Signal Space Cooperative Communication for Single Relay Model

by

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Abstract

In this paper, a single-hop single-relay system with a direct link between the source and the destination is considered when the relay operates in the half-duplex mode. Motivated by the concept of signal space diversity, this paper introduces *signal space cooperation*, in which cooperation between the source and the relay is achieved using a novel constellation design approach. In this approach, the original constellation is expanded so that each member of the new constellation inherits its components from at least two members of the original constellation. The expanded constellation enables the relay to extract the required information in order to cooperate in the relay phase, and it helps the destination to effectively combine received signals during the broadcast phase and the relay phase. The analytical study of the proposed scheme leads to the development of two design criteria for the constellation expansion approach. The proposed design criteria aim at maximizing the relay role in the cooperative scheme by increasing the performance of the source-relay link. In this way, the source and the relay can effectively cooperate in order to maximize the overall performance of the system. The signal space cooperative scheme can be used for any constellation size without incurring significant complexity overhead to the system. Numerical results depict superior performance in comparison with other cooperative schemes such as the adaptive decode and forward and the distributed turbo code schemes.

Index Terms

Index Terms - Signal space diversity, cooperative communication, constellation design.

I. Introduction

OVER the past decade, wireless communication has evolved in various ways. The next generation of wireless systems should service more users while supporting mobility and very high data rates..

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These new requirements necessitate efficient use of available resources to provide acceptable service quality.

Spatial diversity techniques are known to increase system reliability without sacrificing the available bandwidth or time [1]. On the other hand, due to the size and power limitations of mobile devices, antenna diversity is not always practically feasible. Cooperative diversity [2] is an alternative way of providing spatial diversity when the multiple antenna structure is not an option. These schemes offer diversity in an interesting way based on the fact that other users in the network are able to overhear the transmitted signal and forward the information to the destination through different paths. Hence, the destination enjoys several versions of the original signal (or functions of the original signal) that have been received through independent fading channels. Consequently, higher diversity is achieved at the destination end which leads to better system performance.

The basic idea of relaying was introduced by van der Meulen in [3]. Later, Cover and El Gamal investigated the relay channels from the information theoretic perspective [4]. They proposed schemes that provide achievable rates for general one-hop relay models and can be applied to the Additive White Gaussian Noise (AWGN) channels. However, to benefit from cooperative communication in fading channels, new cooperative schemes were required that can provide diversity gain.

The concept of cooperation among users to provide diversity was first introduced by Sendonaris *et al.* in [2] and [5]. These authors showed that cooperative schemes can effectively enhance the system's robustness against fading. Later, Laneman *et al.* derived the formulation for cooperative diversity based on two cooperative schemes, namely Amplify and Forward (AF), and Decode and Forward (DF) [6]. Another scheme called coded cooperation was introduced by Hunter and Nostratinia [7], [8]. This scheme combines channel coding and cooperative communication to enhance the performance of decoding at the destination by providing additional parities through the relay channel. A simple case of the coded cooperation can be built using Rate Compatible Punctured Convolutional (RCPC) codes [9] as each codeword is punctured into two subsets that are transmitted from the source's and the cooperative user's antennas, respectively.

Since coded cooperative schemes consist of two code components, turbo codes are natural fits in such a framework. The application of the distributed turbo codes for cooperative communication was investigated in [10] and [11]. In an approach called Distributed Parallel Concatenated Convolutional Code

(DPCCC), the source encodes the original message and forwards the encoded block to the relay. The relay, decodes the received bits using a Viterbi decoder, passes the decoded message through an interleaver, and re-encodes the message before retransmission to the destination. Punctured concatenated convolutional codes are also another form of turbo cooperative codes that are used in cooperative communication [12], [13]. Coded cooperative schemes often demand complex iterative decoding at the destination and the relay. Moreover, these methods usually require low rate mother codes that reduces the system's spectral efficiency.

In [14] a cooperative approach based on signal superposition was studied. In this approach, each cooperative user transmits a superposition of its own data with other users' data that was received in previous time slots. This approach is sensitive to power allocation since each user may use different power allocation strategies when data symbols are superposed. The same technique was combined with coded cooperation in [15]. In this way, each user sends both its own and its partner's incremental parity bits with hierarchical error protection during the relay phase of the communication. Consequently, a larger constellation should be used for the relay phase to provide enough spectral efficiency. Recently, the notion of constellation re-arrangement was adopted from the Hybrid Automatic Repeat-reQuest (HARQ) techniques [16] and used in cooperative networks [17]. The idea is to re-arrange the signal points of the original constellation at the relay so that the destination can benefit from the combination of the source and the relay signals. However, the problem of finding the optimal symbol mapping at the relay becomes extremely complex as the constellation size increases.

Signal space diversity is another type of diversity that is achieved in the modulation signal space. This concept was pioneered by Belfiore and Boulle [18]. The idea is to spread information carried by each signal point among all components of that particular signal point and send each component through an independent realization of the channel [19]. To satisfy such a condition, a component interleaver/deinterleaver pair is required [20]. In this way, good channel realizations for some of the signal components can compensate for the effect of possible deep fade situations over the rest of the components. The diversity order of a constellation is bounded by the minimum number of distinct components between any two signal points that are affected by independent realizations of the channel.

Signal space diversity techniques have the capability to improve the performance of M-QAM constellations over fading channels [21]. Moreover, using number field theory, a set of algebraic tools was developed in order to build proper constellations that match fading channel characteristics [22]. The aforementioned algebraic tools, enable signal space diversity techniques to go beyond the conventional two-dimensional signal sets, and extend the concept of signal space diversity to multi-dimensional signal sets [23].

The combination of spatial diversity with signal space diversity techniques was addressed in [24]. In an approach called Coordinate Interleaved Space Time Code (CISTC), components of different signal points are interleaved and the result is applied to a space time encoder to achieve spatial diversity [25]. Later, the coordinate interleaved approach was extended to multiple antenna relay networks [26]. Unlike multiple antenna systems, in cooperative schemes full cooperation between the source and the relay is possible if the relay can successfully decode the original message. Hence, as the performance of the source-relay link degrades, the diversity gain of the cooperative scheme diminishes. This challenging problem makes cooperative schemes that use signal space diversity, vulnerable to the source-relay link performance [27], [28].

In this paper, the Signal Space Cooperative (SSC) scheme based on signal space diversity is proposed. To this end, the original constellation is expanded so that each member of the new constellation inherits its components from at least two members of the original constellation. The expanded constellation enables the relay to extract the required information from the source signal and send additional information to the destination. Based on the analytical study of the SSC scheme, two design criteria are developed in order to maximize the likelihood of decoding the original message at the relay, and to improve the destination's performance. To compare the performance of the proposed scheme with other cooperative schemes, the Adaptive Decode and Forward (ADF) [6] and punctured Parallel Concatenated Convolution Code (PCCC) [13] are selected. In fact, the PCCC scheme benefits from the same idea of splitting the original signal between the source and the relay. However, the proposed scheme in this paper performs the signal partitioning in the system signal space while the PCCC scheme applies the partitioning in code domain. The numerical results depict that the SSC scheme outperforms the aforementioned schemes. It is also shown that the proposed scheme can be generalized to larger constellations without incurring significant complexity overhead to the system.

The rest of the paper is organized as follows: The system model is introduced in section II followed by a detailed description of the SSC scheme in section III. System performance is then analyzed in section

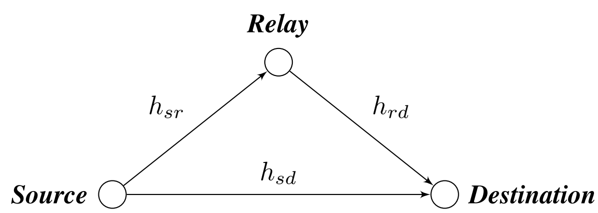

Fig. 1. Single-hop single-relay model

IV, in which the design criteria are derived. The numerical results are stated in section V. Finally, section VI concludes the paper.

II. System Model

In this paper a single-hop single-relay system (Figure 1) with a direct link between the source and the destination is considered. It is assumed that the relay operates in half duplex mode, so it cannot receive and transmit at the same time. In this way, each block of transmission is split into two transmission phases namely the broadcast phase and the relay phase. In the broadcast phase, the source broadcasts the signal to both the relay and the destination. In the relay phase, either the source or the relay sends an additional signal to help the destination to decode the transmitted message. The selection is based on the relay's success in decoding the transmitted message from the broadcast phase signal. If the relay fails to decode the transmitted message at the end of the broadcast phase (e.g. corrupted messages can be detected using Code Redundancy Check codes concatenated with the original message), the relay sends a feedback to the source indicating its failure to decode the transmitted message. These assumptions are consistent with advanced wireless standards such as IEEE 802.16 [29] that are equipped with ARQ schemes and CRC protected frames.

All channels in the system are assumed to be quasi-static block fading. Therefore, the channel coefficients remain constant during the course of each block of transmission. Channel coefficients are drawn from independent zero mean circularly symmetric Gaussian random variables and perfect Channel State Information (CSI) at the receiver is also assumed. Based on the above assumptions, the channel model for source-relay, source-destination and relay-destination links after compensating for the effect of the channel phase can be written as:

$$y_{sr} = \sqrt{E_s} h_{sr} s_s + z_{sr},$$
$$y_{sd} = \sqrt{E_s} h_{sd} s_s + z_{sd},$$
$$y_{rd} = \sqrt{E_r} h_{rd} s_r + z_{rd},$$

respectively. y_{sr}, y_{sd}, y_{rd} are the received signals and s_s, s_r are the transmitted signals from the source and the relay, respectively. Here the transmitted signals from the source and the relay are assumed to be normalized such that $\mathbb{E}[|s_s|^2] = \mathbb{E}[|s_r|^2] = 1$. z_{sr}, z_{sd}, z_{rd} are independent zero mean circularly symmetric additive white Gaussian noise with variance N_0. The channel coefficients h_{sr}, h_{sd}, h_{rd} are assumed to be zero mean independent Rayleigh random variables with unit variance and probability density function $f(h)$ as follows:

$$f(h) = 2he^{-h^2}. \quad (1)$$

III. Signal Space Cooperative Scheme

In this section, the proposed signal space cooperative scheme is described. To this end, constellation expansion algorithm that enables the relay to effectively cooperate during the course of transmission is proposed. Then, the broadcast phase and the relay phase signals are described followed by a discussion on how the destination decodes the received signal at the end of each transmission block.

A. Constellation Expansion

Signal space diversity is proposed in [19] to exploit diversity gain in a system when each component of the transmitted signal is affected by independent channel fading. It is shown that to achieve the maximum diversity gain, any two signal points in the system constellation must have the maximum number of distinct components. In other words, both the in-phase component and the quadrature component of the transmitted signal have to carry enough information to uniquely represent the original signal. In the single relay model, the above conditions can be met using the independent paths from the source and the relay to the destination. In this way, the source and the relay cooperate by sending different components of the original signal to the destination. As is shown in the following, by expanding the signal constellation the source and the relay can cooperate effectively to achieve signal space diversity.

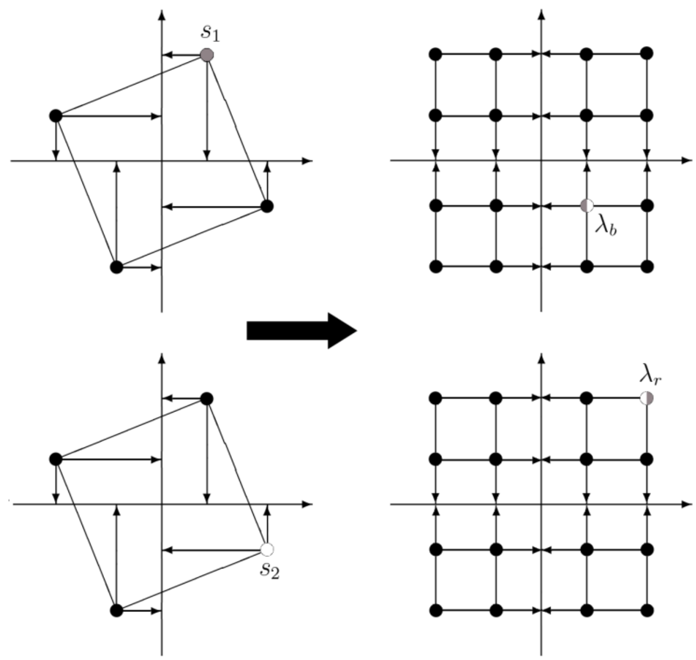

Fig. 2. Constellation expansion in two-dimensional signal space

Let S be a constellation generated by applying a transformation Θ to an ordinary constellation (e.g. QPSK constellation). In the two-dimensional signal space, there exist rotations that provide the aforementioned properties for the signal space diversity [19]. Hence, the transformation Θ can be defined as follows:

$$\Theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \tag{2}$$

where θ is the angle of rotation in the two-dimensional signal space[1]. Therefore, every signal points in S is uniquely distinguishable from its in-phase and quadrature components. In order to transmit different components of the constellation points through independent paths, a component interleaver/deinterleaver pair is used at the source and the destination. Let s_1 and s_2 be two members of the constellation S. Hence, $s_1 = \Re(s_1) + j\Im(s_1)$ and $s_2 = \Re(s_2) + j\Im(s_2)$, where $\Re(\cdot)$ and $\Im(\cdot)$ are the in-phase and quadrature components of the corresponding signal points, respectively. The new constellation points λ_b

[1]A comprehensive list of the transformation Θ for multi-dimensional signal spaces can be found in [30]

and λ_r are formed by interleaving the quadrature components of s_1 and s_2 as follows:

$$\lambda_b = \Re(s_1) + j\Im(s_2), \tag{3}$$

$$\lambda_r = \Re(s_2) + j\Im(s_1). \tag{4}$$

Clearly, λ_b and λ_r belong to the expanded constellation Λ defined as follows:

$$\Lambda = \Re(S) \times \Im(S), \tag{5}$$

where $\times$ denotes the Cartesian product of two sets. It is noted that each member of the expanded constellation consists of two components each of which uniquely identifies a particular member of S. Hence, decoding a member of the expanded constellation results in decoding two different members of the original constellation (e.g. s_1 and s_2). Figure 2, depicts how the expanded constellation points λ_b and λ_r are formed from two signal points in a rotated QPSK constellation.

B. Broadcast Phase

In the broadcast phase, both the destination and the relay receive the signal transmitted by the source. Hence, the purpose of designing the broadcast phase signal is two-fold. On the one hand, it should help the relay to decode the original message and cooperate in the relay phase. On the other hand, the destination should benefit from combining the broadcast phase and the relay phase signals at the end of the transmission block.

Let $\mathbf{s} = (s_1, s_2)$ be a pair of signal points from the original constellation (i.e. $s_1, s_2 \in S$), that corresponds to the source message. To address the aforementioned constraints in the SSC scheme, the broadcast phase signal λ_b is generated based on Equation (3). In fact, λ_b carries enough information to uniquely represent both s_1 and s_2 if decoded correctly at the relay. Figure 3 depicts how the broadcast phase and also the relay phase signals are generated from the signal points in $\mathbf{s}$. The block diagram shows how the source and the relay use particular components of each signal point in $\mathbf{s}$ and transmit the generated signals during the broadcast phase or the relay phase. Thus, the channel model for the broadcast phase of the SSC scheme can be written as:

$$y_{sd} = \sqrt{E_s} h_{sd}\, \lambda_b + z_{sd}, \tag{6}$$

$$y_{sr} = \sqrt{E_s} h_{sr}\, \lambda_b + z_{sr}. \tag{7}$$

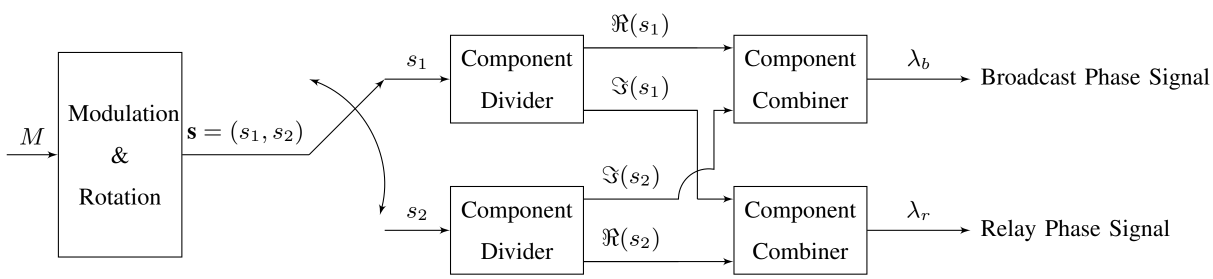

Fig. 3. Broadcast phase and relay phase signal generation

C. *Relay Phase*

At the end of the broadcast phase, the relay should be able to obtain enough information to cooperate in the relay phase. It is noted that the broadcast phase signal consists of signal points from the expanded constellation that uniquely represent the source message. Hence, the relay can decode the source message from the broadcast phase signal and generate the relay phase signal λ_r based on Equation (4). The relay phase signal (λ_r) is also a member of the expanded constellation and contains sufficient information to identify the original signal points in $\mathbf{s}$.

However, if the relay fails to decode the transmitted message, it will not able to generate λ_r correctly. In that case, if the relay insists on cooperating based on its erroneous detection, its signal may add to the destination's ambiguity rather than improving the system's performance. In the following section, a design criterion is introduced that addresses this effect which is caused by quality of the source-relay link.

A feedback strategy, that enables the relay to inform the source about its failure in decoding the transmitted message, is also integrated into the SSC scheme. The idea behind this technique is to opportunistically benefit from good channel realizations of the source-destination link when the source-relay channel is in a deep fade state. In this way, upon reception of the negative acknowledge message (Nack) from the relay, the source takes control of the relay phase and sends the relay phase signal to the destination. Therefore, the destination is supposed to receive λ_r during the relay phase, regardless of the relay success or failure in decoding the transmitted message. Hence, the channel model for the relay phase can be described as:

$$y_{rd} = \sqrt{E_d} h_d \, \lambda_r + z_{rd}, \tag{8}$$

where $h_d = h_{rd}$ and $E_d = E_r$ if the relay participates in cooperation and $h_d = h_{sd}$ and $E_d = E_s$ otherwise.

D. Decoding at the Destination

At the end of each transmission block, the destination combines received signals during the broadcast phase and the relay phase in order to decode the transmitted message. By comparing the received signals at the destination, it is observed that the relay phase signal contains components of the original signal that are not included in the broadcast phase signal. Therefore, from the destination point of view, different components of each member of the original signal (i.e. **s**), are affected by independent channel fading. This means that the primary condition of signal space diversity has been met, and the SSC scheme can exploit full system diversity as is shown in the next section.

To decode the original message, the destination reorders the received components so that the corresponding components of each signal point in **s** join together. Let $\mathbf{r} = (r_1, r_2)$ be the destination's signal after reordering the received components. Hence,

$$\begin{aligned} r_1 &= \Re(y_{sd}) + j\Im(y_{rd}), \\ r_2 &= \Re(y_{rd}) + j\Im(y_{sd}), \end{aligned} \tag{9}$$

where y_{sd} and y_{rd} are the received signals at the destination during the broadcast phase and the relay phase, respectively. Finally, the destination applies a maximum likelihood detector [31] on the reordered signal to detect the source message. The destination's maximum likelihood decision rule can be written as follows:

$$\begin{aligned} \hat{s}_1 &= \arg\min_{s \in S} \ (|\Re(r_1) - \sqrt{E_s} h_{sd} \Re(s)|^2 + |\Im(r_1) - \sqrt{E_d} h_d \Im(s)|^2), \\ \hat{s}_2 &= \arg\min_{s \in S} \ (|\Re(r_2) - \sqrt{E_d} h_d \Re(s)|^2 + |\Im(r_2) - \sqrt{E_s} h_{sd} \Im(s)|^2), \end{aligned}$$

where $h_d = h_{rd}$ and $E_d = E_r$ if the relay participates in cooperation and $h_d = h_{sd}$ and $E_d = E_s$ otherwise.

IV. Performance Analysis

In this section, the performance of the SSC scheme is analyzed and the design criteria are derived in order to maximize the system's overall performance. Without loss of generality and for simplicity, the analysis is performed on a per-symbol basis and the average signal energy of both the source and the relay are assumed to be the same (i.e. $E_s = E_r$).

A. Upper Bound on the Error Probability

Let $\mathbf{h} = (h_{sr}, h_{sd}, h_{rd})$ be a set of channel realizations for a block of transmission. The conditional error probability for the SSC scheme can be written as:

$$P_{e|\mathbf{h}} = P_{r|h_{sr}} P_{f|h_{sd}} + (1 - P_{r|h_{sr}}) P_{coop|h_{sd},h_{rd}}.$$

where $P_{r|h_{sr}}$ is the conditional error probability of the relay in decoding the source message at the end of the broadcast phase. $P_{f|h_{sd}}$ represents the destination conditional error probability when the source participates in the relay phase due to the relay failure in decoding the source message. $P_{coop|h_{sd},h_{rd}}$ is the conditional probability of error at the destination when the relay cooperates in the relay phase. The error probability of the system can be obtained by averaging the conditional error probability over fading channel coefficients. Thus:

$$\begin{aligned} P_e &= \int P_{e|\mathbf{h}} f(\mathbf{h})\ d\mathbf{h} \\ &= \int P_{r|h_{sr}} f(h_{sr}) dh_{sr} \int P_{f|h_{sd}} f(h_{sd}) dh_{sd} + \\ &\int (1 - P_{r|h_{sr}}) f(h_{sr}) dh_{sr} \iint P_{coop|h_{sd},h_{rd}} f(h_{sd}) f(h_{rd}) dh_{sd} dh_{rd} \\ &= P_r.P_f + (1 - P_r).P_{coop} \end{aligned} \tag{10}$$

In the following, an upper bound for P_e is obtained by bounding all the error probabilities in (10). Then the upper bound is used to derive the design criteria that maximize the performance of the SSC scheme.

Let $E(s, \hat{s})$ be the error event that the decoder selects $\hat{s}$ in favor of the transmitted signal s. Assuming all signal points in the original constellation are equiprobable, using the union bound the $P_{r|h_{sr}}$ can be bounded as follows:

$$
\begin{aligned}
P_{r|h_{sr}} &= \frac{1}{|\mathit{\Lambda}|}\sum_{s\in\mathit{\Lambda}}\bigcup_{\hat{s}\neq s}E(s,\hat{s}|h_{sr}),\\
&\leq \frac{1}{|\mathit{\Lambda}|}\sum_{s\in\mathit{\Lambda}}\sum_{\hat{s}\neq s}P(s\rightarrow\hat{s}|h_{sr}). \qquad (11)
\end{aligned}
$$

where $P(s \rightarrow \hat{s}|h_{sr})$ denotes the conditional pair-wise error probability of choosing $\hat{s}$ at the relay, assuming s is transmitted. Since the channel coefficients are assumed to be constant during each transmission block, $P(s \rightarrow \hat{s}|h_{sr})$ can be written as follows [31]:

$$
P(s\rightarrow\hat{s}|h_{sr}) = Q\left(\sqrt{\frac{E_s h_{sr}^2|s-\hat{s}|^2}{2N_0}}\right) \qquad (12)
$$

Hence,

$$
\begin{aligned}
P_{r|h_{sr}} &\leq \frac{1}{|\Lambda|}\sum_{s\in\Lambda}\sum_{\hat{s}\neq s}Q\left(\sqrt{\frac{E_s h_{sr}^2|s-\hat{s}|^2}{2N_0}}\right)\\
&\leq (|\Lambda|-1)Q\left(\sqrt{\frac{E_s h_{sr}^2 d_{min}(\Lambda)}{2N_0}}\right)\\
&\overset{(a)}{\leq} (|\Lambda|-1)\exp\left(-\frac{E_s h_{sr}^2 d_{min}(\Lambda)}{4N_0}\right) \qquad (13)
\end{aligned}
$$

where $d_{min}(\Lambda) = \min_{s,\hat{s}\in\Lambda}(|s-\hat{s}|^2)$ and (a) comes from the Chernov bound for the Gaussian tail function $Q(x)$ [31]. Therefore,

$$
\begin{aligned}
P_r &= \int P_{r|h_{sr}} f(h_{sr})dh_{sr}\\
&\leq \int (|\Lambda|-1)\exp\left(-\frac{E_s h_{sr}^2 d_{min}(\Lambda)}{4N_0}\right)2h_{sr}e^{-h_{sr}^2}\ dh_{sr}\\
&= \frac{|\Lambda|-1}{1+\frac{E_s d_{min}(\Lambda)}{4N_0}}. \qquad (14)
\end{aligned}
$$

Using a similar approach, the upper bound for P_f can be written as:

$$
P_f \leq \frac{|S|-1}{1+\frac{E_s d_{min}(S)}{4N_0}} \qquad (15)
$$

where S is the original constellation.

The relay cooperation in the relay phase is possible, if it can uniquely decode the transmitted message. However, the broadcast phase signal contains only one component from each signal point in the original signal $\mathbf{s}$. Thus, in order to enable the relay to decode the original message at the end of the broadcast phase, *any signal point $s \in S$ must have unique in-phase and quadrature components that distinguish s from any other signal point $\hat{s} \in S$*. In other words, the relay should be able to decode signal **s** from signal components that are included in the broadcast phase signal (either the in-phase or the quadrature component).

From (5) it can be observed that the expanded constellation is formed from all possible combinations of in-phase and quadrature components of signal points in the original constellation S. Therefore, for any couple of distinct signal points s and $\hat{s}$ from the original constellation, one can find two distinct signal points u and $\hat{u}$ in the expanded constellation with the same quadrature components, while they inherit their in-phase components from s and $\hat{s}$, respectively. Hence, the minimum distance of the expanded constellation can be bounded as follows:

$$\begin{aligned} d_{min}(\Lambda) &\leq |u-\hat{u}|^2 \\ &= (\Re(u)-\Re(\hat{u}))^2+(\Im(u)-\Im(\hat{u}))^2 \\ &= (\Re(u)-\Re(\hat{u}))^2 \\ &= (\Re(s)-\Re(\hat{s}))^2 \quad \forall s,\hat{s}\in S \end{aligned} \tag{16}$$

Using a similar approach:

$$d_{min}(\Lambda) \leq (\Im(s)-\Im(\hat{s}))^2 \quad \forall s,\hat{s}\in S. \tag{17}$$

To derive the corresponding upper bound for P_{coop}, the union bound can be applied to the conditional error probability $P_{coop|h_{sd},h_{rd}}$. Hence,

$$P_{coop|h_{sd},h_{rd}} \leq \frac{1}{|S|}\sum_{s\in S}\sum_{\hat{s}\neq s} P(s\rightarrow\hat{s}|h_{sd},h_{rd}). \tag{18}$$

Combining (17) and (16), the conditional pairwise error probability in (18) is given as follows:

$$\begin{aligned} P(s\rightarrow\hat{s}|h_{sd},h_{rd}) &= Q\left(\sqrt{\frac{E_s}{2N_0}(h_{sd}^2(\Re(s)-\Re(\hat{s}))^2+h_{rd}^2(\Im(s)-\Im(\hat{s}))^2)}\right) \\ &\leq Q\left(\sqrt{\frac{E_s}{2N_0}(h_{sd}^2 d_{min}(\Lambda)+h_{rd}^2 d_{min}(\Lambda))}\right) \\ &\overset{(b)}{\leq} \exp\left(-\frac{E_s d_{min}(\Lambda)(h_{sd}^2+h_{rd}^2)}{4N_0}\right), \end{aligned} \tag{19}$$

where (b) follows from Chernov bound. Hence, the upper bound on the error probability can be expressed as:

$$\begin{aligned} P_{coop} &= \iint P_{coop|h_{sd},h_{rd}} f(h_{sd},h_{rd}) dh_{sd} dh_{rd} \\ &\leq \frac{1}{|S|} \iint \sum_{s\in S}\sum_{\hat{s}\neq s} \exp\left(-\frac{E_s d_{min}(\Lambda)(h_{sd}^2+h_{rd}^2)}{4N_0}\right) \\ &\quad \times 2h_{sd}\ e^{-h_{sd}^2}\ .\ 2h_{rd}\ e^{-h_{rd}^2}\ dh_{sd}\ dh_{rd} \\ &= \frac{|S|-1}{\left(1+\frac{E_s d_{min}(\Lambda)}{4N_0}\right)^2}. \end{aligned} \tag{20}$$

B. Diversity Gain Analysis

To study the diversity gain of the SSC scheme, the asymptotic expression of the error probability is derived. As SNR increases (i.e. $SNR = \frac{E_s}{N_0} \gg 1$), according to Equation (14) P_r becomes negligible. Thus, the error probability of Equation (10) can be written as follows:

$$\begin{aligned} P_e &\leq P_r.P_f + P_{coop} \\ &\leq \frac{(|\Lambda|-1)(|S|-1)}{\left(1+\frac{E_s d_{min}(\Lambda)}{4N_0}\right)\left(1+\frac{E_s d_{min}(S)}{4N_0}\right)} + \frac{|S|-1}{\left(1+\frac{E_s d_{min}(\Lambda)}{4N_0}\right)^2} \\ &\simeq \frac{(|\Lambda|-1)(|S|-1)}{\left(\frac{E_s d_{min}(\Lambda)}{4N_0}\right)\left(\frac{E_s d_{min}(S)}{4N_0}\right)} + \frac{|S|-1}{\left(\frac{E_s d_{min}(\Lambda)}{4N_0}\right)^2} \\ &= \left(\frac{E_s}{4N_0}\right)^{-2}\left(\frac{(|\Lambda|-1)(|S|-1)}{d_{min}(\Lambda)d_{min}(S)} + \frac{|S|-1}{d_{min}(\Lambda)^2}\right) \end{aligned} \tag{21}$$

From (21), it is observed that the error probability of the SSC scheme scales with SNR^{-2} meaning that the proposed scheme achieves the diversity order of two. In more details, this scheme can exploit full system diversity regardless of the source-relay channel condition, provided the transmitted signal at the broadcast phase carries enough information for the relay to uniquely decode the transmitted message. Hence, the diversity gain design criterion for the SSC scheme is proposed as follows:

- *Diversity gain design criterion:* To exploit system full diversity, any couple of distinct signal points $s, \hat{s} \in S$, should have different in-phase and quadrature components that distinguish s from $\hat{s}$.

C. Coding Gain

In the relaying schemes, the best performance is achieved when the relay can cooperate effectively during the communication process. The relay's capability to cooperate during each course of transmission is directly affected by the source-relay channel performance. The coding gain design criterion is proposed to select an appropriate transformation Θ in order to maximize the relay's contribution to the system's overall performance. The basic idea is to increase the probability of decoding the original message at the relay during the broadcast phase. According to the SSC scheme, the source uses the expanded constellation (Λ) to generate the broadcast phase signal. Hence, the likelihood of decoding the original message at the relay is directly affected by the performance of the expanded constellation. On the other hand, maximizing the minimum distance among the signal points in the expanded constellation, can effectively reduce the error probability at the relay and contributes to the overall system performance. This observation is supported by the Equation (21). Therefore, the coding gain design criterion is proposed as follows:

- *Coding gain design criterion:* Among transformations that satisfy the diversity gain design criterion, the one that maximizes the minimum distance among signal points of the expanded constellation should be selected.

D. A Case Study: Constellation Design for QPSK Signal Set

To get better understanding of how the design criteria can be applied in the system design, an example of a system with a QPSK constellation is presented. Figure 4 shows how the rotation Θ is applied to an ordinary QPSK constellation to build the rotated constellation S and the expanded constellation Λ. To satisfy the diversity gain design criterion, each member of the rotated constellation S, must have unique in-phase and quadrature components. In other words:

$$|\Re(S)| = |\Im(S)| = |S|,$$

$$|\Lambda| = |\Re(S)| \times |\Im(S)| = |S|^2.$$

To apply the coding gain design criterion, the minimum distance of the expanded constellation needs to be maximized. It is easy to show that the expanded constellation is symmetric with respect to both the real and the imaginary axes. Therefore, for the system setup in Figure 4, two distance parameters

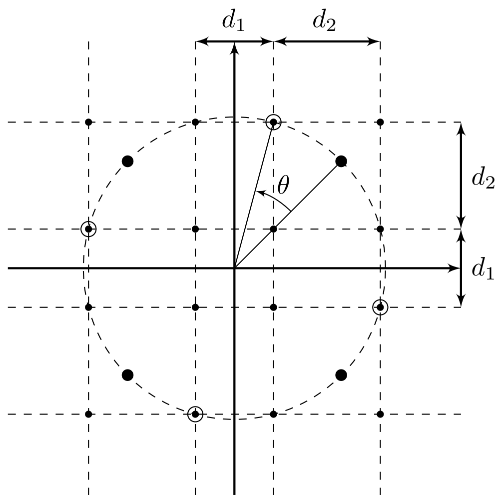

Fig. 4. Constellation expansion using a QPSK generator set : (•) QPSK Constellation, (○) Rotated - QPSK Constellation, and (.) Expanded Constellation.

d_1 and d_2 are the only parameters needed to be considered. d_1 and d_2 are defined as:

$$d_1 = 2r\cos(\theta + \frac{\pi}{4}), \quad (22)$$

$$d_2 = r\sin(\theta + \frac{\pi}{4}) - r\cos(\theta + \frac{\pi}{4}), \quad (23)$$

$r = \sqrt{E_s}$. Furthermore, because of the system's symmetry with respect to the real and the imaginary axes, all possible values of d_1 and d_2 are observed as θ changes in the range from $0°$ to $45°$. Hence, to apply the coding gain design criterion, it is sufficient to find a rotation angle between $0°$ and $45°$ that maximizes the minimum distance of the expanded constellation. Evidently, such a rotation angle must satisfy the diversity gain design criterion for the SSC scheme to perform properly. Hence:

$$\begin{aligned}\theta &= \arg \max_{\theta:|\Lambda|=|S|^2} d_{min}(\Lambda) \\ &= \arg \max_{\theta:|\Lambda|=|S|^2} \min(d_1, d_2) \\ &= \begin{cases} \arg\max(d_1) & d_1 \leq d_2 \quad (\ i.e.\ \theta \in [26.6, 45)\ ) \\ \arg\max(d_2) & d_2 \leq d_1 \quad (\ i.e.\ \theta \in (0, 26.6]\ ) \end{cases}\end{aligned}$$

Since d_1 is a decreasing function of θ and d_2 is an increasing function of θ in the corresponding domains,

TABLE I

ROTATION ANGLES FOR DIFFERENT CONSTELLATIONS

Constellations	Design Criteria θ (degree)	Product Distance Approach θ (degree)
QPSK	26.6°	31.7°
8-PSK	13.8°	22.5°
16-QAM	14.0°	31.7°
64-QAM	7.1°	31.7°

the solution of the aforementioned optimization problem can be written as follows:

$$\begin{aligned}\theta &= \arg \max_{\theta:|\Lambda|=|S|^2} d_{min}(\Lambda) \\ &= 26.6^\circ.\end{aligned}$$

As the constellation size increases, the problem of finding a rotation angle that maximizes the minimum distance of the expanded constellation becomes more complex. However, such a rotation angle depends only on the constellation shape and it does not change with SNR or channel condition. Therefore, once it is found it can be used along the course of communication. A list of rotation angles that are derived based on the proposed design criteria is shown in Table I. These angles are calculated by exhaustive search to select the best rotation that maximizes the minimum distance of the expanded constellation and, satisfies the diversity gain design criterion concurrently.

According to the table, it is evident that the suggested rotation angles for the SSC scheme depend on the original constellations. It is also observed that the proposed design criteria select different rotation angles when compared with those that are calculated based on maximizing the smallest product distance between any two points of the constellation [31]. This difference comes from the fact that the proposed coding gain design criterion aims at maximizing the relay role in the cooperative scheme, while the product distance approach aims at maximizing the overall performance when the relay is successful in decoding the message from the broadcast phase signal. The product distance approach performs much better if the relay enjoys an ideal source-relay link. The effect of choosing different rotation angles on the system performance is studied in section V using simulation results.

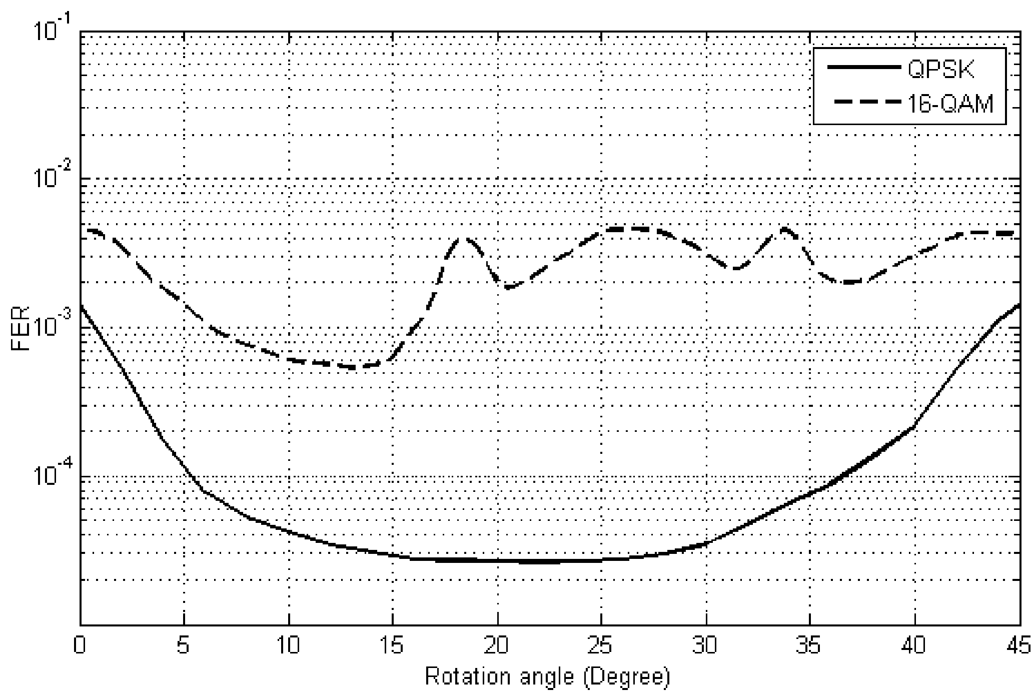

Fig. 5. Performance of the SSC scheme with different rotation angles: QPSK and 16-QAM constellations, SNR = 30dB

V. Numerical Results

Through performance evaluation, a frame of 260 information bits concatenated with 16 bits of CRC code is considered in order to enable the relay to identify corrupted frames. Each frame is encoded with a $R = 1/3$ convolutional channel code which is characterized by a generator matrix G=[05;05;07] in octet notation [13]. It is noted that the SSC scheme does not rely on the channel code to perform. However, to be more consistent with practical communication system models, the simulation model is designed so that the transmitter uses the channel code to transmit its signal. The source and the relay are assumed to have the same transmit power. Full channel state information at the receiver is also assumed.

The adaptive decode and forward (ADF) approach [6] is used as the base line in all simulation scenarios. In this approach the source sends the main message in the broadcast phase for both the relay and the destination. In case the relay is able to correctly recover the source message, it transmits an additional copy of the original signal in the relay phase. Otherwise, the source re-transmits its signal to the destination during the relay phase of the transmission block. In order to maintain the same spectral

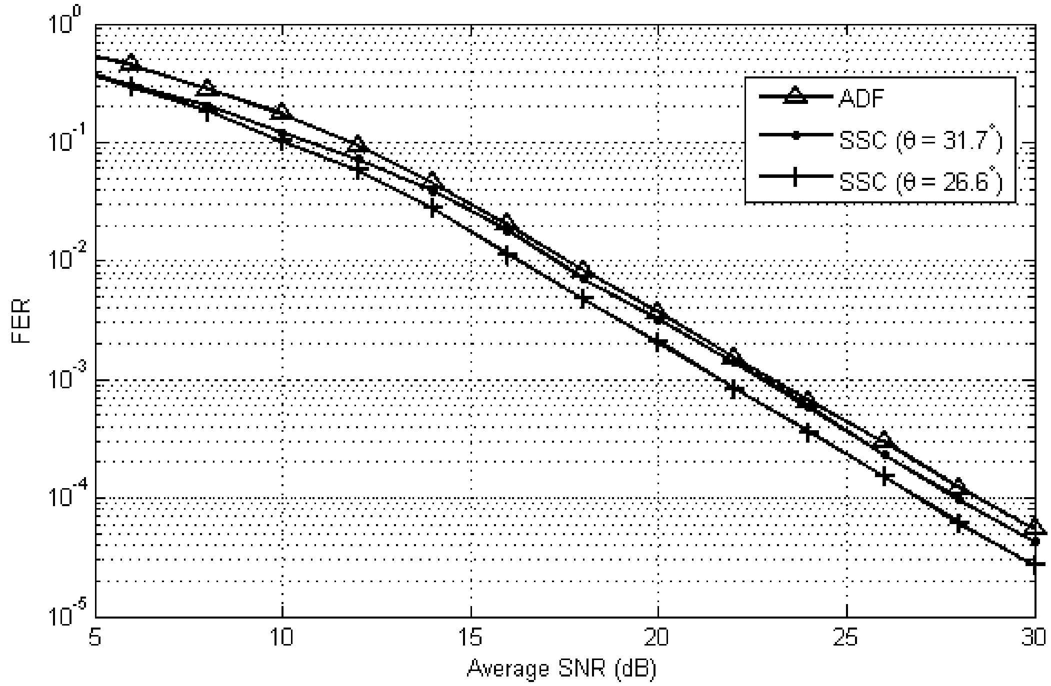

Fig. 6. FER performance of the SSC scheme for the rotation angles based on the proposed design criteria and product distance approach: QPSK constellation

efficiency and rate, the size of the constellation of the ADF approach should be equal to the size of the expanded constellation Λ in the SSC scheme. In addition, the performance of the SSC scheme is compared to the punctured PCCC cooperative scheme [13]. The rate 1/3 punctured PCCC is generated by the aforementioned convolutional encoder and then punctured to generate a rate 2/3 code to be transmitted in the broadcast phase. The complementary punctured subset is generated in order to be sent during the relay phase using an interleaved version of the decoded message at the end of the broadcast phase. Both the broadcast phase and the relay phase signals are combined at the destination to increase the likelihood to correctly decode the source message. In order to decode the transmitted message at the destination, the received codeword is passed to an iterative decoder with eight iteration cycles.

The effect of the rotation angle θ on the performance of the SSC scheme is shown in Figure 5. The graph depicts the variation at the destination's frame error rate for different rotation angles at SNR = 30dB. From the graph, it is observed that the effect of the rotation angle becomes more complex as the size of the constellation increases. Figure 5 supports the proposed design criteria since the best performance is achieved at the same rotation angle that is suggested by the proposed design criteria (See Table I). For the QPSK constellation, all of the presented rotation angles except for the two extreme points (i.e. $\theta = 0°$ and $\theta = 45°$) satisfy the diversity gain design criterion. Therefore, the variation is due to the coding gain design criterion and the minimum distance of the expanded constellation. On the other hand, for the 16-QAM constellation, more rotation angles violate the diversity gain design criterion (e.g. $\theta = 18.3°$ and $\theta = 26.5°$) causing a significant drop in the system's overall performance. Such a violation occurs when there exist at least two distinct signal points in the original constellation S with the same in-phase or quadrature components. This phenomena forces some signal points in the expanded constellation Λ to overlap. Consequently, the relay capability to recover the original message from the broadcast phase signal is disrupted leading to constant failure for the relay to cooperate in the relay phase.

Figure 6 depicts the frame error rate of the SSC scheme and ADF scheme for a QPSK constellation in a Rayleigh fading channel. According to the graph, the SSC scheme outperforms the ADF while both achieving a diversity gain of two. Figure 6 also depicts the effect of the rotation angle θ on the system's overall performance. From the graph, it is concluded that the system with the rotation angle that satisfies the proposed design criteria (i.e. $\theta = 26.6°$), performs much better than the system using the rotation angle calculated based on the product distance approach (i.e. $\theta = 31.7°$).

The performance of the SSC scheme for 16-QAM constellation is illustrated in Figure 7. The graph shows that the SSC scheme holds its superior performance when compared with the ADF scheme as the constellation size increases. Based on Figure 7, it is observed that the effect of choosing a proper rotation angle intensifies as the constellation size increases. In more detail, the system with the rotation angle from the design criteria (i.e. $\theta = 14°$) outperforms the system that utilizes the product distance criterion (i.e. $\theta = 31.7°$) by more than 3dB as SNR increases. These results support the proposed design criteria and outline the effect of proper selection of the transformation Θ over the system's overall performance.

Figure 8 depicts the performance comparison of the SSC scheme with the punctured PCCC cooperative scheme. From the graph it is observed that the SSC scheme outperforms the punctured PCCC cooperative scheme in low to medium SNRs while both schemes show a competitive performance as the SNR increases. It is mentioned that the punctured PCCC cooperative scheme requires iterative decoding

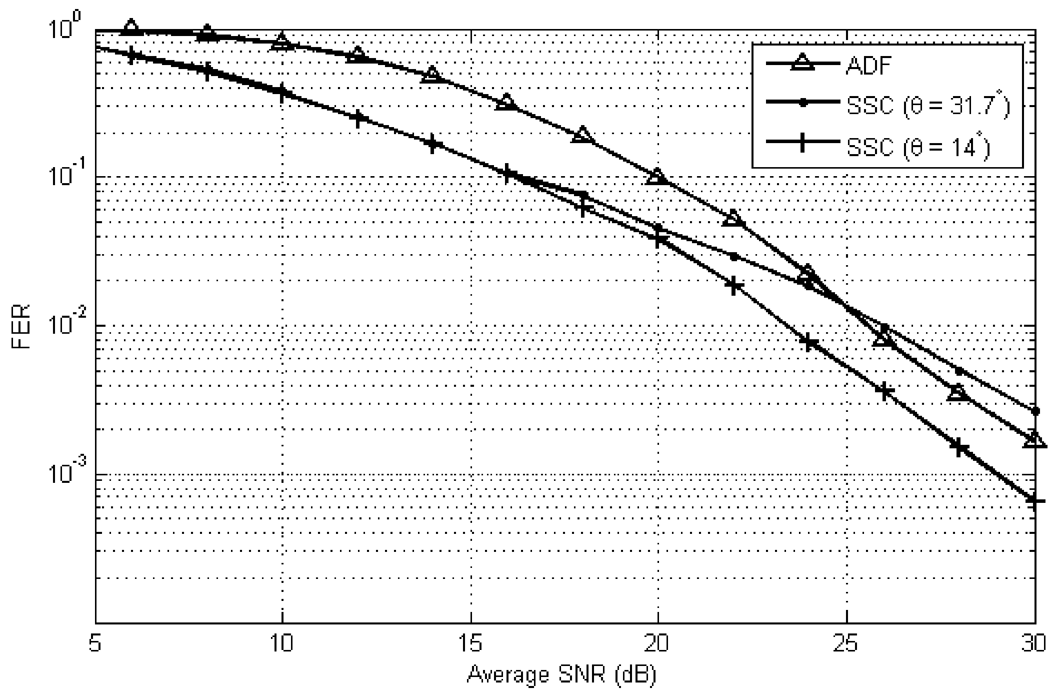

Fig. 7. FER performance of the SSC scheme for the rotation angles based on the proposed design criteria and product distance approach: 16-QAM constellation

at the destination that may cause an excessive decoding delay and increases power consumption. In this paper, the performance of the SSC scheme is evaluated with simple viterbi decoding algorithm at the relay and the destination however, it is possible to integrate a complex iterative decoder with the SSC scheme and improve the performance of the system even further if required. The performance of the SSC scheme and the punctured PCCC cooperative scheme with perfect source-relay link is presented in Figure 9. According to the graph, as the source-relay link performance improves, the SSC scheme outperforms the puncture PCCC cooperative scheme by 1 dB even for relatively high SNRs. In other words, in an ideal cooperation case, the SSC scheme performs much better than the punctured PCCC cooperative scheme both from the frame error rate and the complexity perspectives.

As mentioned earlier, the SSC scheme does not rely on channel coding to perform since the cooperation is achieved in the system signal space. In [17] a trans-modulation approach was proposed in which the relay reassigns the constellation points so that the destination performance is improved. In

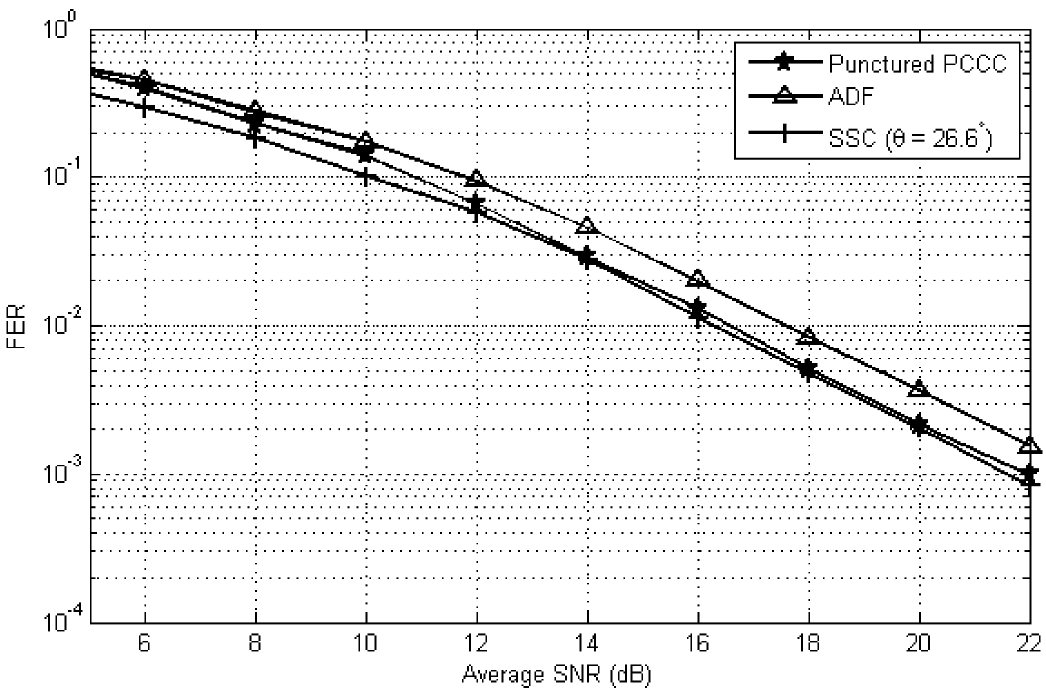

Fig. 8. FER performance comparison of the SSC scheme with ADF and Punctured PCCC: QPSK constellation

other words, the relay decodes the broadcast phase signal and send the matching signal point from the relabeled constellation to the destination. The reassignment at the relay is designed so that by combining the original and the relabeled constellation points the destination's performance improves. Figure 10 depicts the performance comparison of the SSC scheme and the trans-modulation scheme based on the constellation reassignment that is proposed in [17]. The numerical results of Figure 10 are calculated based on the uncoded signal transmission from the source and the relay. Moreover, the broadcast phase and the relay phase signals contains only one symbol during each course of the transmission. The trans-modulation approach uses a QAM-16 constellation which has to be the same size as the expanded constellation Λ in the SSC scheme in order to maintain the same spectral efficiency and rate.

According to Figure 10, the SSC scheme outperforms the trans-modulation scheme regardless of the source-relay link performance. In other words, the SSC scheme provides a gain of about 1 dB when compared with the trans-modulation scheme. In addition, from the graph it is observed that the BER performance of the trans-modulation scheme, when the relay enjoys a better average received SNR than

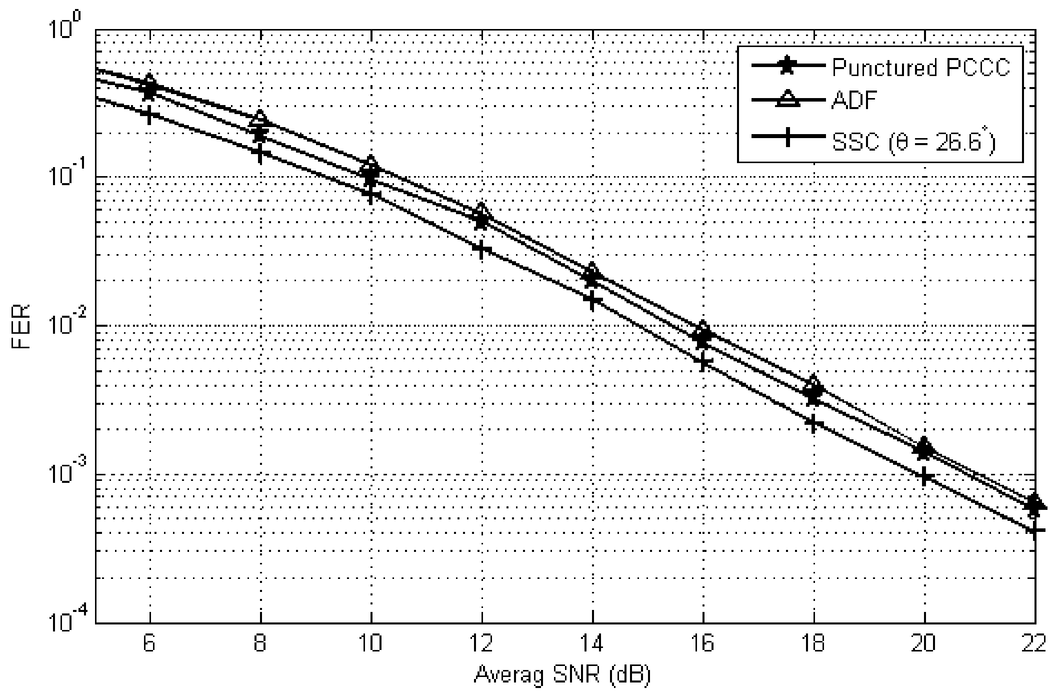

Fig. 9. FER performance comparison of the SSC scheme with ADF and Punctured PCCC (Perfect source-relay link): QPSK constellation

the destination (i.e. $SNR_{sr} = 10SNR_{sd}$), is comparable to the performance of the SSC scheme when the source-relay link has the same condition as the source-destination link.

VI. Conclusion

In this paper, a cooperative scheme based on signal space diversity was proposed in which the cooperation is achieved in modulation signal space. In this way, a constellation expansion method that enables the relay to recover the original message from the broadcast phase signal was proposed. Based on the recovered message, the relay can generate additional information to be sent to the destination during the relay phase. The broadcast and the relay phase signals were designed to boost the likelihood of decoding the original message at the destination when combined together. Moreover, a feedback strategy was proposed to combat performance degradation from the relay failure in decoding the original message during the broadcast phase. The proposed signal space cooperative framework may be extended to larger constellation designs without incurring any significant complexity to the system. The analytical and the

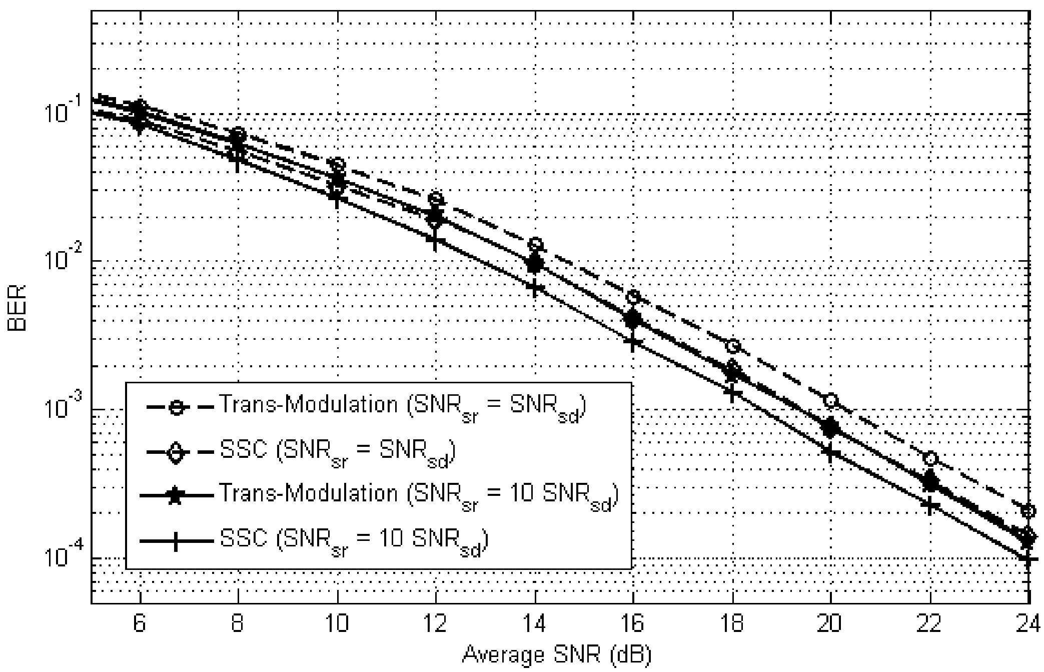

Fig. 10. BER performance comparision of the SSC scheme and the trans-modulation scheme for different source-relay link conditions

simulation results show that the proposed scheme can exploit full system diversity regardless of the source-relay link performance.

Based on the analytical study of the proposed scheme, two design criteria were proposed in order to maximize the system overall performance. The numerical results verify that the proposed design criteria can improve the performance of the SSC scheme especially when the constellation size increases. The simulation results also show that the proposed scheme improves the system performance when compared with repetition coding and distributed turbo coded cooperative schemes both from error probability and complexity perspectives.

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