\triangleq\mathbb{E}\left[\left(\beta_{ji}^{(t)}\right)^2\right]$.

Proof: See Appendix I. ■

Lemma 3 *The maximum effective throughput is achieved at* $\lambda=o(n)$ *and the strong interference regime which is defined as* $\mathbb{E}[I_i^{(t)}]=\omega(1)$, $i\in\mathbb{N}_n$.

Proof: Suppose that $\lambda \neq o(n)$ which implies that $\lambda = \Omega(n)$. Using (38), we have

$$\begin{aligned}
\mathfrak{T}_i &\leq q_n \Delta_n \mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{N_0}\right)\middle| h_{ii}^{(t)} > \tau_n\right] && (41)\\
&\overset{(a)}{\leq} q_n \Delta_n \log\left(1+\frac{\mathbb{E}\left[h_{ii}^{(t)}\middle| h_{ii}^{(t)} > \tau_n\right]}{N_0}\right) && (42)\\
&= q_n \Delta_n \log\left(1+\frac{\tau_n+1}{N_0}\right), && (43)
\end{aligned}$$

where (a) comes from the concavity of $\log(.)$ function and *Jensen's inequality*, $\mathbb{E}\left[\log x\right] \leq \log(\mathbb{E}\left[x\right])$, $x > 0$. Following (22) - (24), it is revealed that $\Delta_n \leq \min\left(1, \frac{1}{\lambda q_n}\right)$ for all packet arrival processes. Substituting in (43), we have

$$\begin{aligned}
\mathfrak{T}_i &\leq \frac{1}{\lambda}\log\left(1+\frac{\log\lambda+1}{N_0}\right)\\
&\sim \frac{\log\log\lambda}{\lambda}, && (44)
\end{aligned}$$

which follows from the fact that the maximum value of $q_n \Delta_n \log\left(1+\frac{\tau_n+1}{N_0}\right)$ with the condition of $\Delta_n \leq \min\left(1, \frac{1}{\lambda q_n}\right)$ is attained at $q_n = \frac{1}{\lambda}$. Noting that $\lambda = \Omega(n)$, we have $\mathfrak{T}_i \leq \Theta\left(\frac{\log\log n}{n}\right)$.

Now, suppose that $\lambda = o(n)$ but $\mathbb{E}[I_i^{(t)}] \neq \omega(1)$, or equivalently, $\mathbb{E}[I_i^{(t)}] = O(1)$ for some i. Since $\mathbb{E}[I_i^{(t)}] = (n-1)\hat{\alpha} q_n \Delta_n$, the condition $\mathbb{E}[I_i^{(t)}] = O(1)$ implies that there exists a constant c such that $q_n \Delta_n \leq \frac{c}{n}$. Noting (22) - (24), it follows that either $\Delta_n \sim \frac{1}{\lambda q_n}$ or $\Delta_n = \Theta(1)$. In the first case, the condition $q_n \Delta_n \leq \frac{c}{n}$ implies that $n \leq c\lambda$ which cannot hold due to the assumption of $\lambda = o(n)$. Therefore, we must have $q_n \leq \frac{c'}{n}$, for some constant c'. Substituting in (43) yields

$$\begin{aligned}
\mathfrak{T}_i &\leq \frac{c'}{n}\log\left(1+\frac{\tau_n+1}{N_0}\right)\\
&\overset{(a)}{\leq} \frac{c'}{n}\log\left(1+\frac{\log(n/c')+1}{N_0}\right)\\
&= \Theta\left(\frac{\log\log n}{n}\right), && (45)
\end{aligned}$$

where (a) results from the fact that $q_n \log\left(1+\frac{\tau_n+1}{N_0}\right)$ is an increasing function of q_n and reaches its maximum at the boundary which is $\frac{c'}{n}$.

In the sequel, we present a lower-bound on the effective throughput of link i in the region $\lambda = o(n)$ and $\mathbb{E}[I_i^{(t)}] = \omega(1)$ and show that this lower-bound beats the upper-bounds derived in the other regions, proving the desired result. For this purpose, using (38), we write

$$\begin{aligned} \mathfrak{T}_i &\overset{(a)}{\geq} q_n \Delta_n \log\left(1 + \frac{\tau_n}{\mathbb{E}\left[I_i^{(t)} \middle| h_{ii}^{(t)} > \tau_n\right] + N_o}\right) \\ &\overset{(b)}{=} q_n \Delta_n \log\left(1 + \frac{\tau_n}{(n-1)\hat{\alpha} q_n \Delta_n + N_o}\right) \\ &\overset{(c)}{\approx} q_n \Delta_n \log\left(1 + \frac{\tau_n}{(n-1)\hat{\alpha} q_n \Delta_n}\right), \end{aligned} \tag{46}$$

where (a) follows from the convexity of the function $\log(1 + \frac{b}{x+a})$ with respect to x and Jensen's inequality, (b) results from the independency of $I_i^{(t)}$ from $h_{ii}^{(t)}$, and (c) follows from neglecting the term N_0 with respect to $(n-1)\hat{\alpha} q_n \Delta_n$ due to the strong interference assumption. Setting $q_n = \frac{\log^2 n}{n}$ and $\lambda = \frac{n}{\log^2 n}$, it is easy to check that $\frac{\tau_n}{(n-1)\hat{\alpha} q_n \Delta_n} = o(1)$ and hence, $\log\left(1 + \frac{\tau_n}{(n-1)\hat{\alpha} q_n \Delta_n}\right) \approx \frac{\tau_n}{(n-1)\hat{\alpha} q_n \Delta_n}$ which gives the effective throughput as $\frac{\tau_n}{(n-1)\hat{\alpha}} = \Theta\left(\frac{\log n}{n}\right)$ which is greater than the throughput obtained in the other regimes. ■

Due to the result of Lemma 3, we restrict ourselves to the case of $\lambda = o(n)$ and the strong interference regime in the rest of the paper.

Lemma 4 *Let us assume $0 < \alpha \leq 1$ is fixed and we are in the strong interference regime (i.e., $\mathbb{E}\left[I_i^{(t)}\right] = \omega(1)$). Then with probability one (w. p. 1), we have*

$$I_i^{(t)} \sim (n-1)\hat{\alpha} q_n \Delta_n, \tag{47}$$

as $n \to \infty$. More precisely, substituting $I_i^{(t)}$ by $(n-1)\hat{\alpha} q_n \Delta_n$ does not change the asymptotic effective throughput of the network.

Proof: See Appendix II. ■

Lemma 5 *The effective throughput of the network for large values of n can be obtained as*

$$\mathfrak{T}_{\text{eff}} \approx n q_n \Delta_n \log\left(1 + \frac{\tau_n}{n \hat{\alpha} q_n \Delta_n}\right). \tag{48}$$

Proof: Using (38), the effective throughput of the network in the asymptotic case of $n \to \infty$ is obtained as

$$\begin{aligned}
\mathfrak{T}_{\text{eff}} &= \sum_{i=1}^{n} \mathfrak{T}_i && (49)\\
&\overset{(a)}{\approx} nq_n\Delta_n \mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{(n-1)\hat{\alpha}q_n\Delta_n+N_0}\right)\middle| h_{ii}^{(t)}>\tau_n\right] && (50)\\
&\overset{(b)}{\approx} nq_n\Delta_n \mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{n\hat{\alpha}q_n\Delta_n}\right)\middle| h_{ii}^{(t)}>\tau_n\right], && (51)
\end{aligned}$$

where (a) results from the strong interference assumption and Lemma 4, and (b) follows from approximating $(n-1)\hat{\alpha}q_n\Delta_n+N_0$ by $n\hat{\alpha}q_n\Delta_n$ due to the strong interference assumption and large values of n. A lower-bound on (51) can be written as

$$\mathfrak{T}_{\text{eff}}^{l} = nq_n\Delta_n \log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right). \tag{52}$$

Furthermore, due to the concavity of $\log(.)$ function and Jensen's inequality, an upper-bound on $\mathfrak{T}_{\text{eff}}$ can be given as

$$\begin{aligned}
\mathfrak{T}_{\text{eff}}^{u} &= nq_n\Delta_n \log\left(1+\frac{\mathbb{E}\left[h_{ii}^{(t)}\middle| h_{ii}^{(t)}>\tau_n\right]}{n\hat{\alpha}q_n\Delta_n}\right)\\
&= nq_n\Delta_n \log\left(1+\frac{\tau_n+1}{n\hat{\alpha}q_n\Delta_n}\right). && (53)
\end{aligned}$$

In order to prove that the above upper and lower bounds have the same scaling, it is sufficient to show that the optimum threshold value (τ_n) is much larger than one. For this purpose, we note that if $\tau_n = O(1)$, then the effective throughput of the network will be upper-bounded by

$$\begin{aligned}
\mathfrak{T}_{\text{eff}} &\overset{(a)}{\leq} \frac{\tau_n+1}{\hat{\alpha}} && (54)\\
&= O(1), && (55)
\end{aligned}$$

where (a) follows from $\log(1+x) \leq x$. In other words, the effective throughput of the network does not scale with n, while the throughput of $\Theta(\log n)$, as will be shown later, is achievable. This suggests that the optimum threshold value must grow with n, and hence, the bounds given in (52) and (53) are asymptotically equal to $nq_n\Delta_n \log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right)$ and this completes the proof of the lemma. ■

Lemma 6 *The maximum effective throughput of the network is obtained in the region that* $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$.

Proof: Rewriting the expression of the effective throughput of the network from (48) and noting the fact that $\log(1+x) \leq x$, for $x \geq 0$, we have

$$\begin{aligned}\mathfrak{T}_{\text{eff}} &\approx nq_n\Delta_n \log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right)\\ &\leq \frac{\tau_n}{\hat{\alpha}}.\end{aligned} \tag{56}$$

It can be shown that if the condition $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$ is not satisfied, the ratio $\frac{\log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right)}{\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}}$ is strictly less than one. Having $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$ results in $\log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right) \approx \frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}$ yielding the upper-bound $\frac{\tau_n}{\hat{\alpha}}$. This means that to achieve the maximum throughput, the interference should not only be strong but also be much larger than τ_n. ■

Having the expression for the effective throughput of the network in (48), in the next theorem, we find the optimum value of q_n (or equivalently τ_n) in terms of n and λ for the aforementioned packet arrival processes, i.e.:

$$\hat{q}_n = \arg\max_{q_n} \mathfrak{T}_{\text{eff}}. \tag{57}$$

As shown in the proof of Lemma 5, since the optimum threshold value is much larger than one, the optimizer $\hat{q}_n$ is sufficiently small, i.e., $\hat{q}_n = o(1)$.

Theorem 1 *Assuming the Poisson packet arrival process and large values of n, the optimum solution for (57) is obtained as*

$$q_n^{PAP} = \delta\frac{\log^2 n}{n} \tag{58}$$

for some constant δ. Furthermore, the maximum effective throughput of the network asymptotically scales as $\frac{\log n}{\hat{\alpha}}$, for $\lambda = o\left(\frac{n}{\log n}\right)$.

Proof: See Appendix III. ■

Theorem 2 *Assuming the Bernoulli packet arrival process and large values of n, the optimum solution for (57) is obtained as*

$$q_n^{BAP} = \delta \frac{\log^2 n}{n} \tag{59}$$

for some constant δ. Furthermore, the maximum effective throughput of the network asymptotically scales as $\frac{\log n}{\hat{\alpha}}$, for $\lambda = o\left(\frac{n}{\log n}\right)$.

Proof: See Appendix IV. ∎

Theorem 3 *Assuming a deterministic packet arrival process, the optimum solution of (57) and the corresponding maximum effective throughput of the network are asymptotically obtained as*

i) $q_n^{CAP} = \delta \frac{\log^2 n}{n}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = o\left(\frac{n}{\log^2 n}\right)$,
ii) $q_n^{CAP} = \delta' \frac{\log^2 n}{n}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = \Theta\left(\frac{n}{\log^2 n}\right)$,
iii) $q_n^{CAP} = \frac{\log\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right)}{\lambda}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda = o\left(\frac{n}{\log n}\right)$,
for some constants δ and δ'.

Proof: See Appendix V. ∎

The above theorems imply that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, despite the packet arrival process. Note that this value is the same as the sum-rate scaling of the same network with backlogged users [26], which is an upper-bound on the effective throughput of the current setup. In other words, the effect of the real-time traffic in the throughput (which is captured in the full buffer probability) is asymptotically negligible. However, we did not consider the effect of dropping on the calculations. In the subsequent section, we include the dropping probability in the analysis and find the maximum effective throughput of the network such that the dropping probability approaches zero.

IV. Delay Analysis

In this section, we analyze the delay characteristics of the underlying network in terms of the number of links (n) and λ. First, we formulate the packet dropping probabilities based on the aforementioned packet arrival processes. Then, we derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Lemma 7 *Let us denote the packet dropping probability of a link i, $i \in \mathbb{N}_n$, for the Poisson, Bernoulli and constant arrival processes as $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\}$, $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\}$ and $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\}$, respectively. Then,*

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \frac{1}{1+\lambda\log(1-q_n)^{-1}}, \tag{60}$$

$$\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \frac{(1-q_n)(\lambda q_n)^{-1}}{1+(1-q_n)(\lambda q_n)^{-1}}, \tag{61}$$

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = (1-q_n)^{\lambda}. \tag{62}$$

Proof: Recalling $t_{A_k}^{(i)}$ as the time instant of the k^{th} packet arrival into the buffer of link i, each user i is active at time slot $t \geq t_{A_k}^{(i)}$ only when $h_{ii}^{(t)} > \tau_n$. In other words, assuming the buffer is full, no transmission (or no service) occurs in each slot with probability $1-q_n$. From (5) and (8)-(12), since the time duration between subsequent packet arrivals is $x_k^{(i)}$, the packet dropping probability for a link i is obtained as

$$\mathbb{P}\left\{\mathscr{B}_i\right\} = \mathbb{E}\left[(1-q_n)^{x_k^{(i)}}\right], \tag{63}$$

where the expectation is computed with respect to $x_k^{(i)}$. For the PAP, since $x_k^{(i)}$ is an exponential random variable, (63) can be simplified as

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \int_0^{\infty} \frac{1}{\lambda}(1-q_n)^x e^{-\frac{1}{\lambda}x} dx \tag{64}$$

$$= \frac{1}{1+\lambda\log(1-q_n)^{-1}}. \tag{65}$$

Also for the BAP, $x_k^{(i)}$ is a geometric random variable with parameter $\rho = \dfrac{1}{\lambda}$. Thus, (63) can be simplified as

$$\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \sum_{m=1}^{\infty}(1-q_n)^m \rho (1-\rho)^{m-1} \tag{66}$$

$$= \frac{\rho}{1-\rho}\sum_{m=1}^{\infty}\left[(1-q_n)(1-\rho)\right]^m \tag{67}$$

$$\stackrel{(a)}{=} \frac{(1-q_n)(\lambda q_n)^{-1}}{1+(1-q_n)(\lambda q_n)^{-1}}, \tag{68}$$

where (a) comes from the following geometric series:

$$\sum_{m=1}^{\infty} x^m = \frac{x}{1-x}, \qquad |x| < 1. \tag{69}$$

According to Fig. 2-a, $x_k^{(i)}$ for the CAP is a deterministic quantity and is equal to λ. Thus, we have

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = (1-q_n)^{\lambda}. \tag{70}$$

It should be noted that (65), (68) and (70) are valid for every value of $q_n \in [0,1]$. In particular, in the extreme case of $q_n = 1$, $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$. ■

We are now ready to prove the main result of this section. In the next theorem, we derive the necessary conditions on λ, such that the corresponding packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Theorem 4 *For the optimum q_n obtained in Theorems 1-3 resulting in the maximum effective throughput of the network,*

i) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = 0$, *if* $\lambda^{PAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{PAP} = o\left(\frac{n}{\log n}\right)$,

ii) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$, *if* $\lambda^{BAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{BAP} = o\left(\frac{n}{\log n}\right)$,

iii) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = 0$, *if* $\lambda^{CAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{CAP} = o\left(\frac{n}{\log n}\right)$.

Proof: i) From (60), we have

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \frac{1}{1-\lambda^{PAP}\log(1-q_n^{PAP})}. \tag{71}$$

It follows from (71) that achieving $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \epsilon$ results in

$$\begin{aligned}\lambda_\epsilon^{PAP} &= \frac{1-\epsilon^{-1}}{\log(1-q_n^{PAP})} \\ &\overset{(a)}{\approx} \frac{\epsilon^{-1}-1}{q_n^{PAP}},\end{aligned} \tag{72}$$

where (a) comes from $q_n^{PAP} = o(1)$ and the following approximation:

$$\log(1-z) \approx -z, \quad |z| \ll 1. \tag{73}$$

Noting the fact that the optimum value of q_n^{PAP} scales as $\Theta\left(\frac{\log^2 n}{n}\right)$, having $\lambda^{PAP} = \omega\left(\frac{n}{\log^2 n}\right)$ results in $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = 0$. On the other hand, from Theorem 1, the condition $\lambda^{PAP} = o\left(\frac{n}{\log n}\right)$ is required to achieve the maximum $\mathfrak{T}_{\text{eff}}$, and this completes the proof of the first part of the Theorem.

ii) It is realized from (61) that achieving $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \epsilon$ results in

$$\begin{aligned}\lambda_\epsilon^{BAP} &= \frac{1}{q_n^{BAP}}\left[(1-q_n^{BAP})\epsilon^{-1} - (1-q_n^{BAP})\right] \\ &\approx \frac{\epsilon^{-1}}{q_n^{BAP}},\end{aligned} \tag{74}$$

for small enough ϵ. Noting the fact that the optimum value of q_n^{BAP} scales as $\Theta\left(\frac{\log^2 n}{n}\right)$, having $\lambda^{BAP} = \omega\left(\frac{n}{\log^2 n}\right)$ results in $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$. On the other hand, from Theorem 2, $\lambda^{BAP} = o\left(\frac{n}{\log n}\right)$ guarantees achieving the maximum effective throughput of the network.

iii) From (62), we have

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = e^{\lambda^{CAP}\log(1-q_n^{CAP})} \tag{75}$$

$$\overset{(a)}{\approx} e^{-q_n^{CAP}\lambda^{CAP}} \tag{76}$$

where (a) follows from (73) for $q_n^{CAP} = o(1)$. To achieve $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = \epsilon$, we must have

$$\lambda_\epsilon^{CAP} = \frac{1}{q_n^{CAP}}\log\epsilon^{-1}. \tag{77}$$

It follows from (76) that setting $q_n^{CAP}\lambda^{CAP} = \omega(1)$ makes $e^{-q_n^{CAP}\lambda^{CAP}} \to 0$. Using part (iii) in Theorem 3, it turns out that choosing $\lambda^{CAP} = \omega\left(\frac{n}{\log^2 n}\right)$ satisfies $q_n^{CAP}\lambda^{CAP} = \omega(1)$

which yields $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = 0$. We also need the condition $\lambda^{CAP} = o\left(\frac{n}{\log n}\right)$ to ensure achieving the maximum effective throughput of the network. ∎

Remark 1- It is worth mentioning that the delay-bound (λ) in each link for the CAP scales the same as that of the PAP and the BAP. However, $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\}$ decays faster than $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\}$ and $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\}$, when n tends to infinity.

An interesting conclusion of Theorem 4 is the possibility of achieving the maximum effective throughput of the network while making the dropping probability approach zero. More precisely, there exists some $\epsilon \ll 1$ such that $\mathbb{P}\left\{\mathscr{B}_i\right\} \le \epsilon$, $\forall i \in \mathbb{N}_n$, while achieving the maximum $\mathfrak{T}_{\text{eff}}$ of $\frac{\log n}{\hat{\alpha}}$. This is true for all arrival processes. However, for arbitrary values of ϵ, there is a tradeoff between increasing the throughput, and decreasing the dropping probability and the delay-bound (λ). This tradeoff is studied in the next section.

V. Delay-Throughput Tradeoff

In this section, we study the tradeoff between the effective throughput of the network and other performance measures, i.e., the dropping probability and the delay-bound (λ) for different packet arrival processes. In particular, we would like to know how much degradation will be enforced in the throughput by introducing the other constraints, and how much this degradation depends on the packet arrival process.

A. Tradeoff Between Throughput and Dropping Probability

In this section, we assume that a constraint $\mathbb{P}\left\{\mathscr{B}_i\right\} \le \epsilon$ must be satisfied for the dropping probability. It can be easily shown that the constraint $\mathbb{P}\left\{\mathscr{B}_i\right\} \le \epsilon$ is equivalent to $\mathbb{P}\left\{\mathscr{B}_i\right\} = \epsilon$. The aim is to characterize the degradation on the effective throughput of the network in terms of ϵ for different packet arrival processes. First, we consider PAP.

Looking at the equations (22) and (60), it turns out that $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \Delta_n^{PAP}$. Hence, the condition $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \epsilon$ is translated to $\Delta_n^{PAP} = \epsilon$. Therefore, using (48), the effective throughput of the network can be written as

$$\mathfrak{T}_{\text{eff}} \approx n q_n \epsilon \log\left(1 + \frac{\tau_n}{n\hat{\alpha} q_n \epsilon}\right). \tag{78}$$

From the above equation, it can be realized that the effective throughput of the network is equal to the average sum-rate of the network with $n\epsilon$ users in the case of backlogged users, which is given in [26] as $\frac{\log(n\epsilon)}{\hat{\alpha}}$ for the case of $n\epsilon \gg 1$ or $\epsilon = \omega(\frac{1}{n})$. Also, the optimum value of q_n is shown to scale as $\delta\frac{\log^2(n\epsilon)}{n\epsilon}$ for some constant δ and hence, the optimum value of λ is given as $\frac{\epsilon^{-1}}{q_n} = \frac{n}{\delta\log^2(n\epsilon)}$. Let us denote $\Delta\mathfrak{T}_{\text{eff}}$ as the degradation in the effective throughput of the network, which is defined as the difference between the maximum effective throughput in the case of no constraint on $\mathbb{P}\{\mathscr{B}_i\}$ (Theorem 1-3) and the case with constraint on $\mathbb{P}\{\mathscr{B}_i\}$. Using Theorem 1, $\Delta\mathfrak{T}_{\text{eff}}$ for the PAP can be written as

$$\begin{aligned}\Delta\mathfrak{T}_{\text{eff}} &\approx \frac{\log n}{\hat{\alpha}} - \frac{\log(n\epsilon)}{\hat{\alpha}}\\ &= \frac{\log(\epsilon^{-1})}{\hat{\alpha}},\end{aligned} \tag{79}$$

for $\epsilon = \omega\left(\frac{1}{n}\right)$[11]. Moreover, for values of ϵ such that $\log(\epsilon^{-1}) = o(\log n)$, it can be shown that the scaling of the effective throughput of the network is not changed, i.e., $\mathfrak{T}_{\text{eff}} \sim \frac{\log n}{\hat{\alpha}}$.

For the BAP, and using (23) and (61), we have

$$\begin{aligned}\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} &= \frac{1-q_n}{1+(\lambda-1)q_n}\\ &\overset{(a)}{\approx} \frac{1}{1+(\lambda-1)q_n}\\ &= \Delta_n^{BAP},\end{aligned} \tag{80}$$

where (a) follows from the fact that $q_n = o(1)$. Therefore, similar to the case of the PAP, we have $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} \approx \Delta_n^{BAP} = \epsilon$ and as a result, the rest of the arguments hold. In particular,

$$\Delta\mathfrak{T}_{\text{eff}} \approx \frac{\log(\epsilon^{-1})}{\hat{\alpha}}. \tag{81}$$

For the CAP, and using (24) and (62), we have

$$(1-q_n)^{\lambda} = \epsilon \quad \Longrightarrow \lambda q_n \approx \log(\epsilon^{-1}), \tag{82}$$

[11] In the case of $\epsilon = O(\frac{1}{n})$, it is easy to see that the effective throughput of the network does not scale with n.

which gives

$$\Delta_n^{CAP} = \frac{1-(1-q_n)^{\lambda}}{\lambda q_n} \tag{83}$$

$$\approx \frac{1}{\log(\epsilon^{-1})}. \tag{84}$$

Hence, using (48), the effective throughput of the network can be written as

$$\mathfrak{T}_{\text{eff}} \approx \frac{n}{\log(\epsilon^{-1})} q_n \log\left(1 + \frac{\tau_n}{\frac{n}{\log(\epsilon^{-1})}\hat{\alpha} q_n}\right), \tag{85}$$

which is equal to the average sum-rate of a network with $\frac{n}{\log(\epsilon^{-1})}$ backlogged users and is asymptotically equal to $\frac{\log\left(\frac{n}{\log(\epsilon^{-1})}\right)}{\hat{\alpha}}$, for values of ϵ satisfying $\log(\epsilon^{-1}) = o(n)$. Therefore, the degradation in the effective throughput of the network for the CAP can be expressed as

$$\Delta\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}} - \frac{\log\left(\frac{n}{\log(\epsilon^{-1})}\right)}{\hat{\alpha}}$$

$$= \frac{\log\log(\epsilon^{-1})}{\hat{\alpha}}. \tag{86}$$

Comparing the expressions of $\Delta\mathfrak{T}_{\text{eff}}$ for the Poisson, Bernoulli and constant packet arrival processes, it follows that the degradation in the effective throughput of the network in the cases of PAP and BAP both grow logarithmically with ϵ^{-1}, while in the case of CAP it grows double logarithmically. In other words, the degradation in the throughput in the cases of the PAP and BAP is much more substantial compared to the CAP. This fact is also observed in the simulation results in the next sections.

B. Tradeoff Between Throughput and Delay

In this section, we aim to find the tradeoff between the effective throughput of the network and the delay-bound (λ), for a given constraint on the dropping probability, i.e., $\mathbb{P}\{\mathscr{B}_i\} \leq \epsilon$.

1) PAP: Using (22) and (60), it follows that for a given λ and $\epsilon \ll 1$, we have

$$q_n \approx \frac{\epsilon^{-1}}{\lambda},$$

$$\Longrightarrow \tau_n \approx \log(\lambda\epsilon), \tag{87}$$

and

$$q_n\Delta_n \approx \frac{1}{\lambda}. \tag{88}$$

Substituting $q_n\Delta_n$ and τ_n from the above equations in (48) yields

$$\mathfrak{T}_{\text{eff}} \approx \frac{n}{\lambda}\log\left(1+\frac{\lambda\log(\lambda\epsilon)}{n\hat{\alpha}}\right). \tag{89}$$

It can be verified that $\mathfrak{T}_{\text{eff}}$ has a global maximum at $\lambda_{opt}^{PAP} \approx \frac{n\hat{\alpha}}{\log^2(n\hat{\alpha}\epsilon^{-1})}$. In other words, for $\lambda < \lambda_{opt}^{PAP}$, there is a tradeoff between the throughput and delay, meaning that increasing λ results in increasing both the throughput and delay. However, the increase in the throughput is logarithmic while the delay increases linearly with λ. It should be noted that the region $\lambda > \lambda_{opt}^{PAP}$ is not of interest, since increasing λ from λ_{opt}^{PAP} results in decreasing the throughput and increasing the delay which is not desired.

2) BAP: Due to the similarity between the values of $\mathbb{P}\{\mathscr{B}_i\}$ and Δ_n for the PAP and the BAP, the results obtained for the PAP are also valid for the BAP.

3) CAP: Using (24) and (62), it follows that for a given λ and $\epsilon \ll 1$, we have

$$\begin{aligned} q_n &\approx \frac{\log(\epsilon^{-1})}{\lambda}, \\ \Longrightarrow \tau_n &\approx \log\left(\frac{\lambda}{\log(\epsilon^{-1})}\right), \end{aligned} \tag{90}$$

and

$$q_n\Delta_n \approx \frac{1}{\lambda}. \tag{91}$$

As can be observed, all the results for the cases of PAP and BAP are extendable to the case of CAP by substituting ϵ^{-1} with $\log(\epsilon^{-1})$. In particular, the optimum value for λ can be written as $\lambda_{opt}^{CAP} \approx \frac{n\hat{\alpha}}{\log^2(n\hat{\alpha}\log(\epsilon^{-1}))}$, and for $\lambda < \lambda_{opt}^{CAP}$, the effective throughput of the network can be given as $\mathfrak{T}_{\text{eff}} \approx \frac{1}{\hat{\alpha}}\log\left(\frac{\lambda}{\log(\epsilon^{-1})}\right)$. This means that in the region $\lambda < \lambda_{opt}^{CAP}$, which is the region of interest, there is a tradeoff between the throughput and delay such that by increasing λ, $\mathfrak{T}_{\text{eff}}$ increases logarithmically, while the delay increases linearly with λ. Furthermore, comparing the value of λ_{opt} for the PAP and BAP with the CAP, it is realized that $\lambda_{opt}^{CAP} > \lambda_{opt}^{PAP}$. This fact is also observed in the simulations.

VI. Numerical Results

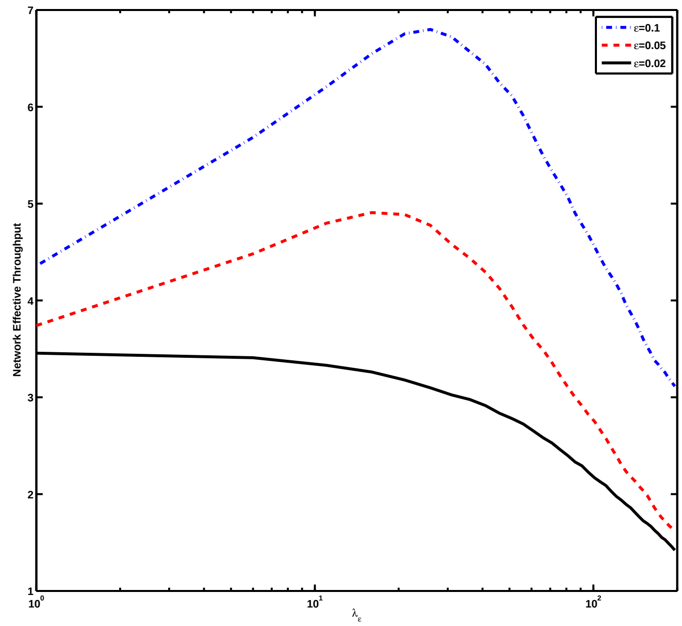
In this section, we present some numerical results to evaluate the tradoff between the effective throughput of the network and other performance measures, i.e., dropping probability and the delay-bound (λ) for different packet arrival processes. For this purpose, we assume that all users in the network follow the threshold-based on-off power allocation policy. In addition, the shadowing effect is assumed to be *lognormal* distributed with mean $\varpi = 0.5$, variance 1 and $\alpha = 0.4$. Furthermore, we assume that $n = 500$ and $N_0 = 1$.

Figures 3 and 4 show the effective throughput of the network versus λ_ϵ for the PAP, BAP and CAP and different values of ϵ. It is observed from these figures that for a given constraint on the dropping probability (e.g., $\epsilon = 0.05$), and for $\lambda < \lambda_{opt}$, increasing λ results in increasing both the throughput and delay. However, the increase in the throughput is logarithmic while the delay increases linearly with λ as expected. Also, increasing λ from λ_{opt} results in decreasing the throughput and increasing the delay which is not desired. Furthermore, comparing the value of λ_{opt} for the PAP and BAP with the CAP, it is realized that $\lambda_{opt}^{CAP} > \lambda_{opt}^{PAP}$ and $\lambda_{opt}^{CAP} > \lambda_{opt}^{BAP}$, as expected.

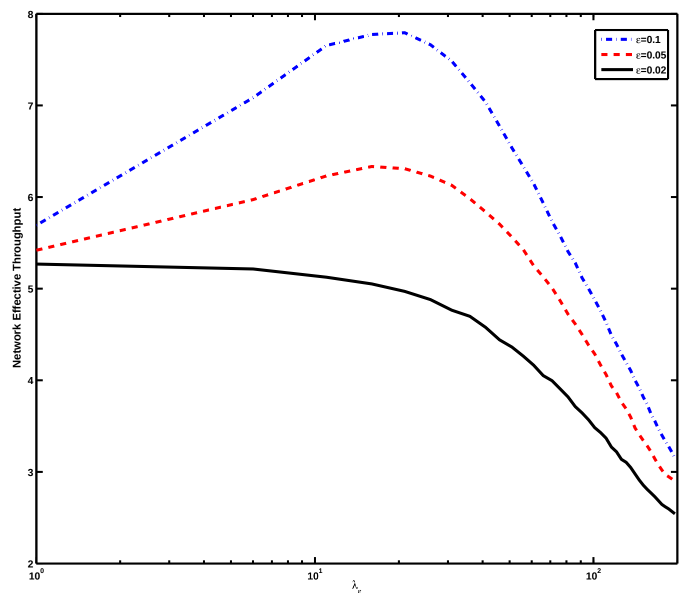
To evaluate the degradation in the effective throughput of the network in terms of dropping probability, we plot $\mathfrak{T}_{\text{eff}}$ versus $\log \epsilon^{-1}$ for different packet arrival processes in Fig. 5. It can be seen that the degradation in the throughput in the cases of the PAP and BAP is much more substantial compared to the CAP, as expected. Hence, the performance of the underlying network with the CAP is better than that of the PAP and BAP from the delay-throughput tradeoff points of view.

VII. Conclusion

In this paper, the delay-throughput tradeoff of a single-hop wireless network in terms of the number of links (n), and under the shadowing effect with parameters (α, ϖ) was analyzed. It was proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. Then, the delay characteristics of the underlying network in terms of a packet dropping probability was presented. Also, the necessary



(a)



(b)

Fig. 3. Effective throughput of the network vs. λ_ϵ for $N_0 = 1$, $n = 500$, $\alpha = 0.4$, and different values of ϵ a) PAP and b) BAP.

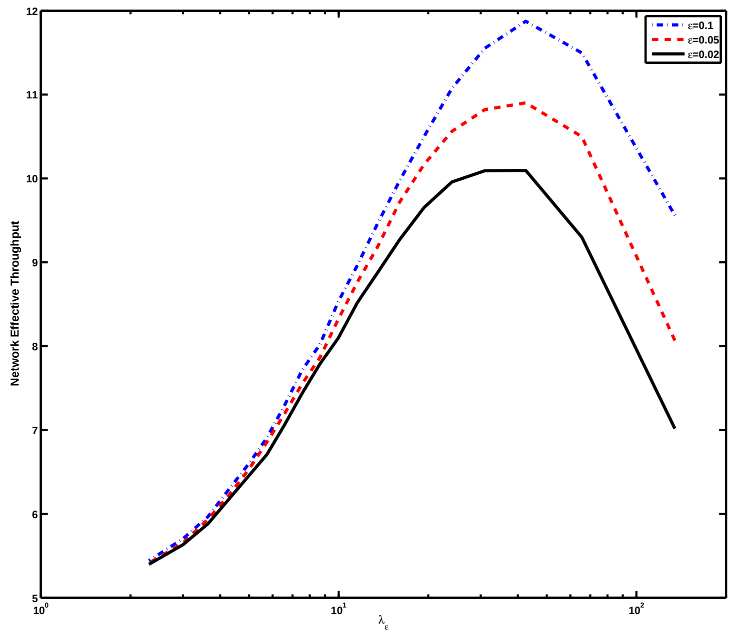


Fig. 4. Effective throughput of the network vs. λ_ϵ for the CAP and $N_0 = 1$, $n = 500$, $\alpha = 0.4$, and different values of ϵ.

conditions in the asymptotic case of $n \to \infty$ was derived such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, the tradeoff between the effective throughput of the network and delay-bounds for different packet arrival processes was studied. It was shown from the numerical results that the performance of the deterministic packet arrival process is better than that of the Poisson and the Bernoulli packet arrival processes, from the delay-throughput tradoff points of view.

APPENDIX I
PROOF OF LEMMA 2

Let us define $\chi_j^{(t)} \triangleq \mathcal{L}_{ji}^{(t)} p_j^{(t)}$, where $\mathcal{L}_{ji}^{(t)}$ is independent of $p_j^{(t)}$, for $j \neq i$. Note that

$$\mathbb{P}\left\{p_j^{(t)} = 1\right\} = \mathbb{P}\left\{h_{jj}^{(t)} > \tau_n,\ \mathscr{C}_j^{(t)}\right\} \quad \text{(A-1)}$$

$$\stackrel{(a)}{=} q_n \Delta_n, \quad \text{(A-2)}$$

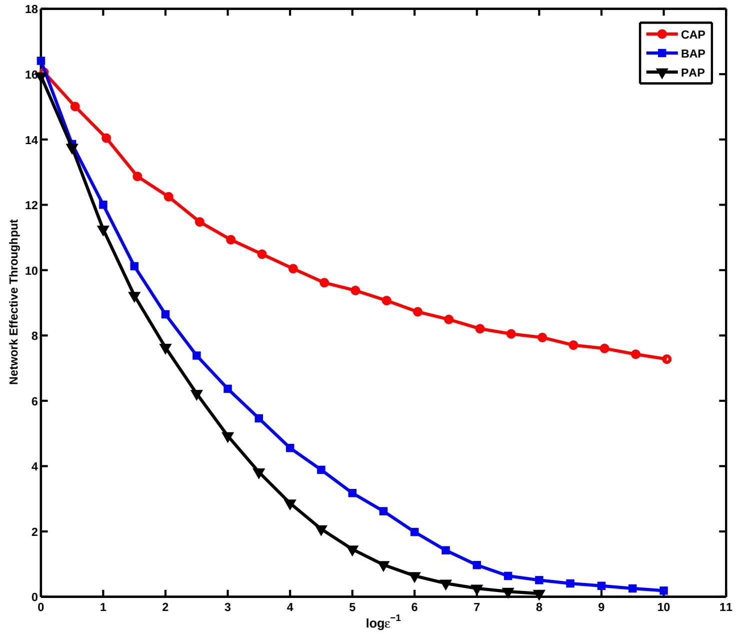


Fig. 5. Effective throughput of the network vs. $\log \epsilon^{-1}$ for different packet arrival processes and $N_0 = 1$, $n = 500$, $\alpha = 0.4$.

where (a) follows from (19). Thus for the on-off power scheme, we have

$$\mathbb{E}\left[p_j^{(t)}\right] = q_n \Delta_n. \tag{A-3}$$

Under a quasi-static Rayleigh fading channel model, it is concluded that $\chi_j^{(t)}$s are independent and identically distributed (i.i.d.) random variables with

$$\mathbb{E}\left[\chi_j^{(t)}\right] = \mathbb{E}\left[\mathcal{L}_{ji}^{(t)} p_j^{(t)}\right] = \hat{\alpha} q_n \Delta_n, \tag{A-4}$$

$$\text{Var}\left[\chi_j^{(t)}\right] = \mathbb{E}\left[\left(\chi_j^{(t)}\right)^2\right] - \mathbb{E}^2\left[\chi_j^{(t)}\right] \tag{A-5}$$

$$\overset{(a)}{\leq} 2\alpha\kappa q_n \Delta_n - (\hat{\alpha} q_n \Delta_n)^2, \tag{A-6}$$

where $\mathbb{E}\left[\left(h_{ji}^{(t)}\right)^2\right] = 2$, $\mathbb{E}\left[\left(\beta_{ji}^{(t)}\right)^2\right] \triangleq \kappa$ and $\hat{\alpha} \triangleq \alpha\varpi$. Also, (a) follows from the fact that $\left(p_j^{(t)}\right)^2 \leq p_j^{(t)}$. Thus, $\mathbb{E}\left[\left(p_j^{(t)}\right)^2\right] \leq \mathbb{E}\left[p_j^{(t)}\right] = q_n \Delta_n$. The interference $I_i^{(t)} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \chi_j^{(t)}$ is a random variable with mean μ_n and variance ϑ_n^2, where

$$\mu_n \triangleq \mathbb{E}\left[I_i^{(t)}\right] = (n-1)\hat{\alpha} q_n \Delta_n, \tag{A-7}$$

$$\vartheta_n^2 \triangleq \text{Var}\left[I_i^{(t)}\right] \leq (n-1)(2\alpha\kappa q_n \Delta_n - (\hat{\alpha} q_n \Delta_n)^2) \leq (n-1)(2\alpha\kappa q_n \Delta_n). \tag{A-8}$$

Appendix II
Proof of Lemma 4

Using Lemma 2 and the *Central Limit Theorem* [34, p. 183], we obtain

$$\mathbb{P}\left\{|I_i^{(t)} - \mu_n| < \psi_n\right\} \approx 1 - Q\left(\frac{\psi_n}{\vartheta_n}\right) \tag{B-1}$$

$$\overset{(a)}{\geq} 1 - e^{-\frac{\psi_n^2}{2\vartheta_n^2}}, \tag{B-2}$$

for all $\psi_n > 0$ such that $\psi_n = o\left(n^{\frac{1}{6}}\vartheta_n\right)$. In the above equations, the $Q(.)$ function is defined as $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$, and (a) follows from the fact that $Q(x) \leq e^{-\frac{x^2}{2}}$, $\forall x > 0$. Selecting $\psi_n = (nq_n\Delta_n)^{\frac{1}{8}}\sqrt{2}\vartheta_n$, we obtain

$$\mathbb{P}\{|I_i^{(t)} - \mu_n| < \psi_n\} \geq 1 - e^{-(nq_n\Delta_n)^{\frac{1}{4}}}. \tag{B-3}$$

Therefore, defining $\varepsilon \triangleq \frac{\psi_n}{\mu_n} = O\left((nq_n\Delta_n)^{-\frac{3}{8}}\right)$, we have

$$\mathbb{P}\{\mu_n(1-\varepsilon) \leq I_i^{(t)} \leq \mu_n(1+\varepsilon)\} \geq 1 - e^{-(nq_n\Delta_n)^{\frac{1}{4}}}. \tag{B-4}$$

Noting that $nq_n\Delta_n \to \infty$, it follows that $I_i^{(t)} \sim \mu_n$, with probability one.

Appendix III
Proof of Theorem 1

Taking the first-order derivative of (48) with respect to τ_n yields

$$\frac{\partial \mathfrak{T}_{\text{eff}}}{\partial \tau_n} \overset{(a)}{=} nq_n\left[\frac{\partial \Delta_n}{\partial \tau_n} - \Delta_n\right]\log\left(1 + \frac{\tau_n}{n\hat{\alpha} q_n\Delta_n}\right) + nq_n\frac{(1+\tau_n)\Delta_n - \tau_n\frac{\partial\Delta_n}{\partial\tau_n}}{n\hat{\alpha} q_n\Delta_n + \tau_n} \tag{C-1}$$

$$\overset{(b)}{\approx} nq_n\left[\frac{\partial \Delta_n}{\partial \tau_n} - \Delta_n\right]\frac{\tau_n}{n\hat{\alpha} q_n\Delta_n} + nq_n\frac{(1+\tau_n)\Delta_n - \tau_n\frac{\partial\Delta_n}{\partial\tau_n}}{n\hat{\alpha} q_n\Delta_n + \tau_n}, \tag{C-2}$$

where (a) comes from $q_n = e^{-\tau_n}$ and $\frac{\partial q_n}{\partial \tau_n} = -q_n$. Also, (b) follows from Lemma 6 and using the approximation $\log(1+x) \approx x$, for $x \ll 1$. Setting (C-2) equal to zero yields

$$n\hat{\alpha} q_n \Delta_n^2 = \left(\Delta_n - \frac{\partial \Delta_n}{\partial \tau_n}\right) \tau_n^2. \tag{C-3}$$

It should be noted that (C-3) is valid for every packet arrival process. Recalling from (22), the full buffer probability for the PAP is given by

$$\Delta_n^{PAP} = \frac{1}{1 + \lambda \log(1-q_n)^{-1}} \tag{C-4}$$

$$\stackrel{(a)}{\approx} \frac{1}{1+\lambda q_n}, \tag{C-5}$$

where (a) follows from the fact that for $q_n = o(1)$, $\log(1-q_n)^{-1} \approx q_n$. In this case, $\frac{\partial \Delta_n^{PAP}}{\partial \tau_n} = \frac{\partial \Delta_n^{PAP}}{\partial q_n} \frac{\partial q_n}{\partial \tau_n} = \frac{\lambda q_n}{(1+\lambda q_n)^2}$, which results in

$$\Delta_n^{PAP} - \frac{\partial \Delta_n^{PAP}}{\partial \tau_n} \approx \frac{1}{(1+\lambda q_n)^2} = \left(\Delta_n^{PAP}\right)^2. \tag{C-6}$$

Thus for the Poisson arrival process, (C-3) can be simplified as

$$n\hat{\alpha} q_n = \tau_n^2. \tag{C-7}$$

It can be verified that the solution for (C-7) is

$$\tau_n^{PAP} = \log n - 2\log\log n + O(1). \tag{C-8}$$

Using $q_n = e^{-\tau_n}$, we conclude that

$$q_n^{PAP} = \delta \frac{\log^2 n}{n}, \tag{C-9}$$

for some constant δ.

To satisfy the condition of lemma 6, we should have

$$\frac{\tau_n}{n\hat{\alpha} q_n \Delta_n^{PAP}} \ll 1, \tag{C-10}$$

Using (C-5), (C-8), and (C-9), it yields

$$\lambda^{PAP} = o\left(\frac{n}{\log n}\right). \tag{C-11}$$

Thus, the maximum effective throughput of the network obtained in (48) can be written as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}}. \tag{C-12}$$

Appendix IV

Proof of Theorem 2

Using (23), we have $\frac{\partial \Delta_n^{BAP}}{\partial \tau_n} = \frac{\partial \Delta_n^{BAP}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n} = -q_n \frac{\partial \Delta_n^{BAP}}{\partial q_n} = \frac{q_n(\lambda-1)}{(1+(\lambda-1)q_n)^2}$. In this case,

$$\Delta_n^{BAP} - \frac{\partial \Delta_n^{BAP}}{\partial \tau_n} = \frac{1}{(1+(\lambda-1)q_n)^2} = \left(\Delta_n^{BAP}\right)^2. \tag{D-1}$$

Thus for the Bernoulli arrival process, (C-3) can be simplified as

$$n\hat{\alpha} q_n = \tau_n^2. \tag{D-2}$$

It can be observed that (D-2) is exactly equal to (C-7) and hence, its solution can be written as

$$\tau_n^{BAP} = \log n - 2\log\log n + O(1), \tag{D-3}$$

and

$$q_n^{BAP} = \delta \frac{\log^2 n}{n}, \tag{D-4}$$

for some constants δ. Similarly, the maximum effective throughput of the network for the BAP is obtained as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}}, \tag{D-5}$$

which is achieved under the condition

$$\lambda^{BAP} = o\left(\frac{n}{\log n}\right). \tag{D-6}$$

Appendix V

Proof of Theorem 3

Using (24), we have

$$\frac{\partial \Delta_n^{CAP}}{\partial \tau_n} = \frac{\partial \Delta_n^{CAP}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n} \tag{E-1}$$

$$= -q_n \frac{\partial \Delta_n^{CAP}}{\partial q_n} \tag{E-2}$$

$$= \frac{1-(1-q_n)^\lambda}{\lambda q_n} - (1-q_n)^{\lambda-1} \tag{E-3}$$

$$= \Delta_n^{CAP} - (1-q_n)^{\lambda-1}. \tag{E-4}$$

Hence, $\Delta_n^{CAP} - \frac{\partial \Delta_n^{CAP}}{\partial \tau_n} = (1-q_n)^{\lambda-1}$. In this case, (C-3) can be simplifies as

$$n\hat{\alpha} q_n \frac{\left[1-(1-q_n)^{\lambda}\right]^2}{(\lambda q_n)^2} = (1-q_n)^{\lambda-1}\tau_n^2. \tag{E-5}$$

or

$$n\hat{\alpha} = \frac{\tau_n^2 \lambda^2 q_n (1-q_n)^{\lambda-1}}{\left[1-(1-q_n)^{\lambda}\right]^2}. \tag{E-6}$$

Since $q_n = o(1)$, we have $(1-q_n)^{\lambda-1} = e^{(\lambda-1)\log(1-q_n)} \overset{(a)}{\approx} e^{-\lambda q_n}$, and $1-(1-q_n)^{\lambda} \overset{(b)}{\approx} 1-e^{-\lambda q_n}$. It should be noted that (a) and (b) are valid under the condition $\frac{\lambda q_n^2}{2} = o(1)$ [12]. Thus, (E-6) can be simplified as

$$n\hat{\alpha} = \frac{\tau_n^2 \lambda^2 q_n e^{-\lambda q_n}}{\left[1-e^{-\lambda q_n}\right]^2}, \tag{E-7}$$

or

$$\frac{\nu \log \nu^{-1}}{(1-\nu)^2} = \Psi, \tag{E-8}$$

where $\nu \triangleq e^{-\lambda q_n}$ and $\Psi \triangleq \dfrac{n\hat{\alpha}}{\tau_n^2 \lambda}$. For this setup, we have the following cases:

Case 1: $\Psi \gg 1$

It is realized from (E-8) that for $\Psi \gg 1$, $\nu = 1-\epsilon$, where $\epsilon = o(1)$. Thus, (E-8) can be simplified as

$$\Psi \approx \frac{\log(1-\epsilon)^{-1}}{\epsilon^2} \tag{E-9}$$

$$\overset{(a)}{\approx} \frac{\epsilon}{\epsilon^2} \tag{E-10}$$

$$= \frac{1}{\epsilon}, \tag{E-11}$$

where (a) follows from the Taylor series expansion $\log(1-z) = -\sum_{k=1}^{\infty} \dfrac{z^k}{k} \approx -z, \ \ |z| \ll 1$. Since $\nu \triangleq e^{-\lambda q_n}$ and $\nu = 1-\epsilon$, we have

$$e^{-\lambda q_n} = 1 - \frac{1}{\Psi}, \tag{E-12}$$

$$\Longrightarrow q_n \overset{(a)}{\approx} \frac{1}{\Psi\lambda} = \frac{\tau_n^2}{n\hat{\alpha}}, \tag{E-13}$$

[12] As we will show the condition $\frac{\lambda q_n^2}{2} = o(1)$ is satisfied for the optimum q_n and the corresponding λ.

where (a) follows from the fact that as $\lambda q_n = o(1)$, we have $e^{-\lambda q_n} \approx 1 - \lambda q_n$. It can be verified that the solution for (E-13) is

$$\tau_n^{CAP} = \log n - 2\log\log n + O(1). \tag{E-14}$$

Using $q_n = e^{-\tau_n}$, we conclude that

$$q_n^{CAP} = \delta \frac{\log^2 n}{n}, \tag{E-15}$$

for some constant δ.

The above results are valid for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2 \lambda} \gg 1$ or $\lambda = o\left(\frac{n}{\log^2 n}\right)$. Also, it can be verified that $\frac{\lambda q_n^2}{2} = o(1)$, and therefore the approximations $(1-q_n)^{\lambda-1} \approx e^{-\lambda q_n}$ and $1-(1-q_n)^{\lambda} \approx 1 - e^{-\lambda q_n}$ are valid in this region.

To satisfy the condition of Lemma 6, we must have

$$\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} \ll 1. \tag{E-16}$$

From (24), (E-14) and noting that as $\lambda = o\left(\frac{n}{\log^2 n}\right)$, $\left[1-(1-q_n)^{\lambda}\right] \approx 1 - e^{-\lambda q_n} \approx \lambda q_n$, we can write

$$\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} \approx \frac{\lambda \log n}{n\hat{\alpha}\left[1-(1-q_n)^{\lambda}\right]} \tag{E-17}$$

$$\approx \frac{\log n}{n\hat{\alpha} q_n}$$

$$= O\left(\frac{1}{\log n}\right), \tag{E-18}$$

which means that the condition of Lemma 6 is automatically satisfied in this region. Thus, the maximum effective throughput of the network obtained in (48) can be simplified as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log n}{\hat{\alpha}}. \tag{E-19}$$

Case 2: $\Psi = \Theta(1)$

From (E-8) which gives $\frac{\nu \log \nu^{-1}}{(1-\nu)^2} = \Psi = \Theta(1)$, we conclude that $\nu \triangleq e^{-\lambda q_n} = \Theta(1)$. Thus,

$$q_n = \frac{c_1}{\lambda} \tag{E-20}$$

$$\stackrel{(a)}{=} \frac{c_2 \tau_n^2}{n\hat{\alpha}} \tag{E-21}$$

where c_1 and c_2 are constants and (a) follows from $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2\lambda} = \Theta(1)$. It can be verified that the solution for (E-21) is

$$\tau_n^{CAP} = \log n - 2\log\log n + O(1). \tag{E-22}$$

$$q_n^{CAP} = \delta' \frac{\log^2 n}{n}, \tag{E-23}$$

for some constant δ'.

The above results are valid for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2\lambda} = \Theta(1)$ or $\lambda = \Theta\left(\frac{n}{\log^2 n}\right)$. Also, it can be verified that $\frac{\lambda q_n^2}{2} = o(1)$, and therefore, the approximations $(1-q_n)^{\lambda-1} \approx e^{-\lambda q_n}$ and $1-(1-q_n)^{\lambda} \approx 1-e^{-\lambda q_n}$ are valid in this region.

Similar to the argument in Case 1, the condition of Lemma 6 is satisfied, and therefore, the maximum effective throughput of the network is obtained as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log n}{\hat{\alpha}}. \tag{E-24}$$

Case 3: $\Psi \ll 1$

It is concluded from (E-8) that $\frac{\nu\log\nu^{-1}}{(1-\nu)^2} = \Psi$, where $\Psi = o(1)$. In this case, $\nu = o(1)$, and therefore, $\nu\log\nu^{-1} \approx \Psi$. The solution for this equation is $\nu \approx \frac{\Psi}{\log(\Psi)^{-1}}$. In other words,

$$e^{-\lambda q_n} \approx \frac{\frac{n\hat{\alpha}}{\lambda\tau_n^2}}{\log\left(\frac{\lambda\tau_n^2}{n\hat{\alpha}}\right)}. \tag{E-25}$$

Thus,

$$\lambda q_n \approx \log\left(\frac{\lambda\tau_n^2}{n\hat{\alpha}}\right) + \log\log\left(\frac{\lambda\tau_n^2}{n\hat{\alpha}}\right) \tag{E-26}$$

$$\overset{(a)}{\approx} \log\left(\frac{\lambda\tau_n^2}{n\hat{\alpha}}\right), \tag{E-27}$$

where (a) follows from $\lambda q_n = \omega(1)$ which comes from $\nu = o(1)$. The solution for the above equation can be written as $\tau_n = \log\lambda - f(\lambda)$ or $q_n = \frac{e^{f(\lambda)}}{\lambda} = o(1)$, where we assume $f(\lambda) = o(\log\lambda)$. Substituting in (E-27), we obtain

$$e^{f(\lambda)} = \log\left(\frac{\lambda(\log\lambda - f(\lambda))^2}{n\hat{\alpha}}\right) \tag{E-28}$$

$$= \log\left(\frac{\lambda\log^2\lambda}{n\hat{\alpha}}\right) + 2\log\left(1 - \frac{f(\lambda)}{\log\lambda}\right) \tag{E-29}$$

$$\overset{(a)}{\approx} \log\left(\frac{\lambda\log^2\lambda}{n\hat{\alpha}}\right), \tag{E-30}$$

where (a) follows from the fact $f(\lambda) = o(\log \lambda)$. Thus, using $\tau_n = \log \lambda - f(\lambda)$, it yields

$$\tau_n^{CAP} = \log \lambda - \log \log \left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}} \right). \tag{E-31}$$

It should be noted that (E-31) is derived from (E-25) for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2 \lambda} \ll 1$. This translates the condition $\frac{n\hat{\alpha}}{\tau_n^2 \lambda} \ll 1$ to $\frac{n\hat{\alpha}}{\lambda \log^2 \lambda} \ll 1$, which incurs that $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$.

Also, in the following we show that the condition $\frac{\lambda q_n^2}{2} = o(1)$ is satisfied. It follows from (E-27) that

$$\begin{aligned} \lambda q_n^2 &= \frac{\log^2\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right)}{\lambda} && \text{(E-32)} \\ &\stackrel{(a)}{\leq} \frac{\log^2\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right)}{\lambda} && \text{(E-33)} \\ &\stackrel{(b)}{=} o(1), && \text{(E-34)} \end{aligned}$$

where (a) follows from (E-31) and (b) comes from $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$.

To satisfy the condition of Lemma 6, we must have

$$\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} \ll 1. \tag{E-35}$$

From (24) and (E-31), we can write

$$\begin{aligned} \frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} &\approx \frac{\lambda \log \lambda}{n\hat{\alpha}\left[1 - e^{-\lambda q_n}\right]} && \text{(E-36)} \\ &\stackrel{(a)}{\approx} \frac{\lambda \log \lambda}{n\hat{\alpha}}, && \text{(E-37)} \end{aligned}$$

where (a) follows from $e^{-\lambda q_n} = o(1)$. In order to have $\frac{\lambda \log \lambda}{n\hat{\alpha}} = o(1)$, one must have $\lambda = o\left(\frac{n}{\log n}\right)$. In this case, the maximum effective throughput of the network can be simplified as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log \lambda}{\hat{\alpha}}. \tag{E-38}$$

Noting that λ satisfies $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$ and $\lambda = o\left(\frac{n}{\log n}\right)$, it follows that $\log \lambda \sim \log n$. In other words, $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$.

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References

[1] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. on Vehicular Technology*, vol. 42, no. 4, pp. 641–646, Nov. 1993.

[2] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Commun.*, vol. 13, no. 7, pp. 1341–1348, Sept. 1995.

[3] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. on Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[4] J. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE Journal on Selected Areas in Commun.*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[5] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE Journal on Selected Areas in Commun.*, vol. 25, no. 3, pp. 517–528, April 2007.

[6] N. Jindal, S. Weber, and J. Andrews, "Fractional power control for decentralized wireless networks," in *Proc. Forty-Fifth Annual Allerton Conference*, University of Illinois, IL, USA, Sept. 2007.

[7] K. Jung and D. Shah, "Low delay scheduling in wireless networks," in *Proc. IEEE International Symposium on Information Theory (ISIT'07)*, June 2007, pp. 1396–1400.

[8] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[9] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Trans. on Networking*, vol. 10, no. 4, pp. 477–486, August 2002.

[10] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling in wireless networks," *IEEE Trans. on Information Theory*, vol. 50, no. 6, pp. 1041–1049, June 2004.

[11] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: scaling laws and optimal operation," *IEEE Trans. on Information Theory*, vol. 50, no. 5, pp. 748–767, May 2004.

[12] R. Gowaikar, B. Hochwald, and B. Hassibi, "Communication over a wireless network with random connections," *IEEE Trans. on Information Theory*, vol. 52, no. 7, pp. 2857–2871, July 2006.

[13] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal throughput-delay scaling in wireless networks - part I: The fluid model," *IEEE Trans. on Information Theory*, vol. 52, no. 6, pp. 2568–2592, June 2006.

[14] L. B. Le, E. Hossain, and T. S. Alfa, "Delay statistics and throughput performance for multi-rate wireless networks under multiuser diversity," *IEEE Trans. on Wireless Commun.*, vol. 5, no. 11, pp. 3234–3243, Nov. 2006.

[15] I. Bettesh and S. Shamai, "A low delay algorithm for the multiple access channel with Rayleigh fading," in *Proc. IEEE Personal, Indoor and Mobile Radio Commun.*, Sept. 1998, vol. 3, pp. 1367–1372.

[16] N. Bansal and Z. Liu, "Capacity, delay and mobility in wireless ad-hoc networks," in *Proc. IEEE INFOCOM*, April 2003, pp. 1553–1563.

[17] S. Toumpis and A. J. Goldsmith, "Large wireless networks under fading, mobility, and delay constraints," in *Proc. IEEE INFOCOM*, March 2004, pp. 609–619.

[18] P. K. Gopala and H. El Gamal, "On the throughput-delay tradeoff in cellular multicast," in *Proc. IEEE International Conference on Wireless Networks, Communications and Mobile Computing*, June 2005, vol. 2, pp. 1401–1406.

[19] L. Xiaojun, G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Degenerate delay-capacity tradeoffs in ad-hoc networks with brownian mobility," *IEEE Trans. on Information Theory*, vol. 52, no. 6, pp. 2777–2784, June 2006.

[20] M. J. Neely and E. Modiano, "Capacity and delay tradeoffs for ad-hoc mobile networks," *IEEE Trans. on Information Theory*, vol. 51, no. 6, pp. 1917–1937, June 2005.

[21] M. Sharif and B. Hassibi, "Delay considerations for opportunistic scheduling in broadcast fading channels," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 9, pp. 3353–3363, Sept. 2007.

[22] A. Bayesteh, M. Ansari, and A. K. Khandani, "Throughput and fairness maximization in wireless downlink systems," *Submitted to IEEE Trans. on Information Theory*, Sept. 2007.

[23] C. Comaniciu and H. V. Poor, "On the capacity of mobile ad-hoc networks with delay constraints," *IEEE Trans. on Wireless Commun.*, vol. 5, no. 8, pp. 2061–2071, August 2006.

[24] Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Capacity-delay tradeoff for information dissemination modalities in wireless networks," in *Proc. IEEE International Symposium on Information Theory (ISIT'08)*, Toronto, Canada, July 2008, pp. 677–681.

[25] J. M. Walsh, S. Weber, and C. wa Maina, "Optimal rate delay tradeoffs for multipath routed and network coded networks," in *Proc. IEEE International Symposium on Information Theory (ISIT'08)*, Toronto, Canada, July 2008, pp. 682–686.

[26] J. Abouei, A. Bayesteh, M. Ebrahimi, and A. K. Khandani, "On the throughput maximization in decentralized wireless networks," *Submitted to IEEE Trans. on Information Theory*, Oct. 2008.

[27] M. J. Neely, "Order optimal delay for opportunistic scheduling in multi-user wireless uplinks and downlinks," in *Proc. of 44th Allerton Conference on Communication, Control, and Computing*, Sept. 2006.

[28] D. E. Knuth, "Big omicron and big omega and big theta," in *ACM SIGACT News*, April-June 1967, vol. 8, pp. 18–24.

[29] A. Abdi and M. Kaveh, "On the utility of gamma PDF in modeling shadow fading (slow fading)," in *Proc. IEEE Vehicular Technology Conference (VTC'99)*, May 1998, vol. 3, pp. 2308–2312.

[30] A. Abdi, H. A. Barger, and M. Kaveh, "A simple alternative to the lognormal model of shadow fading in terrestrial and satellite channels," in *Proc. IEEE Vehicular Technology Conference (VTC'01)*, Fall 2001, vol. 4, pp. 2058–2062.

[31] R. G. Gallager, *Discrete Stochastic Processes*, Kluwer Academic Publishers, 1995.

[32] M. Zorzi and R. R. Rao, "On channel modeling for delay analysis of packet communications over wireless links," in *Proc. 36th Annual Allerton Conference*, Sept. 1998.

[33] J. Tang and X. Zhang, "Cross-layer modeling for quality of service guarantees over wireless links," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 12, pp. 4504–4512, Dec. 2007.

[34] Valentin V. Petrov, *Limit Theorems of Probability Theory: Sequences of Indpendent Random Variables*, Oxford University Press, 1995.