Coexistence in Wireless Decentralized Networks

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Abstract

We consider a wireless communication network with a fixed number of frequency sub-bands to be shared among several transmitter-receiver pairs. In traditional frequency division (FD) systems, the available sub-bands are partitioned into disjoint clusters (frequency bands) and assigned to different users (each user transmits only in its own band). If the number of users sharing the spectrum is random, this technique may lead to inefficient spectrum utilization (a considerable fraction of the bands may remain empty most of the time). In addition, this approach inherently requires either a central network controller for frequency allocation, or cognitive radios which sense and occupy the empty bands in a dynamic fashion. These shortcomings motivate us to look for a decentralized scheme (without using cognitive radios) which allows the users to coexist, while utilizing the spectrum efficiently. We consider a frequency hopping (FH) scheme (with $\mathrm{i.i.d.}$ Gaussian code-books) where each user transmits over a selection of sub-bands and hops to another selection (with the same cardinality) from transmission to transmission. Developing an upper bound on the differential entropy of a mixed Gaussian random vector and via entropy power inequality, we offer a lower bound on the achievable rate of each user in the proposed scheme. Thereafter, in a setup where the number of active users and all the channel gains are unknown to transmitters, we obtain the maximum transmission rate per user to ensure a specified outage probability at a given SNR level. We demonstrate that "outage capacity" can be considerably higher in FH than the case of FD for reasonable distributions on the number of active users which guarantees a higher spectral efficiency in FH.

Index Terms

Frequency Hopping, Spectrum Sharing, Decentralized Networks, Mixed Gaussian Interference, ϵ-Outage Capacity

I. INTRODUCTION

Optimal resource allocation is an imperative issue in wireless networks. When multiple users share the same spectrum, the destructive effect of multi-user interference can limit the achievable rates. As such, an effective and low complexity frequency sharing strategy which maximizes the degrees of freedom per user, while mitigating the impact of the multi-user interference is desirable. In frequency division (FD) systems, different users transmit over disjoint frequency bands. Due to practical considerations, such FD systems usually rely on a fixed number of such frequency bands. The main drawback of FD systems is that most of the time the majority of the potential users may be inactive, reducing the resulting spectral efficiency. Reference [1] considers a network of several users with mutual interference. Treating the interference as noise, a central controller computes the optimum power allocation of each link over the spectrum to maximize a global utility function. This leads to the best spectrum sharing strategy for a specific number of users. Clearly, if the number of users changes, the system is not guaranteed to offer the best possible spectral efficiency. In fact, it is shown in [1] that if the crossover gains are sufficiently greater than the forward gains, frequency division is optimum. However, as mentioned earlier, if the number of users sharing the spectrum is random, FD systems can be highly inefficient in terms of the overall spectral efficiency. To avoid the need for a central controller, cognitive radios [2] are introduced which can sense the bands and transmit over an unoccupied portion of the available spectrum. Fundamental limits of wireless networks with cognitive radios are studied in [3]–[7]. Although cognitive radios avoid the use of a central controller, they require methods for frequency sensing and dynamic frequency assignment which add to the overall system complexity. For example, in opportunistic communication, each cognitive device must search for idle regions of the spectrum or spectrum holes which requires sophisticated detection techniques [8]–[10]. Noting the above points, it is desirable to have a decentralized frequency sharing strategy (without the need for cognitive radios) which allows the users to coexist, while utilizing the spectrum efficiently and fairly.

Distributed strategies based on game theoretic arguments have already attracted a great deal of attention [1], [7], [32], [11]–[14]. In [1], the authors introduce a non-cooperative game theoretic framework to investigate the spectral efficiency issue when several users compete over an unlicensed band with no central controller. This approach has been recently followed more in [11] to the case where the users have incomplete information about the channel gains in the network. This setup is more relevant to a fast fading environment. Reference [12] offers a brief overview of game theoretic dynamic spectrum sharing. Repeated non-cooperstive market game methods are adopted in [13] for resource allocation in a decentralized network. Recently, in [14] the interaction between multiple random access networks has been considered from a game theoretic point of view.

Generally speaking, the problem of distributed spectrum sharing is studied from two different viewpoints. Many authors consider dynamic usage of idle parts of the spectrum that has been already allocated to primary licensees [26]–[31]. An elegant approach to make this feasible is utilizing software defined or cognitive radios. In [30], three different operational models for cognitive radios are mentioned, i.e., overlay, underlay and opportunistic interweaving. In [31], motivated by the fact that the interference imposed on the primary users by the secondary users must be below a certain level, the concept of capacity is studied under constraints at the receiver instead of transmitter. Subsequently, authors in [32] show how the interference effect of the secondary users can be mitigated in fading environments as deep fading on the cross over links is to the benefit of the licensees. Another scenario is sharing the spectrum by a certain number of users competing over a certain open bandwidth [34]–[37]. In [37], through an asynchronous distributed pricing scheme, users exchange signals that indicate the negative effect of interference at the receivers. In [36], users affected by the mobility event, self-organize into bargaining groups and adapt their spectrum assignment to approximate a new optimal assignment.

On the other hand, spread spectrum (SS) communications is a natural setup to share the same bandwidth by several users. This area has attracted tremendous attention by different authors during the past decades in the context of centralized uplink/downlink multiuser systems. Appealing characteristics of SS systems have motivated researchers to utilize these schemes in networks without a certain infrastructure, i.e., packet radio or ad-hoc networks [15]. In direct sequence spread spectrum (DSSS) systems, the signal of each user is spread using a pseudo-random noise (PN) code. The challenging point is that the PN codes used by different users must have relatively small cross-correlation properties. In a network without a central controller, if two users use the same spreading code, they will not be capable to recover the data at the receiver side. Distributed code assignment techniques are developed in [38], [39]. In [38], using a greedy approximation algorithm and invoking graph theory, a distributed code assignment protocol is suggested. Another category of research is devoted to devise distributed schemes in the reverse link (uplink) of cellular systems. Distributed power assignments algorithms are proposed in [41], [42]. Reference [43] proposes a distributed scheduling method called “the token-bucket on-off scenario” utilized by autonomous mobile stations where its impact on the overall throughput of the reverse link is investigated. On the other hand, decentralized rate assignments in a multi-sector CDMA wireless network are discussed in [44].

Being a standard technique in spread spectrum communications and due to its interference avoidance nature, frequency hopping is the simplest spectrum sharing method to use in decentralized networks. As different users typically have no prior information about the codebooks of the other users, the most efficient method is avoiding interference by choosing unused channels. As mentioned earlier, searching the spectrum to find spectrum holes is not an easy task due to the dynamic spectrum usage. As such, frequency hopping is a realization of transmission without sensing while avoiding the collisions as much as possible.

Frequency hopping is one of the standard signaling schemes [15] adopted in ad-hoc networks. In short range scenarios, bluetooth systems [19]–[21] are the most popular examples of a wireless personal area network or WPAN. Using frequency hopping over the unlicensed ISM band, a bluetooth system provides robust communication to unpredictable sources of interference. A modification of frequency hopping called dynamic frequency hopping (DFH), selects the hopping pattern based on interference measurements in order to avoid dominant interferers. The performance of a DFH scheme when applied to a cellular system is assessed in [22]–[24]. Frequency hopping is also proposed in [7] in the context of cognitive radios where each cognitive transmitter selects a frequency band but quits transmitting if the band is already occupied by a primary user.

In this paper, we consider a decentralized party of N users sharing u discrete frequency sub-bands via fast frequency hopping. Different transmitters are linked to different receivers through paths with static and non-frequency-selective fading. Each user is assumed to have no prior knowledge about the code-books of the other users. We propose a frequency hopping (FH) strategy in which the i^{th} user selects v_i frequency sub-bands among the u available sub-bands and hops to another set of v_i sub-bands for the next transmission. It is assumed that all users transmit independent Gaussian code-books over their chosen frequency sub-bands.

As each user hops over different subsets of the sub-bands without informing other users about its hopping pattern, sensing the spectrum to track the instantaneous interference is a difficult task. This assumption makes the interference probability density function (PDF) on each frequency sub-band at the receiver side of each user be mixed Gaussian. Since the channel gains have a continuous PDF, the number of Gaussian components in the interference PDF is 2^{N-1} with probability one. It is presumed that each user is able to derive the interference PDF after a sufficiently long training period at the receiver side.

Deriving an upper bound on the differential entropy of a mixed Gaussian random vector and invoking entropy power inequality (EPI), we propose a lower bound on the achievable rate of each user in the FH system. Our proposed lower bound is tight enough to have the same SNR scaling of the achievable rate itself as SNR goes to infinity. Thereafter, in case no transmitter has the necessary knowledge about the channel gains and the number of users in the system, we derive lower bounds on ϵ-outage capacity defined as

$$R(\epsilon) = \sup\{R : \Pr\{\mathrm{Outage}\} < \epsilon\},$$

i.e., the maximum transmission rate for each user ensuring an outage probability below ϵ. Fixing ϵ, simulation results show that $R_{FH}(\epsilon)$ can be much greater than $R_{FD}(\epsilon)$ for regular SNR values and reasonable distributions on the number of active user in the system. We emphasize that the randomness of the number of users contributes to the outage event.

The paper outline is as follows. System model is given in section II. Section III is devoted to derive lower bounds on the achievable rates of users. In section IV, based on the results in section III, we discuss how the users in the FH system fairly share the band while maximizing the outage capacity. Comparison between the FH and FD systems is given in section V through simulation results. In section VII, the case where the number of active users is a global knowledge is considered. We use the notation $f(\gamma) \sim g(\gamma)$ implying $\lim_{\gamma\to\infty} \frac{f(\gamma)}{g(\gamma)} = 1$ throughout the paper.

II. System Model

We consider a communication system with N users[1] where the i^{th} user exploits $v_i(\leq u)$ out of the u sub-bands and spreads its available power, P, equally over these selected sub-bands by transmitting independent Gaussian signals of variance $\frac{P}{v_i}$ over each of the chosen sub-bands. This user hops to another set of v_i frequency sub-bands after each transmission. We denote the achievable rate of the i^{th} user by $\mathscr{R}_i$. The static and non frequency-selective fading coefficient of the link connecting the i^{th} transmitter to the j^{th} receiver is shown by $h_{i,j}$. Each receiver knows already the hopping pattern of its affiliated transmitter. On the other hand, as all users hop over different portions of the spectrum from transmission to transmission, no user is assumed to be capable of tracking the instantaneous interference. This assumption makes the interference plus noise PDF at the receiver side of each user be a mixed Gaussian distribution. In fact, depending on different choices the other users make to select the frequency sub-bands and values of the

[1] Each user consists of a transmitter-receiver pair.

crossover gains, the interference on each frequency sub-band at the receiver side of any user has up to 2^{N-1} power levels. For each i, the channel model for the i^{th} user is as follows:

$$\vec{Y}_i = h_{i,i}\vec{X}_i + \vec{Z}_i \tag{1}$$

where $\vec{X}_i$ is the $u \times 1$ input vector of the i^{th} user and $\vec{Z}_i$ is the noise plus interference vector on the receiver side of the i^{th} user. One may write $p_{\vec{X}_i}(\vec{x}) = \sum_{C \in \mathcal{C}} \frac{1}{\binom{u}{v_i}} g(\vec{x}, C)$ where $g(\vec{x}, C)$ denotes a zero-mean jointly Gaussian distribution of covariance matrix C and the set $\mathcal{C}$ includes all $u \times u$ diagonal matrices where v_i out of the u diagonal elements are $\frac{P}{v_i}$ while the rest are zeros. Denoting the noise plus interference on the j^{th} sub-band at the receiver side of the i^{th} user by $Z_{i,j}$ (the j^{th} component of $\vec{Z}_i$), it is clear that $p_{Z_{i,j}}(z)$ is not dependent on j. This is by the fact that crossover gains are not sensitive to frequency and there is no particular interest to a specific frequency sub-band by any user. We assume there are $L_i + 1$ ($L_i \leq 2^{N-1} - 1$) possible non-zero power levels for $Z_{i,j}$, say $\{\sigma_{i,l}^2\}_{l=0}^{L_i}$. The occurrence probability of $\sigma_{i,l}^2$ is denoted by $a_{i,l}$. Then, $p_{Z_{i,j}}(z)$ is a mixed Gaussian distribution as follows:

$$p_{Z_{i,j}}(z) = \sum_{l=0}^{L_i} \frac{a_{i,l}}{\sqrt{2\pi}\sigma_{i,l}} \exp -\frac{z^2}{2\sigma_{i,l}^2} \tag{2}$$

where $\sigma^2 = \sigma_{i,0}^2 < \sigma_{i,1}^2 < \sigma_{i,2}^2 < \ldots < \sigma_{i,L_i}^2$ (σ^2 is the ambient noise power). In fact, one may write $Z_{i,j} = \sum_{k=1,k\neq i}^{N} \epsilon_{k,j} h_{k,i} X_{k,j} + \nu_{i,j}$ where $X_{k,j}$ is the signal of the k^{th} user sent on the j^{th} sub-band, $\epsilon_{k,j}$ is a Bernoulli random variable showing if the k^{th} user has utilized the j^{th} sub-band and $\nu_{i,j}$ is the ambient noise which is a zero-mean Gaussian random variable with variance σ^2. Obviously, $\Pr\{\epsilon_{k,j} = 1\} = \frac{v_k}{u}$. Also, a quantity of interest would be the following:

$$a_{i,0} = \Pr\{Z_{i,j} \text{ contains no interference}\} = \prod_{k\neq i} \Pr\{\epsilon_{k,j} = 0\} = \prod_{k\neq i} (1 - \frac{v_i}{u}). \tag{3}$$

We notice that for each $l \geq 1$, there exists a $c_{i,l} > 0$ such that $\sigma_{i,l}^2 = \sigma^2 + c_{i,l}P$ where $c_{i,1} < c_{i,2} < \ldots < c_{i,L_i}$. To compute $\mathscr{R}_i$, one may see that for each i, the communication channel of the i^{th} user is a channel with state S_i, the hopping pattern, which is independently changing over different transmissions and is known to both the transmitter and receiver ends of the i^{th} user. The achievable rate of such a channel is given by

$$\mathscr{R}_i = I(\vec{X}_i; \vec{Y}_i | S_i) = \sum_{s_i \in \mathcal{S}_i} \Pr(S_i = s_i) I(\vec{X}_i; \vec{Y}_i | S_i = s_i) \tag{4}$$

where $I(\vec{X}_i; \vec{Y}_i | S_i = s_i)$ is the mutual information between $\vec{X}_i$ and $\vec{Y}_i$ for the specific sub-band selection dictated by $S_i = s_i$. The set $\mathcal{S}_i$ denotes all possible selections of v_i out of the u sub-bands. As $p_{\vec{Z}_i}(\vec{z})$ is a symmetric density function, meaning all its components have the same PDF given in (2), we deduce that $I(\vec{X}_i; \vec{Y}_i | S_i = s_i)$ is independent of s_i. Therefore, we may assume any specific sub-band selection for the i^{th} user in $\mathcal{S}_i$, say the first v_i out of the u sub-bands. Denoting this specific state by s_i^*, we get:

$$\mathscr{R}_i = I(\vec{X}_i; \vec{Y}_i | S_i = s_i^*) \tag{5}$$

In this case, we denote $\vec{Y}_i$ and $\vec{X}_i$ by $\vec{Y}_i(s_i^*)$ and $\vec{X}_i(s_i^*)$ respectively. Obviously, we have:

$$\mathscr{R}_i = I(\vec{X}_i(s_i^*); \vec{Y}_i(s_i^*)) = \mathrm{h}(\vec{Y}_i(s_i^*)) - \mathrm{h}(\vec{Z}_i). \tag{6}$$

III. Lower Bounds on the Rates

In this section, we derive lower bounds on the achievable rates of users. We are looking for a lower bound on $\mathscr{R}_i$ having the following possible properties:

- Achieving the same asymptotic expression as that of $\mathscr{R}_i$.
- Not depending on the interference details as much as possible.
- Being positive and within a reasonable distance to $\mathscr{R}_i$ for all ranges of SNR.

In what follows, we try to find such a lower bound. The idea behind deriving this lower bound is to invoke entropy power inequality (EPI). As we will see, this initial lower bound is not in a closed form as it depends on the entropy of a mixed Gaussian random variable. In appendix A and B, through a careful examination of such an entropy, we obtain an appropriate upper bound on it which leads us to the final lower bound on $\mathscr{R}_i$.

Let us define $\vec{X}'_i$ to be the $v_i \times 1$ signal vector[2] of the i^{th} transmitter which is sent through the first v_i chosen frequency sub-bands. Let $\vec{Y}'_i = h_{i,i}\vec{X}'_i + \vec{Z}'_i$ where $\vec{Z}'_i$ is the noise plus interference vector at the receiver side of the i^{th} user on the first v_i frequency sub-bands. According to EPI, we have:

$$2^{\frac{2}{v_i}\mathrm{h}(\vec{Y}'_i)} \geq 2^{\frac{2}{v_i}\mathrm{h}(h_{i,i}\vec{X}'_i)} + 2^{\frac{2}{v_i}\mathrm{h}(\vec{Z}'_i)}. \tag{7}$$

Dividing both sides by $2^{\mathrm{h}(\vec{Z}'_i)}$, we get:

$$\mathrm{h}(\vec{Y}'_i) - \mathrm{h}(\vec{Z}'_i) \geq \frac{v_i}{2}\log(2^{\frac{2}{v_i}\left(\mathrm{h}(h_{i,i}\vec{X}'_i) - \mathrm{h}(\vec{Z}'_i)\right)} + 1). \tag{8}$$

[2] $\vec{X}'_i$ consists of the first v_i elements of $\vec{X}_i(s_i^*)$.

On the other hand, since $\vec{Y}_i'$ is a subvector of $\vec{Y}_i(s_i^*)$, we have:

$$\mathscr{R}_i = I(\vec{X}_i(s_i^*);\vec{Y}_i(s_i^*)) \geq I(\vec{X}_i';\vec{Y}_i') = \mathrm{h}(\vec{Y}_i') - \mathrm{h}(\vec{Z}_i'). \tag{9}$$

Based on (8) and (9), we get the following lower bound on $\mathscr{R}_i$:

$$\mathscr{R}_i \geq \frac{v_i}{2}\log(2^{\frac{2}{v_i}\left(\mathrm{h}(h_{i,i}\vec{X}_i')-\mathrm{h}(\vec{Z}_i')\right)}+1). \tag{10}$$

As $\vec{Z}_i'$ is a mixed Gaussian vector, there is no closed-form formula for $\mathrm{h}(\vec{Z}_i')$. To circumvent this difficulty, we have to find an appropriate upper bound on $\mathrm{h}(\vec{Z}_i')$. A general upper bound on the entropy of a random vector is the entropy of a Gaussian vector of the same covariance matrix. But, it can be verified that this leads to a lower bound on $\mathscr{R}_i$ which is less than a constant threshold for all values of γ, and hence would not be suitable for our purposes. To find a sufficiently tight upper bound on $\mathrm{h}(\vec{Z}_i')$, we must investigate the exact PDF of $\vec{Z}_i'$. In appendix A, we have shown the following lemma:

Lemma 1 *Let $\vec{Z}$ be a $u\times 1$ mixed Gaussian random vector having the following density:*

$$p_{\vec{Z}}(\vec{z}) = \sum_{l=1}^{L}\frac{a_l}{(2\pi)^{\frac{u}{2}}\sqrt{\det C_l}}\exp -\vec{z}^T C_l^{-1}\vec{z}. \tag{11}$$

Then,

$$\frac{1}{2}\sum_{l=1}^{L}a_l\log\left((2\pi e)^u\det C_l\right) \leq \mathrm{h}(\vec{Z}) \leq \frac{1}{2}\sum_{l=1}^{L}a_l\log\left((2\pi e)^u\det C_l\right)+\mathcal{H} \tag{12}$$

where $\mathcal{H} = -\sum_{l=1}^{L}a_l\log a_l$ is the discrete entropy of $\{a_l\}_{l=1}^{L}$.

Proof: See appendix A. ■

The following lemma deals with a special class of mixed Gaussian random vectors and is quite useful in terms of yielding a tight lower bound on $\mathscr{R}_i$ having desirable properties mentioned above.

Lemma 2 *Let $\vec{Z}$ be a $u\times 1$ mixed Gaussian random vector with different covariance matrices $\{\sigma_l^2 I_u\}_{l=1}^{L}$ and corresponding probabilities $\{a_l\}_{l=1}^{L}$ where $\sigma_1^2 < \cdots < \sigma_L^2$. Then,*

$$\mathrm{h}(\vec{Z}) \leq \frac{1}{2}u\sum_{l=1}^{L}a_l\log(2\pi e\sigma_l^2)+\mathcal{H}-\mathcal{G} \tag{13}$$

where

$$\mathcal{H} = -\sum_{l=1}^{L}a_l\log a_l \tag{14}$$

and

$$\mathcal{G} = \frac{\sigma_1^u}{\sigma_L^u} \sum_{l=2}^{L} a_l \log(1 + \frac{\sigma_L^u}{\sigma_1^u} \frac{\sum_{m=1}^{l-1} a_m}{a_l}). \tag{15}$$

Proof: See appendix B. ■

Using the chain rule for the entropy function, one has the following bound:

$$\mathrm{h}(\vec{Z}_i') \leq \sum_{j=1}^{v_i} \mathrm{h}(Z_{i,j}). \tag{16}$$

Applying the special case of lemma 3 for a scalar mixed Gaussian random variable given in (2), we get:

$$\mathrm{h}(Z_{i,j}) \leq \frac{1}{2} \sum_{l=0}^{L_i} a_{i,l} \log(2\pi e \sigma_{i,l}^2) + \mathcal{H}_i - \mathcal{G}_i \tag{17}$$

where

$$\mathcal{H}_i = -\sum_{l=0}^{L_i} a_{i,l} \log a_{i,l} \tag{18}$$

and

$$\mathcal{G}_i = \frac{\sigma}{\sigma_{i,L_i}} \sum_{l=1}^{L_i} a_{i,l} \log(1 + \frac{\sigma_{i,L_i}}{\sigma} \frac{\sum_{m=0}^{l-1} a_{i,m}}{a_{i,l}}) \tag{19}$$

• $\mathcal{H}_i$ is only a function of $\{a_{i,l}\}_{l=1}^{L_i}$, i.e., it does not depend on the crossover gains. However, $\mathcal{G}_i$ is implicitly a function of all crossover gains as the partial sums $\sum_{m=1}^{l-1} a_{i,m}$ for $2 \leq l \leq L_i$ depend on the ordering of the crossover gains. This will be more investigated in lemma 4.

• The upper bound given on $\mathrm{h}(Z_{i,j})$ replacing $Z_{i,j}$ by a Gaussian random variable of the same variance is equal to to $\frac{1}{2}\log(2\pi e \sum_{l=0}^{L_i} a_{i,l}\sigma_{i,l}^2)$. In contrast, one can see that although by Jensen's inequality $\sum_{l=0}^{L_i} a_{i,l} \log(2\pi e \sigma_{i,l}^2) \leq \log(2\pi e \sum_{l=0}^{L_i} a_{i,l}\sigma_{i,l}^2)$, the upper bound $\frac{1}{2}\log(2\pi e \sum_{l=0}^{L_i} a_{i,l}\sigma_{i,l}^2)$ might still be less than the upper bound $\frac{1}{2}\sum_{l=0}^{L_i} a_{i,l} \log(2\pi e \sigma_{i,l}^2) + \mathcal{H}_i - \mathcal{G}_i$ for relatively small values of P. But, as SNR increases sufficiently, the upper bound in (17) is way tighter than $\frac{1}{2}\log(2\pi e \sum_{l=0}^{L_i} a_{i,l}\sigma_{i,l}^2)$. To see this, one simply has

$$\lim_{P \to \infty} \frac{\frac{1}{2}\sum_{l=0}^{L_i} a_{i,l} \log(2\pi e \sigma_{i,l}^2) + \mathcal{H}_i - \mathcal{G}_i}{\frac{1}{2}\log(2\pi e \sum_{l=0}^{L_i} a_{i,l}\sigma_{i,l}^2)} = 1 - a_{i,0} < 1. \tag{20}$$

This shows that as P increases the bound in (17) is a tighter upper bound than the Gaussian upper bound. By (10), (16) and (17), we have:

$$\mathscr{R}_i \geq \frac{v_i}{2} \log\left(2^{\frac{2}{v_i}\left(\frac{1}{2}\log(2\pi e \frac{|h_{i,i}|^2 P}{v_i})^{v_i} - v_i\left(\frac{1}{2}\sum_{l=0}^{L_i} a_{i,l}\log(2\pi e \sigma_{i,l}^2) + \mathcal{H}_i - \mathcal{G}_i\right)\right)} + 1\right)$$

$$= \frac{v_i}{2} \log \left(\frac{2^{-2\mathcal{H}_i} 2^{2\mathcal{G}_i} |h_{i,i}|^2 \gamma}{v_i \prod_{l=1}^{L_i} (c_{i,l}\gamma + 1)^{a_{i,l}}} + 1 \right). \tag{21}$$

Let us define

$$\mathscr{R}_{i,lb}^{(1)} = \frac{v_i}{2} \log \left(\frac{2^{-2\mathcal{H}_i} 2^{2\mathcal{G}_i} |h_{i,i}|^2 \gamma}{v_i \prod_{l=1}^{L_i} (c_{i,l}\gamma + 1)^{a_{i,l}}} + 1 \right). \tag{22}$$

$\mathscr{R}_{i,lb}^{(1)}$ is a lower bound having desirable properties listed beow.

- It is easily seen that $\mathscr{R}_{i,lb}^{(1)} \sim \frac{1}{2} v_i a_{i,0} \log \gamma$, i.e., $\mathscr{R}_{i,lb}^{(1)}$ has the same asymptotic form [54] as that of $\mathscr{R}_i$.
- $\mathscr{R}_{i,lb}^{(1)}$ is positive and continuous for all values on γ.
- It is notable that as $\prod_{l=1}^{L_i} (c_{i,l}\gamma + 1)^{a_{i,l}} \leq \prod_{l=1}^{L_i} (c_{i,L_i}\gamma + 1)^{a_{i,l}} = (c_{i,L_i}\gamma + 1)^{(1-a_{i,0})}$, one gets a looser version of the bound given in (22) as

$$\mathscr{R}_{i,lb}^{(2)} = \frac{v_i}{2} \log \left(\frac{2^{-2\mathcal{H}_i} 2^{2\mathcal{G}_i} |h_{i,i}|^2 \gamma}{v_i (c_{i,L_i}\gamma + 1)^{1-a_{i,0}}} + 1 \right). \tag{23}$$

- $\mathscr{R}_{i,lb}^{(2)}$ still has the same asymptotic expression as that of $\mathscr{R}_{i,lb}^{(1)}$.
- In case all the signal and noise components are assumed to be circular complex Gaussian (called the complex setup),

$$\mathscr{R}_{i,lb}^{(2)} = v_i \log \left(\frac{2^{-\mathcal{H}_i} 2^{\mathcal{G}_i} |h_{i,i}|^2 \gamma}{v_i (c_{i,L_i}\gamma + 1)^{1-a_{i,0}}} + 1 \right) \tag{24}$$

where $\mathcal{G}_i$ is given by:

$$\mathcal{G}_i = \frac{\sigma^2}{\sigma_{i,L_i}^2} \sum_{l=1}^{L_i} a_{i,l} \log(1 + \frac{\sigma_{i,L_i}^2}{\sigma^2} \frac{\sum_{m=0}^{l-1} a_{i,m}}{a_{i,l}}). \tag{25}$$

Let us consider a "fair" FH system in which $v_i = v$ for all $1 \leq i \leq N$. As $L_i = 2^{N-1} - 1$ with probability one, and each user selects a certain frequency band with probability $\frac{v}{u}$, the collection $\{a_{i,l}\}_{l=0}^{L_i}$ consists of the numbers $(\frac{v}{u})^i (1 - \frac{v}{u})^{N-1-i}$ repeated $\binom{N-1}{i}$ times for $0 \leq i \leq N-1$. Generally, $\mathcal{H}_i$ is only a function of $\{a_{i,l}\}_{l=0}^{L_i}$. On the other hand, $\mathcal{G}_i$ depends on details about the ordering of $\{c_{i,l}\}_{l=0}^{L_i}$. To avoid this, the following lemma introduces a lower bound on $\mathcal{G}_i$ which only depends on c_{i,L_i} i.e., the largest interference crossover gain.

Lemma 3 *In a "fair" FH system, considered in the complex setup,*

$$\mathcal{H}_i = -(N-1)\big(\frac{v}{u} \log \frac{v}{u} + (1 - \frac{v}{u}) \log(1 - \frac{v}{u})\big) \tag{26}$$

and

$$\mathcal{G}_i \geq \mathcal{G}_{i,lb} := \frac{E\Big\{ \log\Big(1 + \big(1 - (\frac{v}{u})^B\big)c_{i,L_i}\gamma\Big)\Big\} - (N-1)\frac{v}{u}\log\frac{v}{u}}{c_{i,L_i}\gamma + 1} \tag{27}$$

where B is a binomial random variable of parameters $(N-1, \frac{v}{u})$

Proof: Let us define $p = \frac{v}{u}$. By the fact stated before to the lemma, we have:

$$\begin{aligned}
\mathcal{H}_i &= -\sum_{i=0}^{N-1}\binom{N-1}{i}p^i(1-p)^{N-1-i}\log(p^i(1-p)^{N-1-i}) \\
&= -\Big(\sum_{i=0}^{N-1} i\binom{N-1}{i}p^i(1-p)^{N-1-i}\Big)\log p \\
&\quad -\Big(\sum_{i=0}^{N-1}(N-1-i)\binom{N-1}{i}p^i(1-p)^{N-1-i}\Big)\log(1-p) \\
&= -(N-1)p\log p - (N-1-(N-1)p)\log(1-p) = (N-1)\mathcal{H}(p, 1-p).
\end{aligned} \tag{28}$$

On the other hand, by (19), we have:

$$\mathcal{G}_i = \frac{1}{c_{i,L_i}\gamma + 1}\sum_{l=1}^{L_i} a_{i,l}\log\big(1 + \frac{\sum_{m=0}^{l-1} a_{i,m}}{a_{i,l}}(c_{i,L_i}\gamma + 1)\big). \tag{29}$$

Computation of $\sum_{m=0}^{l-1} a_{i,m}$ is not an easy task. In fact, it all depends on the ordering of the cross over gains. For example, if $N = 4$,

$$\sum_{m=0}^{2} a_{1,m} = \begin{cases} 2p(1-p)^2 + p^2(1-p) & \text{if } |h_{2,1}|^2 < |h_{3,1}|^2 < |h_{2,1}|^2 + |h_{3,1}|^2 < |h_{4,1}|^2 \\ 3p(1-p)^2 & \text{if } |h_{2,1}|^2 < |h_{3,1}|^2 < |h_{4,1}|^2 < |h_{2,1}|^2 + |h_{3,1}|^2 \end{cases}. \tag{30}$$

To avoid this difficulty of describing $\mathcal{G}_i$, we derive a lower bound on it which is not sensitive to the ordering of crossover gains. Taking each $a_{i,l}$, there is a $0 \leq s \leq N-1$ such that $a_{i,l} = p^s(1-p)^{N-1-s}$. This implies that $a_{i,l}$ corresponds to the interference plus noise power level $\frac{\sum_{j=1}^{s}|h_{k_j,i}|^2}{v}P + \sigma^2$ for some $1 \leq k_1 < \cdots < k_s \leq N$ where $k_j \neq i$ for $1 \leq j \leq s$. Since $\frac{\sum_{j=1}^{s}|h_{k_j,i}|^2}{v}P + \sigma^2 > \frac{\sum_{t\in\mathcal{A}\subsetneq\{1,2,\cdots,s\}}|h_{k_t,i}|^2}{v}P + \sigma^2$ for all sets $\mathcal{A} \subsetneq \{1, 2, \cdots, s\}$, and $\frac{\sum_{t\in\mathcal{A}\subsetneq\{1,2,\cdots,s\}}|h_{k_t,i}|^2}{v}P + \sigma^2$ is itself an interference plus noise power level, we can deduce that its associated probability, $p^{|\mathcal{A}|}(1-p)^{N-1-|\mathcal{A}|}$ is an element in the sequence $(a_{i,0}, a_{i,1}, \cdots, a_{i,l-1})$. Therefore, we come up with the following lower bound:

$$\sum_{m=0}^{l-1} a_{i,m} \geq \sum_{\mathcal{A}\subsetneq\{1,2,\cdots,s\}} p^{|\mathcal{A}|}(1-p)^{N-1-|\mathcal{A}|} = \sum_{s'=0}^{s-1}\binom{s}{s'}p^{s'}(1-p)^{N-1-s'}. \tag{31}$$

Using (31) in (29) yields:

$$\mathcal{G}_i \geq \frac{1}{c_{i,L_i}\gamma+1}\sum_{s=0}^{N-1}\binom{N-1}{s}p^s(1-p)^{N-1-s}\log(1+\frac{\sum_{s'=0}^{s-1}\binom{s}{s'}p^{s'}(1-p)^{N-1-s'}}{p^s(1-p)^{N-1-s}}(c_{i,L_i}\gamma+1))$$

$$=\frac{1}{c_{i,L_i}\gamma+1}\sum_{s=0}^{N-1}\binom{N-1}{s}p^s(1-p)^{N-1-s}\log(1+\frac{\sum_{s'=0}^{s-1}\binom{s}{s'}p^{s'}(1-p)^{s-s'}}{p^s}(c_{i,L_i}\gamma+1))$$

$$=\frac{1}{c_{i,L_i}\gamma+1}\sum_{s=0}^{N-1}\binom{N-1}{s}p^s(1-p)^{N-1-s}\log(1+\frac{1-p^s}{p^s}(c_{i,L_i}\gamma+1))$$

$$=-\frac{1}{c_{i,L_i}\gamma+1}\sum_{s=0}^{N-1}\binom{N-1}{s}sp^s(1-p)^{N-1-s}\log p$$

$$+\frac{1}{c_{i,L_i}\gamma+1}\sum_{s=0}^{N-1}\binom{N-1}{s}p^s(1-p)^{N-1-s}\log(1+(1-p^s)c_{i,L_i}\gamma)$$

$$\frac{E\{\log\left(1+(1-p^B)c_{i,L_i}\gamma\right)\}-(N-1)p\log p}{c_{i,L_i}\gamma+1} \tag{32}$$

where B is a binomial random variable of parameters $(N-1,p)$. ■

From now on, we replace $\mathcal{G}_i$ by $\mathcal{G}_{i,lb}$ in all the expression offered for the lower bounds on $\mathscr{R}_i$. In a "fair" FH system, we denote $a_{i,0}$, $\mathcal{H}_i$ and $\mathcal{G}_{i,lb}$ by $a(v,N)$, $\mathcal{H}(v,N)$ and $\mathcal{G}_{i,lb}(v,N)$ respectively. It is notable that considering the tern $\mathcal{G}_{i.lb}$ results in improvement for relatively lower ranges of SNR. In fact, it is seen that $\mathcal{G}_{i.lb}$ goes to zero as SNR goes to infinity. We will point out this fact again in example 1 given in section V.

On the other hand, let us assume $v_i = u$ for all i, i.e., all users spread their power on the whole spectrum. We call this the full-band spreading scenario or FBS. Then $a_{i,l} = 0$ for $l \leq L_i - 1$ and $a_{i,L_i} = 1$. This yields $a(u,N) = \mathcal{H}(u,N) = \mathcal{G}_{i,lb}(u,N) = 0$. In fact, $\mathscr{R}_{i,lb}^{(2)}$ is tight for $v = u$, i.e., $\mathscr{R}_{i,lb}^{(2)}$ is exactly the achievable rate of the i^{th} user while all users transmit over the whole spectrum. We denote this rate by $\mathscr{R}_{i,FBS}$. In the complex setup, $\mathscr{R}_{i,FBS}$ is given by

$$\mathscr{R}_{i,FBS} = \frac{u}{2}\log\left(\frac{|h_{i,i}|^2\gamma}{u(c_{i,L_i}\gamma+1)}+1\right). \tag{33}$$

IV. System Design, Part I: The Number of Active Users Is Unknown

In this section, we consider the complex setup where signals, ambient noise and channel gains are circular complex Gaussian random variables. In particular, we assume $h_{i,j} \sim \mathcal{CN}(0,1)$. Generally, the number of active users in the system is a random variable N. Each user is assumed to have no knowledge about the channel gains and the number of active users. The FD system is originally designed to service K users based on frequency division multiplexing. Therefore, each user occupies $\frac{u}{K}$ sub-bands upon activation and is not allowed to occupy the sectors that are assigned to other users. However, if the distribution of the number of active users changes, this scenario is highly inefficient on the heels that a considerable portion of the sub-bands is unused. Assume, the number of active users follows the distribution $\vec{q}$ where $q_n = \Pr\{N = n\}$ and $q_1 \geq q_2 \geq \cdots$. This is a reasonable assumption as the probability that two users become active simultaneously is less than the probability that only one active user is in the system, etc. Also, $\Pr\{N > K'\} = 0$ where $K' < K$. Comparison between a "fair" FH system and a FD system is made according to a performance measure called "ϵ-outage capacity" described as follows.

Denoting the transmission rate of the i^{th} user by R, the outage event for this user is

$$\mathcal{O}_i = \{N, \{h_{j,i}\}_{1 \leq j \leq N} : \mathscr{R}_i < R\}. \tag{34}$$

We notice that the randomness of the number of active users is considered in the outage event. Let us define the ϵ-outage capacity as follows:

$$R(\epsilon) := \sup\{R : \Pr\{\mathcal{O}_i\} \leq \epsilon\} \tag{35}$$

, i.e., the maximum transmission rate of a typical user such that its outage probability is below ϵ. We denote this quantity in the frequency division, full-band spreading and frequency hopping by $R_{FD}(\epsilon)$, $R_{FBS}(\epsilon)$ and $R_{FH}(\epsilon)$ respectively.

A. *Computing* $R_{FD}(\epsilon)$

In the FD system, the spectrum is already divided into K non-overlaping units each containing $\frac{u}{K}$ sub-bands. Each user that becomes active occupies one of the units. In this way, there is no interference at all, and the outage event is

$$\mathcal{O}_{i,FD} = \{h_{i,i} : \frac{u}{K} \log(1 + \frac{K|h_{i,i}|^2\gamma}{u}) < R\}. \tag{36}$$

As $h_{i,i} \sim \mathcal{CN}(0,1)$, defining $G_{i,i} := |h_{i,i}|^2$, we have:

$$p_{G_{i,i}}(g) = \begin{cases} \exp(-g) & g \geq 0 \\ 0 & \text{oth.} \end{cases}, \tag{37}$$

i.e., $G_{i,i}$ is an exponential random variable of parameter one. Thus,

$$\Pr\{\mathcal{O}_{i,FD}\} = 1 - \exp\big(\frac{u}{K}\frac{1}{\gamma}(1 - 2^{\frac{KR}{u}})\big). \tag{38}$$

Using this in (35) yields:

$$R_{FD}(\epsilon) = \sup\{R : \exp\big(\frac{u}{K}\frac{1}{\gamma}(1 - 2^{\frac{KR}{u}})\big) > 1 - \epsilon\} = \frac{u}{K}\log\big(1 - \frac{K\gamma}{u}\ln(1-\epsilon)\big). \tag{39}$$

B. *Computing* $R_{FBS}(\epsilon)$

Using (33), the following proposition yields $R_{FBS}(\epsilon)$:

Proposition 1

$$R_{FBS}(\epsilon) = \sup\{R : \exp\big(\frac{u}{\gamma}(1 - 2^{\frac{R}{u}})\big)\sum_{n=1}^{K'} q_n 2^{-\frac{(n-1)R}{u}} > 1 - \epsilon\}. \tag{40}$$

Proof: See appendix C. ∎

$R_{FBS}(\epsilon)$ yields the maximum rate of each user assuming all users spread their power on the whole spectrum while the outage probability per user is maintained below ϵ.

C. *Computing* $R_{FH}(\epsilon)$

Let us consider a "fair" FH system in which $v_i = v$ for all i. In this part, we derive lower bounds on $R_{FH}(\epsilon)$ using different lower bounds $\{\mathscr{R}_{i,lb}^{(k)}\}_{k\in\{1,2\}}$ on $\mathscr{R}_i$ derived in the previous section.

Taking the lower bound $\mathscr{R}_{i,lb}^{(k)}$ on $\mathscr{R}_i$ for any $k \in \{1,2\}$, it is clear that $\mathcal{O}_{i,FH} \subset \{N, \{h_{j,i}\}_{1\leq j\leq N} : \mathscr{R}_{i,lb}^{(k)} < R\}$. This yields[3] $\Pr\{\mathcal{O}_{i,FH}\} \leq \Pr\big\{\mathscr{R}_{i,lb}^{(k)} < R\big\}$, and hence

$$\big\{R : \Pr\big\{\mathscr{R}_{i,lb}^{(k)} < R\big\} \leq \epsilon\big\} \subset \{R : \Pr\{\mathcal{O}_{i,FH}\} \leq \epsilon\}. \tag{41}$$

Defining

$$R_{FH}^{(k)}(\epsilon) := \sup\big\{R : \Pr\big\{\mathscr{R}_{i,lb}^{(k)} < R\big\} < \epsilon\big\}, \tag{42}$$

[3]By $\Pr\{\mathscr{R}_{i,lb}^{(k)} < R\}$, we mean $\Pr\{\{N, \{h_{j,i}\}_{1\leq j\leq N} : \mathscr{R}_{i,lb}^{(k)} < R\}\}$

we get

$$R_{FH}^{(k)}(\epsilon) \leq R_{FH}(\epsilon). \tag{43}$$

In the following subsections we separately compute $\{R_{FH}^{(k)}(\epsilon)\}_{k\in\{1,2\}}$.

1) Computation of $R_{FH}^{(2)}(\epsilon)$: We start with the following definition.

Definition 1 *Let $n \in \mathbb{N}$. For $c \in (0,1]$ and $b > 0$, define the function $\alpha_n(.;b,c) : \mathbb{R}^+ \to \mathbb{R}^+$ as*

$$\alpha_n(z;b,c) = \frac{E\{\log\left(1+b(1-c^B)z\right)\} - (n-1)c\log c}{bz+1} \tag{44}$$

where B is a binomial random variable of parameters $(n-1,c)$. Also, for $b_1 < 0$, $b_2 > 0$ and $c_1, c_2 \in [0,1]$, define the function $\phi_n(b_1,b_2,c_1,c_2)$ as

$$\phi_n(b_1,b_2,c_1,c_2) = \frac{1}{(n-2)!}\int_0^\infty z^{n-2}\exp\Big(b_1(b_2z+1)^{c_1}2^{-\alpha_n(z;b_2,c_2)} - z\Big)dz. \tag{45}$$

Using this class of functions, the following proposition yields $R_{FH}^{(2)}(\epsilon)$.

Proposition 2

$$R_{FH}^{(2)}(\epsilon) = \max_v \sup\{R : q_1\exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}})) + \sum_{n=2}^{K'} q_n\phi_n(b_{1,n},b_{2,n},c_{1,n},c_{2,n}) > 1-\epsilon\} \tag{46}$$

where $b_{1,n} = \frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}})$, $b_{2,n} = \frac{\gamma}{v}$, $c_{1,n} = 1-a(v,n)$ and $c_{2,n} = \frac{v}{u}$.

Proof: See appendix D. ■

The functions ϕ_n appearing in the formulation of $R_{FH}^{(2)}(\epsilon)$ involve numerical integrations. The following corollary, proved in appendix E, yields a closed form lower bound on $R_{FH}^{(2)}(\epsilon)$.

Corollary 1 *Let*

$$R_{FH}^{(3)}(\epsilon) := \max_v \sup\{R : q_1\exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}})) + \sum_{n=2}^{K'} \frac{q_n}{(n-2)!}\exp\Big(b_{1,n}\big((n-1)b_{2,n}+1\big)^{1-a(v,n)}\Big) > 1-\epsilon\}. \tag{47}$$

Then,

$$R_{FH}^{(2)}(\epsilon) \geq R_{FH}^{(3)}(\epsilon) \tag{48}$$

where $b_{1,n} = \frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}})$ and $b_{2,n} = \frac{\gamma}{v}$.

Proof: See appendix E. ■

2) Computation of $R_{FH}^{(1)}(\epsilon)$: We start with the following definition.

Definition 2 *For $b_1 < 0$, $b_2 > 0$ and $c \in [0,1]$, we define*

$$\psi_n(b_1, b_2, c) = \int_{z_1 \geq 0, \cdots, z_{n-1} \geq 0} \exp\left(b_1 2^{-\alpha_n(z_{n-1,n-1}; b_2, c)} \prod_{m=1}^{n-1} \prod_{m'=1}^{\binom{n-1}{m}} (b_2 z_{m,m'} + 1)^{c_{m,n}} - z_{n-1,n-1}\right) dz_1 \cdots dz_{n-1} \tag{49}$$

where for each m, $\{z_{m,m'}\}_{m'=1}^{\binom{n-1}{m}}$ consists of all possible summations of m elements in the set $\{z_i\}_{i=1}^{n-1}$ and $c_{m,n} = c^m (1-c)^{n-1-m}$.

For example,

$$\psi_2(b_1, b_2, c) = \int_0^\infty \exp\left(b_1 2^{-\alpha_2(z; b_2, c)} (b_2 z_1 + 1)^c - z_1\right) dz_1 \tag{50}$$

and

$$\psi_3(b_1, b_2, c) = \int_{z_1, z_2 > 0} \exp\left(b_1 2^{-\alpha_3(z_{2,2}; b_2, c)} \big((b_2 z_1 + 1)(b_2 z_2 + 1)\big)^{c(1-c)} (b_2 z_{2,2} + 1)^{c^2} - z_{2,2}\right) dz_1 dz_2 \tag{51}$$

where $z_{2,2} = z_1 + z_2$ by definition. It is notable that $\psi_2(b_1, b_2, c) = \phi_2(b_1, b_2, c, c)$.

The following proposition offers an expression to compute $R_{FH}^{(1)}(\epsilon)$.

Proposition 3 *Let*

$$R_{FH}^{(1)}(\epsilon; v) = \sup\left\{R : q_1 \exp\left(\frac{v}{\gamma}(1 - 2^{\frac{R}{v}})\right)) + \sum_{n=2}^{K'} q_n \psi_n\big(b_{1,n}, b_{2,n}, c_n\big) > 1 - \epsilon\right\} \tag{52}$$

where $b_{1,n} = \frac{v}{\gamma} 2^{\mathcal{H}(v,n)}(1 - 2^{\frac{R}{v}})$, $b_{2,n} = \frac{\gamma}{v}$ and $c_n = \frac{v}{u}$. Then,

$$R_{FH}^{(1)}(\epsilon) = \max_v R_{FH}^{(1)}(\epsilon; v). \tag{53}$$

Proof: See appendix F. ■

The expression given in (53) is quite complicated. On one hand, the multiple integrals do not have a closed form. On the other hand, the maximization over v must be computed numerically. By the way, $R_{FH}^{(1)}(\epsilon)$ is the best lower bound on $R_{FH}(\epsilon)$ as $\mathscr{R}_{1,lb}^{(1)}$ is the best lower bound we have found on $\mathscr{R}_1$. Fig. 1 shows the three lower bounds $R_{FH}^{(k)}(\epsilon)$ for $k \in \{1, 2, 3\}$ we have developed in a system with $K' = 4$, $(q_1, q_2, q_3, q_4) = (.4, .2, .2, .2)$, $u = 15$ and $\gamma = 100$.

Remark 1 *One may construct a lower bound on $R_{FH}(\epsilon)$ mixing the bounds $R_{FH}^{(1)}(\epsilon)$ and $R_{FH}^{(2)}(\epsilon)$. Let us*

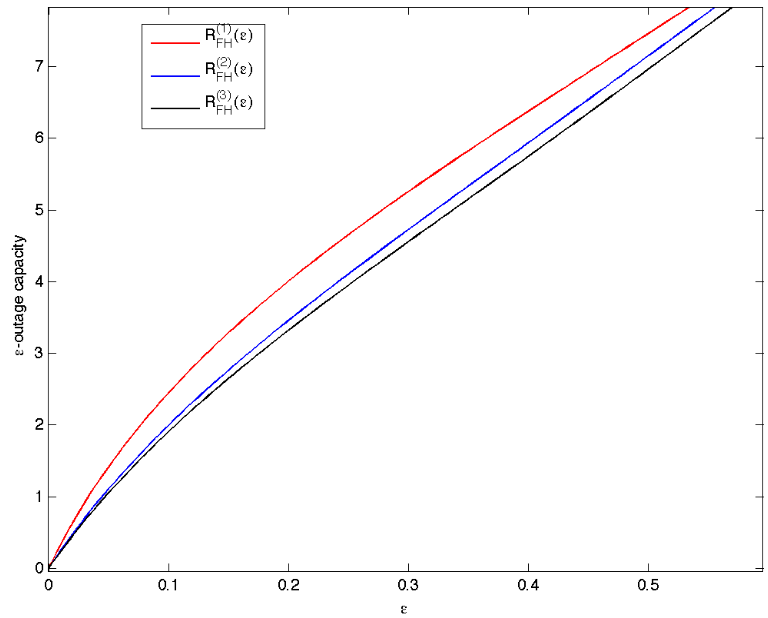

Fig. 1. Depictions of $R_{FH}^{(k)}(\epsilon)$ for $k \in \{1, 2, 3\}$ in a setup where $K' = 4$, $(q_1, q_2, q_3, q_4) = (.4, .2, .2, .2)$, $\gamma = 100$ and $u = 15$.

write $\Pr\{\mathcal{O}_i\}$ *as*

$$\Pr\{\mathcal{O}_i\} = \sum_{n=1}^{K'} \Pr\{\mathcal{O}_i | N = n\} q_n. \tag{54}$$

For $n = 1$*, we have* $\Pr\{\mathcal{O}_i | N = 1\} = 1 - \exp\left(\frac{v}{\gamma}(1 - 2^{\frac{R}{v}})\right)$*. For* $n > 1$*, one can find an upper bound on each* $\Pr\{\mathcal{O}_i | N = n\}$ *using either of the lower bounds* $\mathscr{R}_{i,lb}^{(k)}$ *on* $\mathscr{R}_i$ *for* $k \in \{1, 2\}$*. Using* $\mathscr{R}_{i,lb}^{(1)}$*, we get the following:*

$$\Pr\{\mathcal{O}_i | N = n\} \leq 1 - \psi_n(b_{1,n}, b_{2,n}, c_n). \tag{55}$$

On the other hand, using $\mathscr{R}_{i,lb}^{(2)}$*, we have:*

$$\Pr\{\mathcal{O}_i | N = n\} \leq 1 - \phi_n(b_{1,n}, b_{2,n}, c_{1,n}, c_{2,n}). \tag{56}$$

The bound in (55) is tighter than the bound in (56), but its computation is more involving. Therefore, one may choose the bound (55) for those n *for which* q_n *is relatively larger. Assume for* $n \in \mathcal{N} \subset \{2, \cdots, K'\}$*, we use the bound in (55). Then our recommended upper bound on* $\Pr\{\mathcal{O}_i\}$ *would be*

$$\Pr\{\mathcal{O}_i\} \leq 1 - q_1 \exp\left(\frac{v}{\gamma}(1 - 2^{\frac{R}{v}})\right) - \sum_{n \in \mathcal{N}} q_n \psi_n(b_{1,n}, b_{2,n}, c_n) - \sum_{n \notin \mathcal{N}} q_n \phi_n(b_{1,n}, b_{2,n}, c_{1,n}, c_{2,n}). \tag{57}$$

Defining $R_{FH}^{(4)}(\epsilon;\mathcal{N})$ *as*

$$R_{FH}^{(4)}(\epsilon;\mathcal{N}) =$$

$$\max_{v} \sup\{R : q_1 \exp\big(\frac{v}{\gamma}(1-2^{\frac{R}{v}})\big) + \sum_{n\in\mathcal{N}} q_n\psi_n(b_{1,n}, b_{2,n}, c_n) + \sum_{n\notin\mathcal{N}} q_n\phi_n(b_{1,n}, b_{2,n}, c_{1,n}, c_{2,n}) \geq 1-\epsilon\}, \quad (58)$$

we get the forth type lower bound on $R_{FH}(\epsilon)$ *as*

$$R_{FH}(\epsilon) \geq R_{FH}^{(4)}(\epsilon;\mathcal{N}). \quad (59)$$

V. Discussions and Simulation Results

In this section, we consider different examples to demonstrate cases where the FH system outperforms the FD system, i.e., $R_{FH}(\epsilon) > R_{FD}(\epsilon)$ for an ϵ and at a given SNR. In fact, for any $(K, \{q_n\}_{n=1}^{K'}, u, \epsilon)$, we have three strategies to investigate, i.e., FD, FH and FBS. We have no expression for $R_{FH}(\epsilon)$. However, we have developed the following set of lower bounds on this quantity:

$$R_{FH}(\epsilon) \geq R_{FH}^{(1)}(\epsilon) \geq R_{FH}^{(4)}(\epsilon,\mathcal{N}) \geq R_{FH}^{(2)}(\epsilon) \geq R_{FH}^{(3)}(\epsilon). \quad (60)$$

$R_{FH}^{(1)}(\epsilon)$ is the best lower bound, however, its computation involves multiple integrals of up to order $K'-1$ for any K'. Computing $R_{FH}^{(2)}$ only involves single integrals for all K'. Computation of $R^{(3)}(\epsilon)$ involves no integrals. $R_{FH}^{(4)}(\epsilon,\mathcal{N})$ is a lower bound on $R_{FH}(\epsilon)$ mixing the lower bounds of type one and two. In the following examples, we always assume $K' \leq 4$. This enables us to use our best lower bound $\mathscr{R}_{FH}^{(1)}$ as we encounter double and triple integrals. Let us define

$$f_{FD}(R;\gamma) = \exp\big(\frac{u}{K}\frac{1}{\gamma}(1-2^{\frac{KR}{u}})\big), \quad (61)$$

$$f_{FBS}(R;\gamma) = \exp\big(\frac{u}{\gamma}(1-2^{\frac{R}{u}})\big)\sum_{n=1}^{K} q_n 2^{-\frac{(n-1)R}{u}}, \quad (62)$$

$$f_{FH}^{(2)}(R,v;\gamma) = q_1\exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}})) + \sum_{n=2}^{K} q_n\phi_n(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}}), \frac{\gamma}{v}, 1-a(v,n), \frac{v}{u}) \quad (63)$$

and

$$f_{FH}^{(1)}(R,v;\gamma) = q_1\exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}}))) + \sum_{n=2}^{K} q_n\psi_n\big(v, \frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}}), \frac{\gamma}{v}, \frac{v}{u}\big). \quad (64)$$

Now, by (39), (40), (46) and (53),

$$R_{FD}(\epsilon) = \sup\{R : f_{FD}(R;\gamma) > 1-\epsilon\}, \tag{65}$$

$$R_{FBS}(\epsilon) = \sup\{R : f_{FBS}(R;\gamma) > 1-\epsilon\}, \tag{66}$$

$$R_{FH}^{(k)}(\epsilon) = \max_{v}\sup\{R : f_{FH}^{(k)}(R, v;\gamma) > 1-\epsilon\} \tag{67}$$

for $k \in \{1, 2\}$.

A. FH vs FD

To get a better insight, we include the FBS scenario in all the figures in this section.

Example 1 *In this example, we consider a system with* $u = 20$*,* $K = 10$*,* $K' = 2$ *and* $(q_1, q_2) = (.5, .5)$*. We have sketched the* ϵ*-outage capacity for the FH, FBS and FD systems in fig. 2 for* $\gamma = 21.76\text{dB}$*. In case of* FH*, the best lower bound is obtained for* $v = 9$ *in the given range of* ϵ*. We have included the curve* $R_{FH}^{(1)}(\epsilon; v = 2)$ *for comparison. Also, there are two curves for the case* $v = 5$*. The blue curve considers the term* $\mathcal{H}_i$ *and* $\mathcal{G}_{i,lb}$ *proposed in lemmas 2 and 3. On the other hand, the red curve only considers the term* $\mathcal{H}_i$*. It is seen that the resulted improvement (the lift in the lower bound) by considering the term* $\mathcal{G}_{i,lb}$ *enables us to conclude that for* $\epsilon \in [.13, .17]$*, FH outperforms FBS. Clearly, the red curve which does not consider this improvement is below the curve* $R_{FBS}(\epsilon)$ *for this range of* ϵ*. Also, using the blue curve, it is seen that for all* $\epsilon \geq .12$*, FH outperforms FD. However, if we use the red curve the anticipated advantage of FH over FD is observed for* $\epsilon \geq .14$*. Fig. 3 depicts the same setup at* $\gamma = 30\text{dB}$*. It is seen that the improvement offered by introducing* $\mathcal{G}_{i,lb}$ *fades away as* γ *increases.*

Example 2 *Assume* $u = K = 20$*,* $K' = 3$ *and* $(q_1, q_2, q_3) = (.5, .3, .2)$*. Let* $\gamma = 20\text{dB}$*. Fig. 4 depicts the lower bound on* $R_{FH}(\epsilon)$ *for different hopping patterns, i.e., different values of* v*. It is observed that for smaller values of* v*, we get a stronger lower bound. The curves for* $v = 2$ *and* $v = 3$ *are very close to each other. It is seen that for* $\epsilon < .075$*, taking* $v = 2$ *yields a better performance while for* $\epsilon > .075$*, the hopping pattern must be set at* $v = 3$*. Also, for all* $\epsilon > .075$*, the* ϵ*-outage capacity of FH (by setting* $v = 3$*) is larger than that of* FD*. However, at* $\epsilon = .15$*, the curve* $R_{FBS}(\epsilon)$ *is above* $R_{FH}^{(1)}(\epsilon; v = 3)$*, and hence, we are unable to claim any advantage of FH over FBS in the range* $\epsilon > .15$*. Therefore, a main conclusion of the simulation result in this certain case is that FH outperforms both FD and FBS as long*

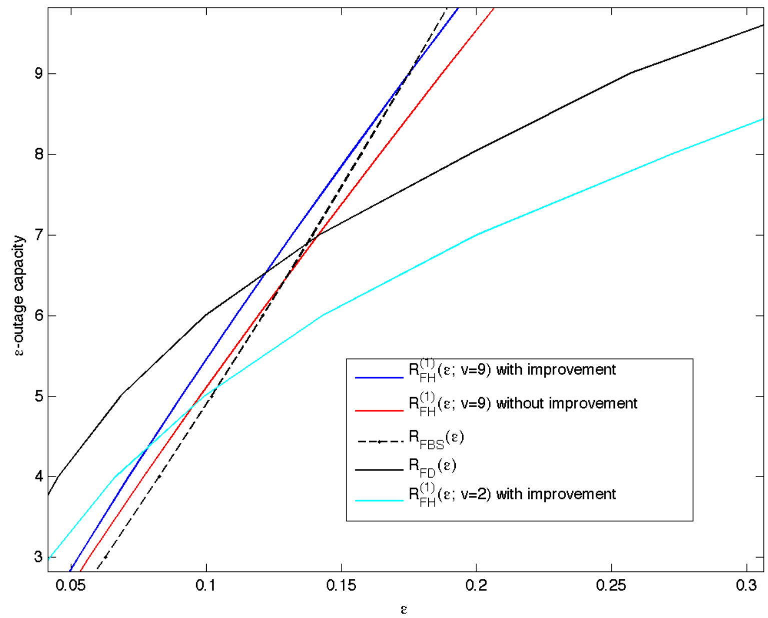

Fig. 2. Comparison of FH and FD for $u = 20$, $K = 10$, $K' = 2$, $\gamma = 21.76$dB and $(q_1, q_2) = (.5, .5)$.

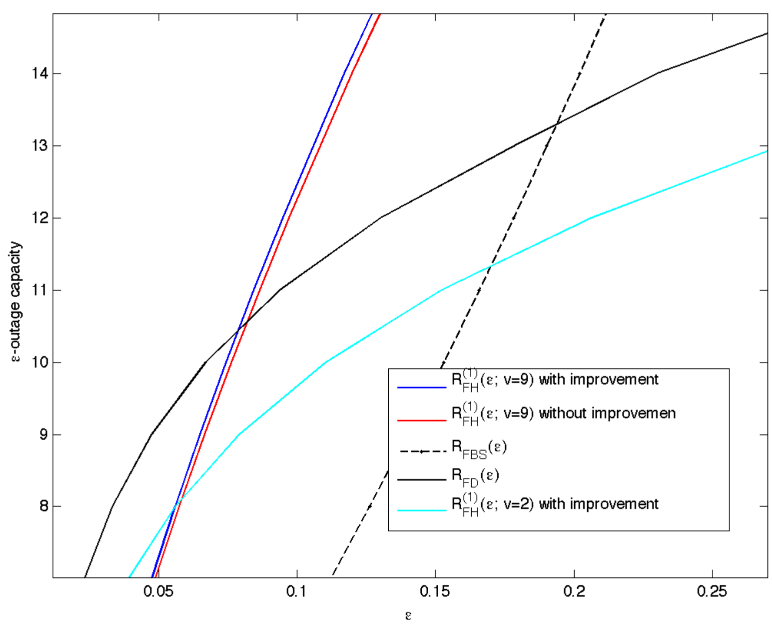

Fig. 3. Comparison of FH and FD for $u = 20$, $K = 10$, $K' = 2$, $\gamma = 30$dB and $(q_1, q_2) = (.5, .5)$.

as $\epsilon \in [.075, .15]$. *Now, let us increase the* SNR *value. Let* $\gamma = 30dB$. *Fig. 5 offers a comparison of* $R^{(1)}_{FH}(\epsilon; v = 2)$ *(hopping pattern set at* $v = 2$*),* $R_{FBS}(\epsilon)$ *and* $R_{FD}(\epsilon)$ *at* $\gamma = 30$dB. *It is seen that for at least all* $\epsilon \in [.01, .2]$*, FH offers a considerably higher performance than the cases of FBS and FD.*

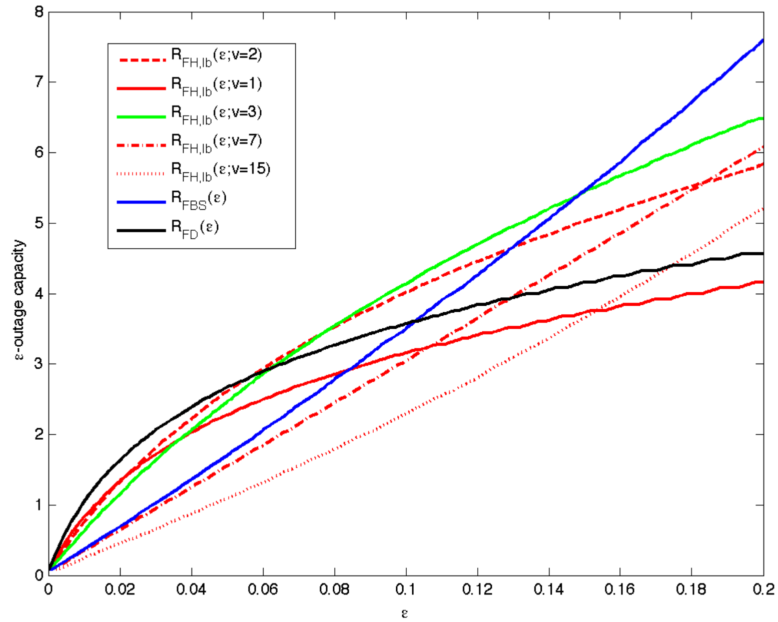

Fig. 4. Comparison of FH and FD for $u = K = 20$, $K' = 3$, $\gamma = 20$dB and $(q_1, q_2, q_3) = (.5, .3, .2)$.

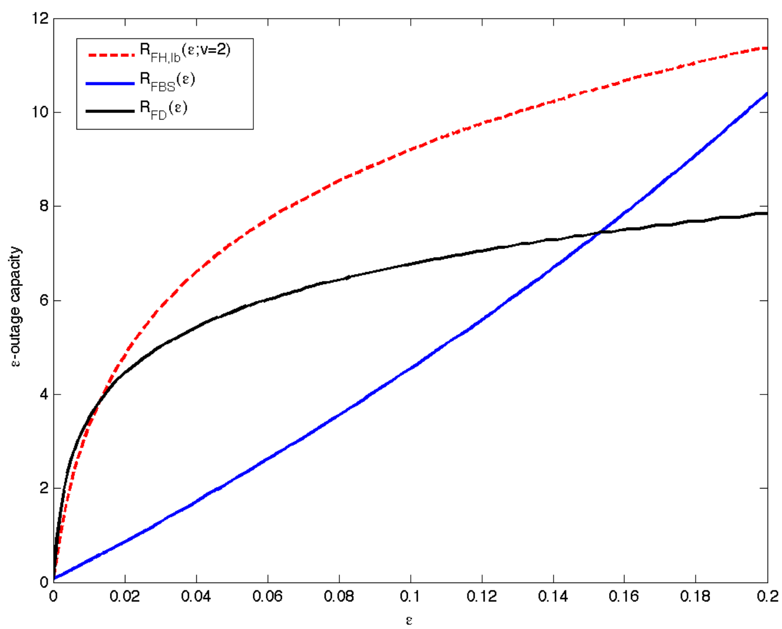

Fig. 5. Comparison of FH and FD for $u = K = 20$, $K' = 3$, $\gamma = 30$dB and $(q_1, q_2, q_3) = (.5, .3, .2)$.

Example 3 *In this example, we propose another method to choose the hopping pattern. In [54], we have determined the sum-rate multiplexing gain in a "fair" FH system with N users as*

$$\mathrm{MG}_{\mathrm{FH}}(N, v) = Nv(1 - \frac{v}{u})^{N-1}. \tag{68}$$

We propose to select v by the following rule:

$$v^* = \arg \max_{1 \le v < u} E\{\mathrm{MG}_{\mathrm{FH}}(N, v)\} = \arg \max_{1 \le v < u} vE\{N(1 - \frac{v}{u})^{N-1}\}. \tag{69}$$

Let us consider the setup in example 2 above. As $u = 20$ and the distribution of N is given by $(q_1, q_2, q_3) = (.5, .3, .2)$, we have:

$$v^* = \arg \max_{1 \le v < 20} v\big(.5 + .6(1 - \frac{v}{20}) + .6(1 - \frac{v}{20})^2\big). \tag{70}$$

Fig. 6 yields the sketch of $E\{\mathrm{MG}_{\mathrm{FH}}(N, v)\}$. It is seen that $v^* = 15$. Clearly, the value of γ has no

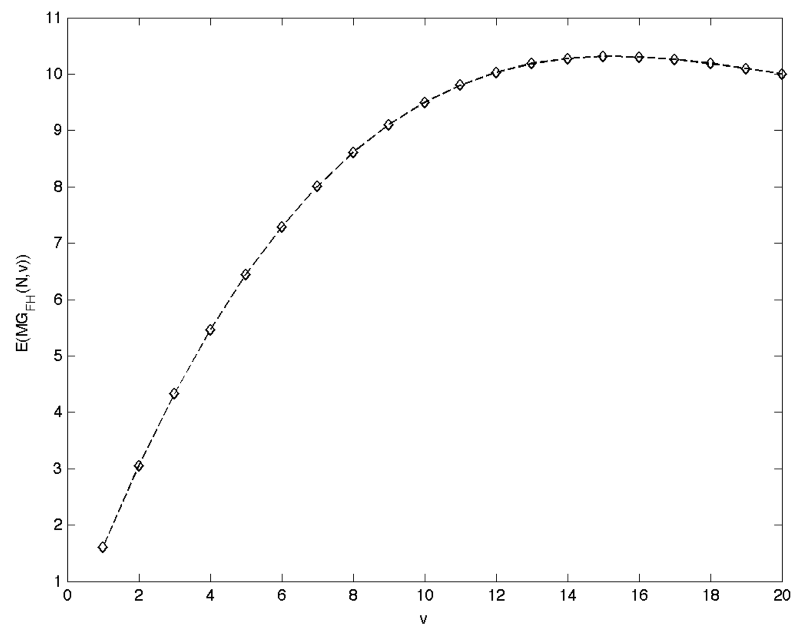

Fig. 6. Sketch of $E\{\mathrm{MG}_{\mathrm{FH}}(N, v)\}$ in terms of v

role to determine v^*. By the results of example 2, at $\gamma = 20\mathrm{dB}$, the ϵ-outage capacity is maximized for $v_{\mathrm{opt}} = 3$ for all $\epsilon \in [.075, .2]$. Fig. 4 depicts both $R_{FH}^{(1)}(\epsilon; v = 3)$ and $R_{FH}^{(1)}(\epsilon; v = 15)$. Comparing these two shows that taking $v = v^* = 15$ implies no advantage in terms of ϵ-outage capacity. However, $\frac{E\{\mathrm{MG}_{\mathrm{FH}}(N,3)\}}{E\{\mathrm{MG}_{\mathrm{FH}}(N,15)\}} = .5340$. In fact, by fig. 6, taking $v = 3$ is far from maximizing $E\{\mathrm{MG}_{\mathrm{FH}}(N, v)\}$.

Finally, we wrap up with an observation about FH and FBS in a busy network of two users, i.e., $q_2 > q_1$.

B. FBS vs FH

Assume $K' = 2$ and $u = 10$. Intuitively, one might expect that for $q_1 > .5$, FBS is the best scenario. On the other hand, if $q_1 \le .5$, i.e., there are two active users in the system with probability more that $\frac{1}{2}$, FH might be better as it partly avoids collisions. This intuition is valid only in higher ranges of SNR. As SNR increases, the FBS scenario leads to saturation of the rates of all active users. On the other hand, the FH system confers each user a certain amount of multiplexing gain which is $v(1-\frac{v}{u})$ in case the two users are active [49]. Let $q_1 = .1$, i.e., the two users become active with probability $.9$. Let us take $\gamma = 20\mathrm{dB}$. Fig. 7 sketches $R_{FH,lb}(\epsilon)$ and $R_{FD}(\epsilon)$. The best lower bound on the FH performance is obtained for $v = 2$. It is seen that the ϵ-outage capacity for FH is considerably larger than that of FBS for all ϵ. Clearly, the same holds for all $\gamma > 20\mathrm{dB}$.

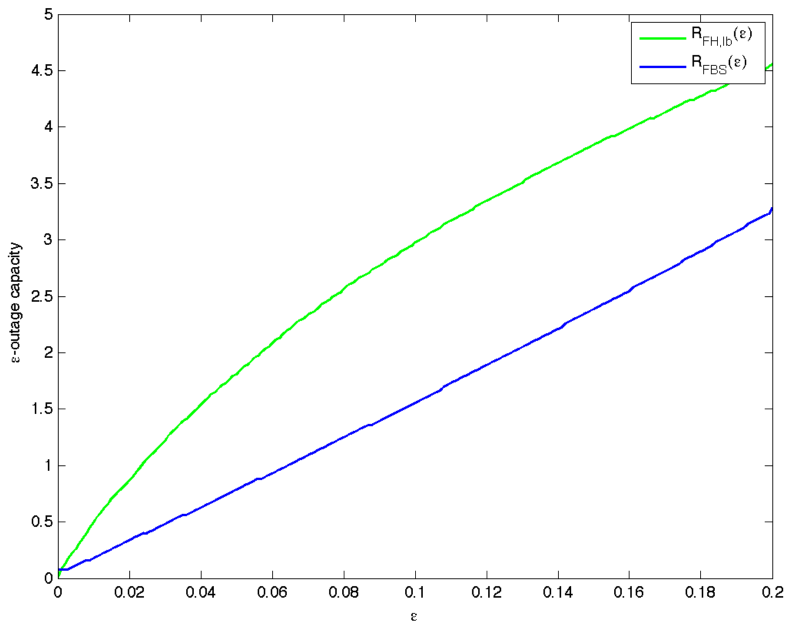

Fig. 7. Comparison of FH and FBS for $u = 10$, $\gamma = 20\mathrm{dB}$, $K' = 2$ and $(q_1, q_2) = (.1, .9)$

VI. Appendix A

In this appendix, we prove lemma 2. Let us consider a general $u \times u$ vector mixed gaussian distribution $p_{\vec{Z}}(\vec{z})$ with different covariance matrices $\{C_l\}_{l=1}^{L}$ and associated probabilities $\{a_l\}_{l=1}^{L}$ given by:

$$p_Z(z) = \sum_{l=1}^{L} a_l g(\vec{z}, C_l) \tag{71}$$

where $g(\vec{z}, C_l) = \frac{1}{(2\pi)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}} \exp -\frac{1}{2}\vec{z}^T C_l^{-1} \vec{z}$. Hence, we get:

$$I := \int p_Z(z) \ln p_Z(z) dz = \sum_{l=1}^{L} J_l \tag{72}$$

where $J_l = a_l \int g(\vec{z}, C_l) \ln p_{\vec{Z}}(\vec{z}) d\vec{z}$ for $1 \leq l \leq L$. To find a proper lower bound on each J_l in this expression, we proceed as follows:

$$J_l = a_l \int g(\vec{z}, C_l) \ln \Big(\sum_{m=1}^{L} a_m g(\vec{z}, C_m) \Big) d\vec{z} \geq a_l \int g(\vec{z}, C_l) \ln \big(a_l g(\vec{z}, C_l) \big) d\vec{z}$$

$$= a_l \ln \big(\frac{a_l}{(2\pi)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}} \big) \int g(\vec{z}, C_l) d\vec{z} - \frac{1}{2} a_l \int \vec{z}^T C_l^{-1} \vec{z} g(\vec{z}, C_l) d\vec{z}$$

$$= a_l \ln \big(\frac{a_l}{(2\pi)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}} \big) - \frac{1}{2} a_l E\{ \vec{Z}_G^T C_l^{-1} \vec{Z}_G \} \tag{73}$$

where $\vec{Z}_G$ is a Gaussian vector with PDF $g(\vec{z}, C_l)$. However,

$$E\{ \vec{Z}_G^T C_l^{-1} \vec{Z}_G \} = E\{ tr \big(\vec{Z}_G^T C_l^{-1} \vec{Z}_G \big) \} = E\{ tr \big(\vec{Z}_G \vec{Z}_G^T C_l^{-1} \big) \}$$

$$= tr \Big(E\{ \vec{Z}_G \vec{Z}_G^T \} C_l^{-1} \Big) = tr I_u = u. \tag{74}$$

Thus,

$$J_l \geq a_l \ln \big(\frac{a_l}{(2\pi)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}} \big) - \frac{u}{2} a_l. \tag{75}$$

On the other hand,

$$\mathrm{h}(\vec{Z}) = -(\log e) I = -(\log e) \sum_{l=1}^{L} J_l$$

$$\leq - \sum_{l=1}^{L} a_l \Big(\log \big(\frac{a_l}{(2\pi)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}} \big) - \frac{u}{2} \log e \Big) = - \sum_{l=1}^{L} a_l \log \frac{a_l}{(2\pi e)^{\frac{u}{2}} (\det C_l)^{\frac{1}{2}}}$$

$$= \frac{1}{2} \sum_{l=1}^{L} a_l \log((2\pi e)^u \det C_l) + \mathcal{H}(a_1, \cdots, a_L) \tag{76}$$

where $\mathcal{H}(a_1, \cdots, a_L) = -\sum_{l=1}^{L} a_l \log a_l$ is the discrete entropy of $\{a_l\}_{l=1}^{L}$. On the other hand, we know that differential entropy is a concave function of the density. Thus,

$$\mathrm{h}(\vec{Z}) \geq \frac{1}{2} \sum_{l=1}^{L} a_l \log((2\pi e)^u \det C_l). \tag{77}$$

This concludes the lemma.

VII. Appendix B

In this appendix, we try to improve the upper bound derived in appendix A on the differential entropy of a mixed Gaussian random vector for a special class of such vectors. Consider the PDF given in (71), where $C_l = \sigma_l^2 I_u$. Assume $\sigma_1^2 < \sigma_2^2 < \cdots < \sigma_L^2$. We modify the bounding in (73) as follows. We know that

$$J_l = a_l \int g(\vec{z}, C_l) \ln \big(\sum_{m=1}^{L} a_m g(\vec{z}, C_m) \big) d\vec{z}. \tag{78}$$

On the other hand,

$$\ln \big(\sum_{m=1}^{l} a_m g(\vec{z}, C_m) \big) = \ln(a_l g(\vec{z}, C_l)) + \ln(1 + \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{g(\vec{z}, C_m)}{g(\vec{z}, C_l)} + \sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{g(\vec{z}, C_m)}{g(\vec{z}, C_l)}). \tag{79}$$

However, the term $\sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{g(\vec{z},C_m)}{g(\vec{z},C_l)} = \sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u} \exp -\big(\frac{1}{2}(\frac{1}{\sigma_m^2} - \frac{1}{\sigma_l^2}) \vec{z}^T \vec{z}\big)$ is always greater than $\sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u}$. Hence,

$$\ln \big(\sum_{m=1}^{l} a_m g(\vec{z}, C_m) \big) \geq \ln(a_l g(\vec{z}, C_l)) + \ln(1 + \sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u} + \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{g(\vec{z}, C_m)}{g(\vec{z}, C_l)}). \tag{80}$$

On the other hand, the term $\sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{g(\vec{z},C_m)}{g(\vec{z},C_l)} = \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u} \exp -\big(\frac{1}{2}(\frac{1}{\sigma_m^2} - \frac{1}{\sigma_l^2}) \vec{z}^T \vec{z}\big)$ is always less than $\sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u}$. Now, we use the following inequality [4]which is valid for any $b > 0$ and $0 \leq x \leq a$,

$$\ln(1 + b + x) \geq (1 - \frac{x}{a}) \ln(1 + b) + \frac{x}{a} \ln(1 + a + b). \tag{81}$$

Utilizing this in the expression on the right hand side of (80), we get:

$$\ln \big(\sum_{m=1}^{l} a_m g(\vec{z}, C_m) \big) \geq \Big(1 - \frac{1}{\nu_l} \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{g(\vec{z}, C_m)}{g(\vec{z}, C_l)} \Big) \ln(1 + \mu_l) + \frac{1}{\nu_l} \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{g(\vec{z}, C_m)}{g(\vec{z}, C_l)} \ln(1 + \nu_l + \mu_l) \tag{82}$$

where $\mu_l = \sum_{m=l+1}^{L} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u}$ and $\nu_l = \sum_{m=1}^{l-1} \frac{a_m}{a_l} \frac{\sigma_l^u}{\sigma_m^u}$. Using this in (78) yields:

$$J_l \geq a_l \int g(\vec{z}, C_l) \ln(a_l g(\vec{z}, C_l)) dz + (a_l - \frac{\sum_{m=1}^{l-1} a_m}{\nu_l}) \ln(1 + \mu_l) + \frac{\sum_{m=1}^{l-1} a_m}{\nu_l} \ln(1 + \nu_l + \mu_l). \tag{83}$$

One may notice that the part $a_l \int g(\vec{z}, C_l) \ln(a_l g(\vec{z}, C_l)) dz$ is the lower bound on J_l used in appendix A. This finally leads to the upper bound of appendix A. But, here, we have an extra term which makes the

[4]One may verify this using Jensen's inequality and concavity of the $\ln(.)$ function.

upper bound tighter. Briefly, we get:

$$\mathrm{h}(\vec{Z}) \leq \frac{1}{2}\sum_{l=1}^{L}\log((2\pi e)^u \det C_l) + \mathcal{H} - \mathcal{G}'' \tag{84}$$

where

$$\mathcal{G}'' = \sum_{l=1}^{L}\Big((a_l - \frac{\sum_{m=1}^{l-1} a_m}{\nu_l})\ln(1+\mu_l) + \frac{\sum_{m=1}^{l-1} a_m}{\nu_l}\ln(1+\nu_l+\mu_l)\Big). \tag{85}$$

$\mathcal{G}''$ is a complicated function of $\{\sigma_l\}_{l=1}^{L}$. To simplify it, one may notice that $\mathcal{G}''$ is an increasing function of μ_l. Hence, using $\mu_l \geq 0$, we get a lower bound on $\mathcal{G}''$, namely $\mathcal{G}'$ given by:

$$\mathcal{G}' = \sum_{l=2}^{L}\frac{\ln(1+\nu_l)}{\nu_l}\sum_{m=1}^{l-1} a_m. \tag{86}$$

On the other hand, using the fact that $\frac{\ln(1+x)}{x}$ is a decreasing function of x, one may obtain a lower bound on $\mathcal{G}'$ by finding an upper bound on ν_l for each l. One option is $\nu_l \leq \frac{\sigma_L^u}{\sigma_1^u}\frac{\sum_{m=1}^{l-1} a_m}{a_l}$. Thus, we come up with the following lower bound on $\mathcal{G}'$

$$\mathcal{G}' \geq \mathcal{G} := \frac{\sigma_1^u}{\sigma_L^u}\sum_{l=2}^{L} a_l \log(1 + \frac{\sigma_L^u}{\sigma_1^u}\frac{\sum_{m=1}^{l-1} a_m}{a_l}). \tag{87}$$

VIII. Appendix C

We first compute $\Pr\{\mathscr{R}_{i,FBS} < R\}$. By (33),

$$\Pr\{\mathscr{R}_{i,FBS} < R\} = \Pr\{u\log(1 + \frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_N)}) < R\} \tag{88}$$

where

$$\mathcal{I}_N = \begin{cases} \sum_{j=1,j\neq i}^{N}|h_{j,i}|^2 & N>1 \\ 0 & N=1 \end{cases}. \tag{89}$$

Therefore,

$$\Pr\{\mathscr{R}_{i,FBS} < R\} = \sum_{n=1}^{K'} q_n \Pr\{\mathscr{R}_{i,FBS} < R | N=n\}$$

$$= q_1\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u}) < R\} + \sum_{n=1,n\neq i}^{N} q_2\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_n)}) < R\}. \tag{90}$$

The first term can be computed easily as:

$$\Pr\{v\log(1+\frac{|h_{i,i}|^2\gamma}{u}) < R\} = 1 - \exp(\frac{u}{\gamma}(1-2^{\frac{R}{u}})). \tag{91}$$

As $|h_{i,i}|^2$ and $\mathcal{I}_n$ are independent random variables for each n, one can write:

$$\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_n)})<R\}=E_{\mathcal{I}_n}\{\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_n)})<R|\mathcal{I}_n\}\} \tag{92}$$

Since $|h_{i,i}|^2$ is an exponential random variable with parameter one,

$$\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_n)})<R|\mathcal{I}_n\}=1-\exp\big(\frac{u}{\gamma}(1-2^{\frac{R}{u}})(1+\frac{\gamma}{u}\mathcal{I}_n)\big). \tag{93}$$

Replacing this in (92) yields:

$$\Pr\{u\log(1+\frac{|h_{i,i}|^2\gamma}{u(1+\frac{\gamma}{u}\mathcal{I}_n)})<R\}=E_{\mathcal{I}_n}\{1-\exp\big(\frac{u}{\gamma}(1-2^{\frac{R}{u}})(1+\frac{\gamma}{u}\mathcal{I}_n)\big)\}$$

$$=1-\exp(\frac{u}{\gamma}(1-2^{\frac{R}{u}}))E_{\mathcal{I}_n}\{\exp\big((1-2^{\frac{R}{u}})\mathcal{I}_n\big)\}=1-\exp(\frac{u}{\gamma}(1-2^{\frac{R}{u}}))2^{-\frac{(n-1)R}{u}} \tag{94}$$

where we have used the fact that $E\{\exp(t\mathcal{I}_n)\}=\frac{1}{(1-t)^{n-1}}$ as $2\mathcal{I}_n\sim\chi^2_{2(n-1)}$. Thus, $R_{FBS}(\epsilon)$ is given by:

$$R_{FBS}(\epsilon)=\sup\{R:\exp\big(\frac{u}{\gamma}(1-2^{\frac{R}{u}})\big)\sum_{n=1}^{K'}q_n2^{-\frac{(n-1)R}{u}}>1-\epsilon\}. \tag{95}$$

IX. Appendix D

Let us compute $\Pr\{\mathscr{R}^{(2)}_{i,lb}<R\}$. By (23),

$$\mathscr{R}^{(2)}_{i,lb}=\frac{v}{2}\log\left(\frac{2^{-\mathcal{H}(v,N)}2^{\mathcal{G}_{i,lb}(v,N)}|h_{i,i}|^2\gamma}{v(\frac{\gamma}{v}\mathcal{I}_N+1)^{1-a(v,N)}}+1\right) \tag{96}$$

where $\mathcal{I}_N$ is given in (89). Thus, following the same lines in appendix C, we have:

$$\Pr\{\mathscr{R}^{(2)}_{i,lb}<R\}=q_1\xi_1+\sum_{n=2}^{K'}q_n\xi_n \tag{97}$$

where

$$\xi_1=1-\exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}})) \tag{98}$$

and

$$\xi_n|_{n\geq2}=1-E_{\mathcal{I}_n}\left\{\exp\left(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}})2^{\alpha_n(\mathcal{I}_N;\frac{\gamma}{v},\frac{v}{u})}(\frac{\gamma}{v}\mathcal{I}_n+1)^{1-a(v,n)}\right)\right\}. \tag{99}$$

Since $2\mathcal{I}_n\sim\chi^2_{2(n-1)}$, we have $p_{\mathcal{I}_n}(z)=\frac{1}{(n-2)!}z^{n-2}\exp -z$. Therefore, (99) can be expressed as

$$\xi_n|_{n\geq2}=1-\phi_n(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}}),\frac{\gamma}{v},1-a(v,n),\frac{v}{u}). \tag{100}$$

Thus,

$$R_{FH}^{(2)} = \max_{v} \sup\{R : q_1 \exp(\frac{v}{\gamma}(1-2^{\frac{R}{v}})) + \sum_{n=2}^{K'} q_n \phi_n(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1-2^{\frac{R}{v}}), \frac{\gamma}{v}, 1-a(v,n), \frac{v}{u}) > 1-\epsilon\}. \quad (101)$$

X. Appendix E

As $b_1 < 0$ and $0 < 2^{-\alpha_n(z;b_2,c_2)} < 1$, we have:

$$\phi_n(b_1, b_2, c_1, c_2) \geq \frac{1}{(n-2)!} \int_0^{\infty} z^{n-2} \exp\left(b_1(b_2 z + 1)^{c_1} - z\right) dz. \quad (102)$$

One may easily check that $\frac{1}{(n-2)!} z^{n-2} \exp(-z)$ is a PDF. Let us define the random variable Z as $p_Z(z) = \frac{1}{(n-2)!} z^{n-2} \exp(-z)$. Thus, (102) can be written as

$$\phi_n(b_1, b_2, c_1, c_2) \geq E\{\exp\left(b_1(b_2 Z + 1)^{c_1}\right)\}. \quad (103)$$

However, as $b_1 < 0$ and $0 < c_1 < 1$, the function $\exp\left(b_1(b_2 Z + 1)^{c_1}\right)$ is a convex function of Z. Hence, applying Jensen's inequality yields

$$\phi_n(b_1, b_2, c_1, c_2) \geq \exp\left(b_1(b_2 E\{Z\} + 1)^{c_1}\right) = \exp\left(b_1\left((n-1)b_2 + 1\right)^{c_1}\right) \quad (104)$$

where we have used $E\{Z\} = n - 1$. Using (104) in (46), we get the desired lower bound.

XI. Appendix F

By (22),

$$\Pr\{\mathscr{R}_{i,lb}^{(1)} < R\} = \Pr\{v \log\left(\frac{2^{-\mathcal{H}(v,N)} 2^{\mathcal{G}_{i.lb}(v,N)} |h_{i,i}|^2 \gamma}{v \prod_{m=1}^{N-1} \prod_{m'=1}^{\binom{N-1}{m}} (\frac{\gamma}{v} \mathcal{J}_{m,m'} + 1)^{c_{m,N}}} + 1\right) < R\} \quad (105)$$

where $c_{m,N} = (\frac{v}{u})^m (1 - \frac{v}{u})^{N-1-m}$ and

$$\mathcal{G}_{i,lb}(v, N) = \alpha_N(\mathcal{J}_{N-1,1}; \frac{\gamma}{v}, \frac{v}{u}). \quad (106)$$

For each m, $\{\mathcal{J}_{m,m'}\}_{m'=1}^{\binom{N-1}{m}}$ consists all possible summations of m elements in the set $\{|h_{j,i}|^2\}_{j=1, j\neq i}^{N}$. Following the same lines as in appendix C, we get:

$$\Pr\{\mathscr{R}_{i,lb}^{(2)} < R\} = q_1 \xi_1 + \sum_{n=2}^{K'} q_n \xi_n, \quad (107)$$

where

$$\xi_1 = 1 - \exp(\frac{v}{\gamma}(1 - 2^{\frac{R}{v}})) \tag{108}$$

and

$$\xi_n|_{n\geq 2} = 1 - E\Big\{ \exp\left(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1 - 2^{\frac{R}{v}})2^{-\alpha_n(\mathcal{J}_{N-1,1};\frac{\gamma}{v},\frac{v}{u})} \prod_{m=1}^{n-1} \prod_{m'=1}^{\binom{n-1}{m}} (\frac{\gamma}{v}\mathcal{J}_{m,m'} + 1)^{a_m(v,n)}\right)\Big\}. \tag{109}$$

As a result, we get

$$R_{FH}^{(1)}(\epsilon) = \max_v \sup\big\{R : q_1 \exp(\frac{v}{\gamma}(1 - 2^{\frac{R}{v}}))) + \sum_{n=2}^{K'} q_n \psi_n\big(\frac{v}{\gamma}2^{\mathcal{H}(v,n)}(1 - 2^{\frac{R}{v}}), \frac{\gamma}{v}, \frac{v}{u}\big) > 1 - \epsilon\big\}. \tag{110}$$

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