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Cooperative Strategies for Half-Duplex Parallel Relay Channel: Simultaneous Relaying versus Successive Relaying

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Abstract

This paper deals with the problem of cooperative communication for a network with multiple half-duplex relays. Two half-duplex relaying protocols, i.e. simultaneous and successive relaying protocols, associated with two possible relay orderings for a half-duplex parallel relay network are proposed. The optimum ordering of the relays and hence the capacity of the Gaussian half-duplex parallel relay network in high SNR scenarios is derived. Furthermore, for the simultaneous relaying protocol a combined Amplify-Forward and Decode-Forward (AF-DF) scheme is devised which gives a better achievable rate with respect to other known schemes in certain ranges of SNR.

I. Introduction

A. Motivation

Wireless communication has evolved considerably beyond simple voice based cellular technology. Several wireless standards such as “2.5”, third, and fourth generation cellular phone systems,

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with data transfer capabilities as a dominant feature, were designed or currently are under development. This tremendous growth in wireless communication has motivated researchers to extend Shannon's Information Theory for a single user channel to some that involves communication among multiple users. In fact, constructing a large-scale wireless data network is very expensive. Therefore, it is important to understand how to efficiently utilize the available power and bandwidth resources.

In this regard, cooperative wireless communication has received significant attention during recent years due to several reasons. First, since the received power decreases rapidly with distance, the idea of multi-hopping is becoming of particular importance. In multi-hopped communication, the source exploits some intermediate nodes as relays. Then, the source sends its message via those relays to the destination. For example, in a sensor network, each node not only transmits its own message, but also acts as a router to pass the message from other nodes in the network. Employing this technique can save battery power and increase the physical coverage area. Second, relays can emulate some kind of distributed transmit antennas to form spatial diversity and combat multi-path fading effect of the wireless channel. For example, in each cell, each user has a partner. Each of the two partners is responsible for transmitting not only its own information, but also the information of the corresponding partner, which it receives. Indeed, each user is attempting to use the other one's antenna; however, this is complicated by the fact that the interuser channel is noisy.

Motivated by practical constraints, half-duplex relays which cannot transmit and receive at the same time and in the same frequency band are of great importance. Here, our goal is to study and analyze the performance limits of the parallel half-duplex relay network.

B. History

Basically relay channel is a three terminal network which was introduced for the first time by Van-der Meulen in 1971 [1]. The most important capacity result of relay channel was reported by Cover and El Gamal [2] where they established the capacity of the discrete memoryless physically degraded relay channels, reversely degraded relay channels, additive white Gaussian noise (AWGN) degraded relay channels with average power constraints, and relay channels with feedback. However, the relay channel did not receive much attention and no major progress was made toward establishing its capacity for a long time. The reasons were the complicated nature of the problem and also lack of interest in network information theory in general during the 1990's when it was viewed as a purely mathematical theory with no practical implications. That viewpoint has changed since then with the successful implementation of several ideas of network information theory and considerable gain in performance of the systems due to application of these techniques. Recent interest in sensor and ad hoc wireless networks has revived the interest in studying AWGN relay networks.

Generally, there are two types of relays:

1. *Full-duplex relays*

   that can transmit and receive at the same time and in the same frequency band.

2. *Half-duplex relays*

   that cannot transmit and receive at the same time and in the same frequency band.

While there might exist some radio-frequency (RF) techniques to facilitate the use of full-duplex relays, exploiting them in general is regarded unrealistic in practical systems, due to the dynamic range of incoming and outgoing signals and the bulk of ferroelectric components like circulators. Therefore, although building RF radios that are capable of receiving and transmitting simultaneously in the same frequency band are not impossible in theory, they require precise and expensive components and elegant design. Hence, M. A. Khojastepour and B. Aazhang in [3] and [4] call half-duplex relay as "*Cheap Relay*" against those full-duplex relay which are really expensive.

Recently, half-duplex relaying has drawn a great deal of attention. Zahedi and El Gamal have considered two different cases of frequency division Gaussian relay channel and derived lower and upper bounds on the capacity, which in turn translates to upper and lower bounds on the minimum required energy per bit for the reliable transmission [5]. They also derived single letter characterization of the capacity of frequency division AWGN relay channel with simple linear relaying scheme [6], [7]. The problem of time division relaying has also been considered by Host-Madsen and Zhang [8]. By considering fading scenarios, taking into account practical constraint on the synchronization between source node and the relay node, and assuming channel state information (CSI), they study upper and lower bounds on the outage capacity and the ergodic capacity. In [9], Y. Liang and V. V. Veeralli present a Gaussian orthogonal relay model, in which source transmits to the relay and destination in channel 1, and the relay transmits to the destination in channel 2, with channels 1 and 2 being orthogonalized in the time-frequency plane in order to satisfy practical constraints (Figure 1).

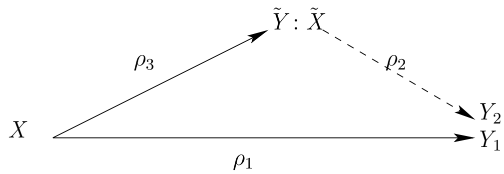

Fig. 1. Orthogonal relay channel model.

They split the total available channel resource (time and bandwidth) into the two orthogonal channels, and considering the resource allocation to the two channels as a design parameter that needs to be optimized. The main focus of their analysis is on the case where the source-to-relay channel is better than the source-to-destination channel. They show that when the SNR of the relay-to-destination channel is less than a given threshold, optimizing resource allocation causes the lower and upper bounds coincide with each other.

There are also several works on multi-relay channel in the literature. Schein in [11] established upper and lower bounds on the capacity of a full-dulex parallel relay channel where the channel consists of a source, two relays and a destination, with no direct link between the sender and the receiver (Figure 2).

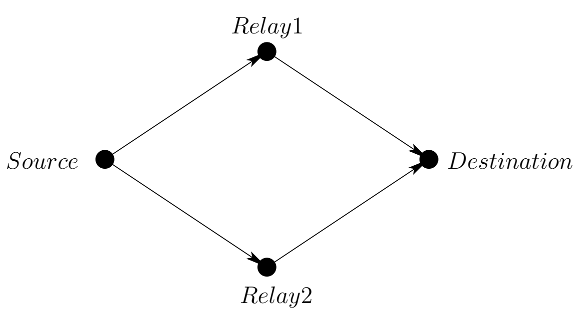

Fig. 2. Parallel relay channel.

The upper bound on the capacity of the parallel relay channel is based on the cut-set upper bound and the lower bounds are based on the block Markov and side-information coding schemes. Generally, the best rate they achieved for the full-duplex Gaussian parallel relay channel is based on Decode-Forward (DF) or Amplify-Forward (AF) with time sharing [11]. Xie and Kumar generalized the block Markov encoding scheme for a network of multiple relays [13]. Asymptotic capacity in the limit as the number of relays tends to infinity was found in [15] and [16] by employing AF at the relays. Another strategy in which relays do not decode a message, but send the compressed received values to the destination, was considered in [2]. Gastpar, Kramer, and Gupta extended compressed and forward scheme to a multiple relay channel by introducing the concept of antenna polling in [17] and [18]. They showed that when the relays are close to the destination, this strategy achieves the antenna-clustering capacity. On the other hand, when relays are close to the source, DF strategy can achieve the capacity in a wireless relay network [19]. In [20] Amichai, Shamai, Steinberg, and Kramer, considered the problem of a nomadic terminal sending information to a remote destination via agents with lossless connections. They investigated the case that these agents do not have any decoding capability, so they have to compress what they receive. They also fully characterized this case for the Gaussian channel. In [21], I. Maric, and R. D. Yates investigated DF and AF schemes in a parallel-relay network. Motivated by applications in sensor networks, they assume large bandwidth resources allowing orthogonal transmissions at the nodes. They characterize optimum resource allocation for AF and DF and showed that the wide-band regime minimizes the energy cost per information bit in DF while AF should work in different regime to get the best rate. In fact, for a network operating in the wide-band regime, there is no benefit from relays employing the AF scheme. Peyman Razaghi and Wei Yu in [22] proposed a parity-forwarding scheme for full-duplex multiple relay. They showed that relay networks can be degraded in more than one way, and parity-forwarding is capacity achieving for a new form of degraded relay networks.

C. Contributions and Relation to Previous Works

In this paper, we study cooperative strategies for a network with a source, a destination, and a set of relays which cooperate with each other to facilitate data transmission from the source to the destination.

In fact, relaying strategies for the network with multiple relays has been discussed in [10]–[14], [16]–[19], [22], [30]–[33]. Schein in [10] and [11] studied the possible coding scheme for a parallel relay channel, which consists of a source, a set of parallel relays, and the final destination. This parallel relay channel is a special case of a multiple relay network, in which source broadcasts its data to all the relays, and the relays transmit their data coherently to the destination. Later on, authors in [12]–[14], and [22] considered different cooperative strategies for general multiple relay network. These works are all dealt with full-duplex relay networks. For half-duplex case, Gastpar in [16] showed that in a Gaussian parallel relay channel with infinite number of relays the optimum coding scheme is AF. Boris Rankov and Armin Wittneben in [30] and [31] further studied the problem of half-duplex relaying in a two-hop communication scenarios. They consider a relaying protocol where two half-duplex relays, either AF or DF, alternately forward messages from the source to the destination. We call this protocol "*Successive Relaying*" protocol in the sequel. Woohyuk Chang, Sae-Young Chung, and Yong H. Lee in [32] proposed a combined Dirty Paper Coding and Block Markov encoding scheme for successive relaying protocol for half-duplex Gaussian parallel relay channel with two relays. Feng Xue, and Sumeet Sandhu in [33] further studied different half-duplex relaying protocols for Gaussian parallel relay channel. They proposed several communication schemes such as multihop with spatial reuse, scale-forward, broadcast-multiaccess with common message, compress-forward, as well as hybrid ones. Since they assumed that there is not any link between the relays, they called their parallel channel as a *Diamond Relay Channel*.

In this work we consider the problem of half-duplex relaying in a network with multiple relays in which there is no direct link between the transmitter and the receiver. We introduce two relaying protocols, i.e. simultaneous relaying protocol versus successive relaying protocol, associated with two possible relay orderings for a half-duplex parallel relay network. For simultaneous relaying, we propose a combined AF-DF scheme which leads to a better achievable rate in certain ranges of SNR and when the first hop limits the overall performance with respect to other previous schemes for parallel relay channel. Furthermore, we show that the optimum relay ordering in high SNR scenarios is achieved by successive relaying protocol. We also independently from [32] propose two different schemes for successive relaying protocol. One of them is based on superposition coding, binning, and block Markov encoding and the other one is based on Dirty Paper Coding. We show that SNR goes to infinity Dirty Paper Coding achieves the capacity of half-duplex Gaussin parallel relay channel with two relays.

The rest of the paper is organized as follows. In section II, the system model is introduced. In section III, the achievable rates for a half-duplex relay network are derived. Upper bound on the capacity of a half-duplex relay network is derived in section IV. Section V is dedicated to the Gaussian half-duplex relay network. Simulation results are presented in section VI. And finally, section VII concludes the paper.

D. Notation

Throughout the paper, lowercase bold letters and regular letters represent vectors and scalars, respectively. For any functions $f(n)$ and $g(n)$, $f(n) = O(g(n))$ is equivalent to $\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| < \infty$, and $f(n) = \Theta(g(n))$ is equivalent to $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where $0 < c < \infty$.

II. System Model

Here we consider a parallel relay channel which consists of a transmitter, a set of relays, and a receiver, and there is not any direct link between the transmitter and the receiver. What we are going to answer in this paper is: what is the best ordering of relays to convey the source's information in a half-duplex parallel relay channel? In general, two different orderings for the relays can be considered: simultaneous relaying versus successive relaying.

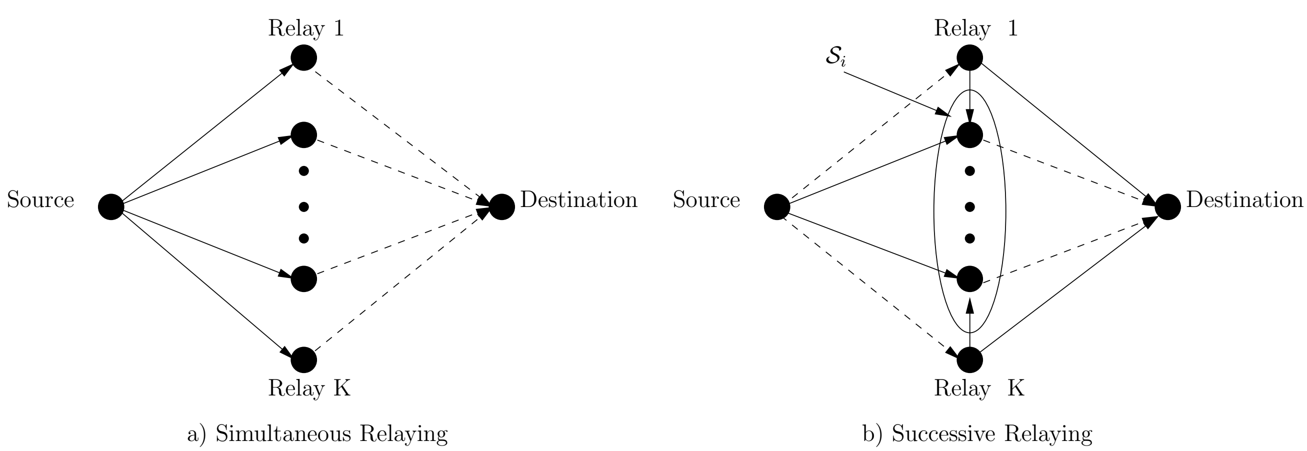

Fig. 3. Simultaneous and Successive Relaying Protocols.

A. Simultaneous Relaying

In this scenario, fig. 3a, information flow from the source to the destination goes as follows. In the first time slot, source broadcasts its signal x to the K relays. Having received noisy versions of the transmitted signal, i.e. $y_1, \cdots, y_K$, all the relays transmit their own signals, i.e. $x_1, \cdots, x_K$, simultaneously to the destination and the destination receives signal y. Signals x

and $y_1, \cdots, y_K$, and signals $x_1, \cdots, x_K$ and y are related through the transitional probabilities $p\left(y_1, \cdots, y_K \mid x\right)$ and $p\left(y \mid x_1, \cdots, x_K\right)$, respectively.

If the total available dimension (time or bandwidth) from the source to the destination is considered as “1”, one can allocate an appropriate portion $\tilde{\alpha}$ of dimension to the first hop and $1-\tilde{\alpha}$ portion to the second hop and optimize the overall performance over this parameter $\tilde{\alpha}$.

For the Gaussian case, there are two coding schemes for this setup: DF and AF. This time /bandwidth optimization over the two hops can be easily implemented by using DF scheme at each relay (we call this kind of DF, *Modified DF*), however, exploiting AF at each relay forces $\tilde{\alpha}=0.5$. This fact is one of the most important motivations for one of the main contribution of this paper which is devising a new combined AF-DF coding scheme.

Although simultaneous relaying protocol is not spectrally efficient for half-duplex scenarios, due to some practical issues it is proposed in IEEE802.16.

B. Successive Relaying

In this protocol, in the bth time slot, $1 \leq b \leq B$, a non-empty subset of relays $\mathcal{S}_i$, ($\mathcal{S}_i \subset \{1, \cdots, K\}, 0 <\mid \mathcal{S}_i \mid< K$), is chosen to listen while the relays belonging to $\{1, \cdots, K\} \backslash \mathcal{S}_i$ are sending the new information to the receiver. During every time slot, except the first and the last one, both the transmitter and the receiver links are active. Hence, in order to maximize the bandwidth usage at both the transmitter and the receiver ends, it is desirable to have a large number of time slots B.

However, in this paper due to the complexity of tackling the protocol for the general number of relays, we only consider the scenario for two relays. In this scenario, within odd/even intervals, the first/second relay is listening to the source and the other relay, whereas the other relay is sending its information to the destination. The information flow of successive relaying protocol is illustrated in fig. 3b.

III. Achievable Rates

A. Simultaneous Relaying Protocol

1) Observe-and-Forward(OF): In the Observe and Forward scheme, what each relay transmits is based only on one symbol it has received(one shot relaying). From the probabilistic point of view, the kth relay at each time “i” transmits x_{ik} according to the probability $p(x_{ik} \mid y_{ik})$. Here

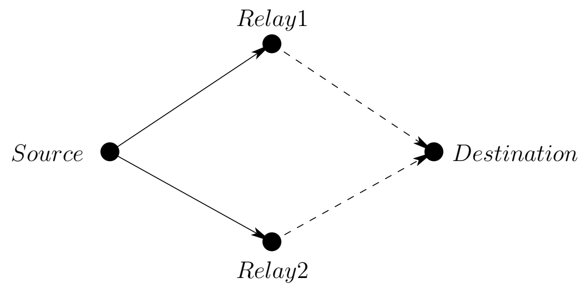

Fig. 4. Simultaneous Relaying Protocol.

we prove that, in order to maximize the information rate from the source to the destination i.e. $I(X;Y)$, a deterministic function should be used at each relay. First we have the following lemma.

Lemma 1 *Each stochastic matrix can be written as a convex combination of permutation matrices.*

Proof: This lemma was proved in [24]. ∎

Now, we have the following theorem:

Theorem 1 *In a parallel relay channel, assuming simultaneous relaying protocol, there exists a series of deterministic functions associated with each relay that maximize the information rate from the source to the destination.*

Proof: From Lemma 1, we can write the transition matrix $[p(x_{ik}|y_{jk})]$, for each relay $k \in \{1, 2, \cdots, K\}$, as follows:

$$[p(x_{ik}|y_{jk})] = \sum_{j=1}^{M_k} p_{k_j} \mathbf{F}_{k_j}, \quad p_{k_j} \geq 0, \quad \sum_{j=1}^{M_k} p_{k_j} = 1. \tag{1}$$

Where $\mathbf{F}_{k_j}$'s are the permutation matrices, which can be considered as deterministic functions, and the coefficients p_{k_j}'s are the associated probabilities with which each deterministic function $\mathbf{F}_{k_j}$ is used at each relay.

So we define K auxiliary random variables $\{\Theta_1, \Theta_2, \cdots, \Theta_k, \cdots, \Theta_K\}$. Each of them can take values in $\{1, \cdots, M_k\}$ with probabilities $\{p_{k_1}, \cdots, p_{k_{M_k}}\}$ associated with the usage of each deterministic function $\{\mathbf{F}_{k_1}, \cdots, \mathbf{F}_{k_{M_k}}\}$ at the kth relay.

Therefore, knowing Θ_k's are independent of X, we have

$$\begin{aligned} & I\left(X;Y\right) \leq I\left(X;Y \mid \Theta_1, \cdots, \Theta_K\right) = \\ & \sum_{i,j,\cdots,l} p_{1_i} p_{2_j} \cdots p_{K_l} I\left(X;Y \mid \Theta_1 = i, \Theta_2 = j, \cdots, \Theta_K = l\right) \\ & \leq \max I\left(X;Y \mid \Theta_1 = i^*, \Theta_2 = j^*, \cdots, \Theta_K = l^*\right). \end{aligned} \tag{2}$$

Hence, there exits K deterministic functions $\{\mathbf{F}_{1_{i^*}}, \mathbf{F}_{2_{j^*}}, \cdots, \mathbf{F}_{K_{l^*}}\}$, which can be used at each relay and maximize the information rate from the source to the destination. ■

2) Modified Decode-and-Forward (DF): Fig. 4 shows simultaneous relaying protocol for two relays. Let us assume that the channel from the source to the first relay is better than the channel from the source to the other relay. Hence, in this situation, transmitter splits its message into *"Private"* and *"Common"* messages.

The *"Private"* message is the message which is decodable only by the first relay whereas the *"Common"* message is the message that can be decoded by both relays.

The achievable rate of this DF scheme is the sum of the rate pairs, (R_p, R_c) associated with the private and common rate, respectively. These pairs should be both in the capacity region of the Broadcast channel (BC) i.e. first hop [23] and the extended Multiple access channel (MAC) i.e. second hop ([25], [26], [27]). By extended MAC, we mean the MAC in which one user knows the other one's message. Hence, the achievable rate for this scheme is:

$$R = R_p + R_c. \tag{3}$$

From the Broadcast Channel(BC) and for the first hop, we have:

$$\begin{aligned} R_p &\leq \tilde{\alpha} I(X;Y_1 \mid U), \\ R_c &\leq \tilde{\alpha} I(U;Y_1). \end{aligned} \tag{4}$$

For the second hop and from the capacity region of the extended MAC [25], we have:

$$\begin{aligned} R_p &\leq (1-\tilde{\alpha}) I(X_1;Y \mid X_2), \\ R_p + R_c &\leq (1-\tilde{\alpha}) I(X_1, X_2;Y). \end{aligned} \tag{5}$$

	$Block1$	$Block2$	$Block3$	$Block4$
$Source$	$\tilde{x}(w_1\|1,1)$	$\tilde{\tilde{x}}(w_2\|w_1,1)$	$\tilde{x}(w_3\|w_2,s_1^1)$	$\tilde{\tilde{x}}(w_4\|w_3,s_2^2)$
$Relay1$		$\tilde{\tilde{x}}_1(w_1\|1),\tilde{\tilde{u}}(1)$		$\tilde{\tilde{x}}_1(w_3\|s_2^2),\tilde{\tilde{u}}(s_2^2)$
$Relay2$	$\tilde{x}_2(1\|1),\tilde{u}(1)$		$\tilde{x}_2(w_2\|s_1^1),\tilde{u}(s_1^1)$	

Fig. 5. Decode-and-forward for successive relaying protocol.

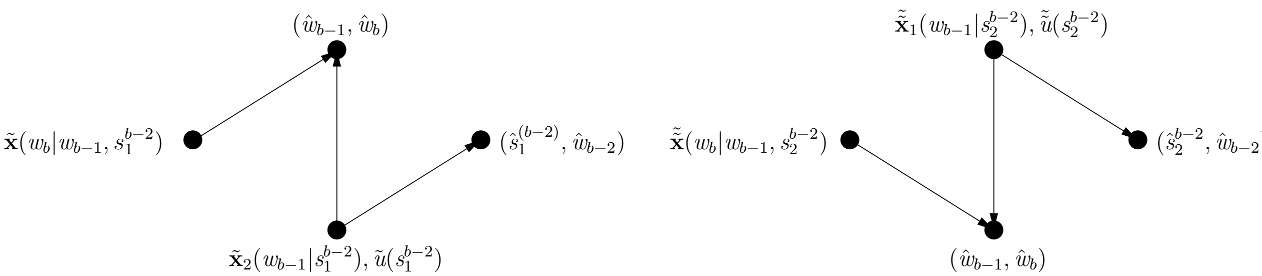

Fig. 6. Successive relaying protocol for two relays.

B. Successive Relaying Protocol

1) Cooperative Coding: In this section, we propose a coding scheme based on binning, superposition coding, and block Markov encoding for a half-duplex parallel relay network with two relays. The extension to a relay network with more than two relays is straightforward. The message w is divided into B blocks $w_1, w_2, \cdots, w_B$ of nR bits each. The transmission is performed in $B+2$ blocks.

Generally, this scheme can be described as follows. In each time slot b, source transmits new message w_b to one of the relays. Each time, one of the relays is receiving data from the source and the other relay, while the other relay is transmitting its information to the silent relay and the destination (Figs. 5 and 6). Each silent relay decodes the transmitted messages w_b and w_{b-1} from the source and the other relay, respectively. On the other hand, each transmitting relay-using the binning function-broadcasts the bin index of the message it has received from the relay during the last interval along the message it has received from the source to the other relay and the destination. Binning function can be defined as follows:

Definition (The Binning Function): Consider a set of integers, $Q = \{1, 2, \ldots, 2^{nR_Q}\}$. Let $Bin = \{S_1, S_2, \ldots, S_{2^{nR_{Bin}}}\}$ denotes a random independent uniform partitioning of elements of Q into $2^{nR_{Bin}}$ subsets $S_1, S_2, \ldots, S_{2^{nR_{Bin}}}$. The binning function $P_{R_{Bin},Bin}(w) : Q \longrightarrow \{1, 2, \ldots, 2^{nR_{Bin}}\}$is defined by $P_{R_{Bin},Bin}(w) = q$ if $w \in S_q$.

As indicated in fig. 5, in the first time slot, source transmits the codeword $\tilde{\mathbf{x}}(w_1|1,1)$ generated according to the probability $p(\tilde{x})$ to the first relay, while the second relay transmits a doubly index codeword $\tilde{\mathbf{x}}_2(1|1)$ and the codeword $\tilde{\mathbf{u}}(1)$ according to the probability $p(\tilde{x}_2|\tilde{u})p(\tilde{u})$ to the first relay and the destination. In the second time slot, source transmits the codeword $\tilde{\tilde{\mathbf{x}}}(w_2|w_1,1)$ to the second relay, and having decoded the message w_1, the first relay broadcasts the codewords $\tilde{\tilde{\mathbf{x}}}_1(w_1|1)$ and $\tilde{\tilde{\mathbf{u}}}(1)$ to the second relay and the destination. It should be noted that the destination cannot decode the message w_1 at the end of this time slot; however, the second relay decodes w_1 and w_2 messages. Using the binning function, it finds the bin index of w_1 according to $s_1^1 = P_{R_1,Bin_1}(w_1)$. In the third time slot, source transmits the codeword $\tilde{\mathbf{x}}(w_3|w_2,s_1^1)$ to the first relay, and the second relay broadcasts the codewords $\tilde{\mathbf{x}}_2(w_2|s_1^1)$ and $\tilde{\mathbf{u}}(s_1^1)$ to the first relay and the destination, respectively. Two types of decoding can be used at the destination, i.e. successive decoding and backward decoding. Successive decoding at the destination can be described as follows. At the end of the third time slot, the destination cannot decode the message w_2; however, having decoded the bin index s_1^1, it can decode the message w_1. On the other hand, backward decoding can be explained as follows. Having received $B+2$ blocks, the final destination starts decoding the intended messages. In the time slot $B+2$, one of the relays transmits the dummy message "1" along with the bin index of the message w_B to the destination. Having received this bin index, the destination decodes it, and then backwardly decodes messages $w_i, \ i = 1, \cdots, B$ and their bin indices.

From now on, each relay does the same job in an alternating fashion. Hence, we have the following theorem:

Theorem 2 *For the half-duplex parallel relay channel, assuming successive relaying, the Block Markov scheme achieves the following rates $R_{BM_{suc}}$, and $R_{BM_{back}}$, using successive and back-*

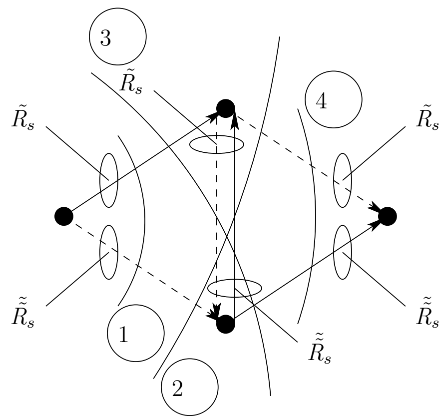

Fig. 7. Information flow transfer for successive relaying protocol for two relays.

ward decoding, respectively:

$$\begin{aligned}
R_{BM_{suc}} &= \tilde{R}_s + \tilde{\tilde{R}}_s \leq \max_{0 \leq \acute{t}_1, \acute{t}_2, \acute{t}_1 + \acute{t}_2 = 1} \min \Big(\\
&\min \left(\acute{t}_1 I \left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2, \tilde{U} \right), \acute{t}_2 I \left(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}} \mid \tilde{\tilde{U}} \right) + \acute{t}_1 I \left(\tilde{U}; \tilde{Y} \right) \right) + \\
&\min \left(\acute{t}_1 I \left(\tilde{X}_2; \tilde{Y} \mid \tilde{U} \right) + \acute{t}_2 I \left(\tilde{\tilde{U}}; \tilde{\tilde{Y}} \right), \acute{t}_2 I \left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1, \tilde{\tilde{U}} \right) \right), \\
&\acute{t}_1 I \left(\tilde{X}, \tilde{X}_2; \tilde{Y}_1 \mid \tilde{U} \right), \acute{t}_2 I \left(\tilde{\tilde{X}}, \tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{U}} \right) \Big). \qquad (6) \\
R_{BM_{back}} &= \tilde{R}_s + \tilde{\tilde{R}}_s \leq \max_{0 \leq \acute{t}_1, \acute{t}_2, \acute{t}_1 + \acute{t}_2 = 1} \min \Big(\\
&\acute{t}_1 I \left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2, \tilde{U} \right) + \acute{t}_2 I \left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1, \tilde{\tilde{U}} \right), \\
&\acute{t}_1 I \left(\tilde{X}_2, \tilde{U}; \tilde{Y} \right) + \acute{t}_2 I \left(\tilde{\tilde{X}}_1, \tilde{\tilde{U}}; \tilde{\tilde{Y}} \right), \\
&\acute{t}_1 I \left(\tilde{X}, \tilde{X}_2; \tilde{Y}_1 \mid \tilde{U} \right), \acute{t}_2 I \left(\tilde{\tilde{X}}, \tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{U}} \right) \Big). \qquad (7)
\end{aligned}$$

Proof: See Appendix A. ■

2) Non-Cooperative Coding: In this scheme, each relay considers the other one's signal as interference. Since the transmitter knows each relay's message, it can apply the Gelfand-Pinsker's coding scheme to transmit its message to each of the relays. In this case we have the following theorem:

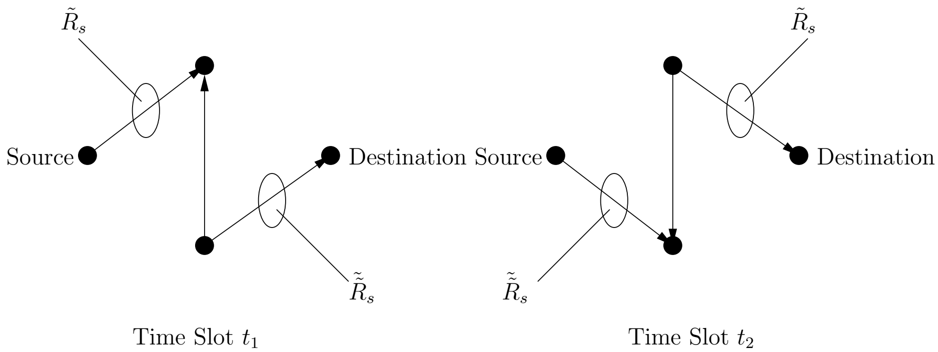

Fig. 8. Dirty paper coding for successive relaying protocol for two relays.

Theorem 3 *For the half-duplex parallel relay channel, assuming successive relaying, the following rate R_{DPC} is achievable:*

$$\begin{aligned} R_{DPC} &\leq \tilde{R}_s + \tilde{\tilde{R}}_s \\ &= \min\left(\acute{t}_1(I(\tilde{U};\tilde{Y}_1) - I(\tilde{U};\tilde{X}_2)), \acute{t}_2 I(\tilde{\tilde{X}}_1;\tilde{\tilde{Y}})\right) + \\ &\quad \min\left(\acute{t}_2(I(\tilde{\tilde{U}};\tilde{\tilde{Y}}_2) - I(\tilde{\tilde{U}};\tilde{\tilde{X}}_1)), \acute{t}_1 I(\tilde{X}_2;\tilde{Y})\right). \end{aligned} \tag{8}$$

with probabilities:

$$p(\tilde{x}_2, \tilde{u}, \tilde{x}) = p(\tilde{x}_2)p(\tilde{u}|\tilde{x}_2)p(\tilde{x}|\tilde{u}, \tilde{x}_2), \tag{9}$$

$$p(\tilde{\tilde{x}}_1, \tilde{\tilde{u}}, \tilde{\tilde{x}}) = p(\tilde{\tilde{x}}_1)p(\tilde{\tilde{u}}|\tilde{\tilde{x}}_1)p(\tilde{\tilde{x}}|\tilde{\tilde{u}}, \tilde{\tilde{x}}_1). \tag{10}$$

Proof: See Appendix B. ■

IV. UPPER BOUNDS

In this section, the upper bound on the parallel relay network with two relays is derived and investigated.

The authors in [28] proposed some upper bounds on achievable rates for the general half-duplex multi-terminal networks. Here we explain their results briefly and apply them to our half-duplex parallel relay network.

Consider a network with *N* nodes. We define the state of the network as a valid partitioning of the nodes of the network into two sets of the *"sender nodes"* and the *"receiver nodes"* such that there is no active link that arrives at a sender node. It is safe to say that the number of possible states *M* of a network with finite number of nodes is finite. Let t_m defines the portion of the time that network is used in state *m*-where $m \in \{1, 2, \ldots, M\}$. The following theorem was proved in [28]:

Theorem 4 *Consider a general network with finite states, M, for which the sequence m_k of the states of the network is known to all nodes. Maximum achievable information rates $\{R^{ij}\}$ from a node set S_1 to a disjoint node set S_2, $S_1, S_2 \subset \{1, 2, \ldots, N\}$ for the proper choice of network state sequence m_k is bounded by:*

$$\sum_{i\in S_1, j\in S_2} R^{ij} \leq \sup_{t_m} \min_{S} \sum_{m=1}^{M} t_m I\left(X^S_{(m)}; Y^S_{(m)} \mid X^{S^c}_{(m)}\right). \tag{11}$$

for some joint probability distribution $p(x^{(1)}, x^{(2)}, \ldots, x^{(N)} \mid m)$ when the minimization is taken over all set $S \subset \{1, 2, \ldots, N\}$ subject to $S \bigcap S_1 = S_1$, $S \bigcap S_2 = \emptyset$ and the supremum is over all the non-negative t_m subject to $\sum_{i=1}^{M} t_m = 1$.

In our network $N = 4$ and since we have $N - 2 = 2$ relays, the number of possible states $M = 2^2$. Four different cuts can be taken into account as in fig. 9. The information flows associated with different cuts at each state are calculated as follows

1. State 1:

-*First Cut:*

$$t_1 I\left(\tilde{X}; \tilde{Y}_1, \tilde{Y} \mid \tilde{X}_2\right) = t_1 I\left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2\right). \tag{12}$$

Note that in deriving (12) we have

$$\tilde{X} \longrightarrow (\tilde{X}_2, \tilde{Y}_1) \longrightarrow \tilde{Y}$$

-*Second Cut:* No information is transferred through this cut.

-*Third Cut:* Similar to the first cut, we have

$$t_1 I\left(\tilde{X}, \tilde{X}_2; \tilde{Y}_1, \tilde{Y}\right) = t_1 I\left(\tilde{X}_2; \tilde{Y}_1, \tilde{Y}\right) + t_1 I\left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2\right). \tag{13}$$

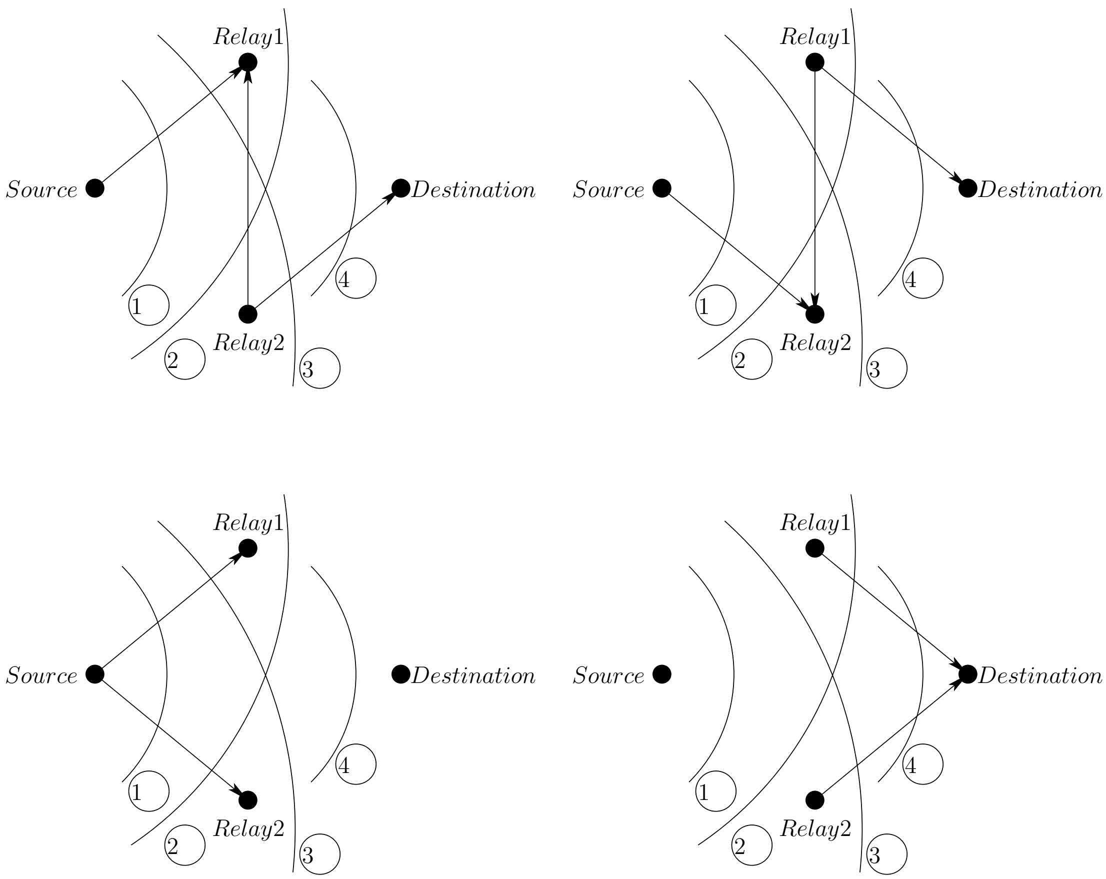

Fig. 9. Possible States for Parallel Relay Channel.

-Fourth Cut:

$$t_1 I\left(\tilde{X}, \tilde{X}_2; \tilde{Y}\right) = t_1 I\left(\tilde{X}_2; \tilde{Y}\right). \tag{14}$$

2. State 2:

Just the same as in state 1, we have the following equations associated with different cuts

-First Cut:

$$t_2 I\left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2, \tilde{\tilde{Y}} \mid \tilde{\tilde{X}}_1\right) = t_2 I\left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1\right). \tag{15}$$

-Second Cut: Similar to the first cut we have

$$t_2 I\left(\tilde{\tilde{X}}, \tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2, \tilde{\tilde{Y}}\right) = t_2 I\left(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2, \tilde{\tilde{Y}}\right) + t_2 I\left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1\right). \tag{16}$$

-Third Cut: No information is transferred through this cut.

-Fourth Cut:

$$t_2 I\left(\tilde{\tilde{X}}, \tilde{\tilde{X}}_1; \tilde{\tilde{Y}}\right) = t_2 I\left(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}}\right). \tag{17}$$

3. State 3:

-First Cut:

$$t_3 I\left(X; Y_1, Y_2, Y\right) = t_3 I\left(X; Y_1, Y_2\right). \tag{18}$$

-Second Cut:

$$t_3 I\left(X; Y_2, Y\right) = t_3 I\left(X; Y_2\right). \tag{19}$$

-Third Cut:

$$t_3 I\left(X; Y_1, Y\right) = t_3 I\left(X; Y_1\right). \tag{20}$$

No information is transferred through the fourth cut.

4. State 4:

-Second Cut:

$$t_4 I\left(X, X_1; Y \mid X_2\right) = t_4 I\left(X_1; Y \mid X_2\right). \tag{21}$$

-Third Cut:

$$t_4 I\left(X, X_2; Y \mid X_1\right) = t_4 I\left(X_2; Y \mid X_1\right). \tag{22}$$

-Fourth Cut:

$$t_4 I\left(X_1, X_2; Y\right). \tag{23}$$

No information is transferred through the first cut.

From (12), (13), (14), (15), (16), (17), (18), (19), (20), (21), (22), and (23), the maximum achievable rate C_{up} of our network is upper bounded as

$$\begin{aligned} C_{up} \leq \min(& t_1 I\left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2\right) + t_2 I\left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1\right) + t_3 I\left(X; Y_1, Y_2\right), \\ & t_2 I\left(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2, \tilde{\tilde{Y}}\right) + t_2 I\left(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1\right) + t_3 I\left(X; Y_2\right) + t_4 I\left(X_1; Y \mid X_2\right), \\ & t_1 I\left(\tilde{X}_2; \tilde{Y}_1, \tilde{Y}\right) + t_1 I\left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2\right) + t_3 I\left(X; Y_1\right) + t_4 I\left(X_2; Y \mid X_1\right), \\ & t_1 I\left(\tilde{X}_2; \tilde{Y}\right) + t_2 I\left(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}}\right) + t_4 I\left(X_1, X_2; Y\right)). \end{aligned} \tag{24}$$

V. Gaussian Case

A. Simultaneous Relaying

In the Gaussian case, assuming simultaneous relaying, in the first time slot source transmits a gaussian codeword $\mathbf{x}$ with zero mean and variance P_s to relay 1 up to relay k. Hence, the kth relay receives

$$\mathbf{y}_k = h_{0k}\mathbf{x} + \mathbf{z}_k. \tag{25}$$

In the second time slot, the kth relay transmits a gaussian codeword $\mathbf{x}_k$ with zero mean and variance P_{r_k} to the final destination. Hence, the destination receives

$$\mathbf{y} = \sum_{k=1}^{K} h_{kK+1}\mathbf{x}_k + \mathbf{z}. \tag{26}$$

where $\mathbf{z}_k$ and $\mathbf{z}$ are additive white gaussian noises with zero mean and variance "1" per dimension, and h_{0k} and h_{kK+1} are channel coefficients from the source to the kth relay and from the kth relay to the final destination, respectively.

1) Decode-and-Forward(DF): Fig. 10 shows simultaneous relaying protocol for a half-duplex Gaussian parallel relay channel. Here, we assume that relay 1 has a better receiving channel than relay 2 (i.e. $h_{01} > h_{02}$). In this situation, transmitter splits its total available power P_s to P_{s-p} and P_{s-c} associated with the *"Private"* and *"Common"* messages, respectively.

From the argument we had in section III-A and (3), (4), and (5) for the Gaussian case the

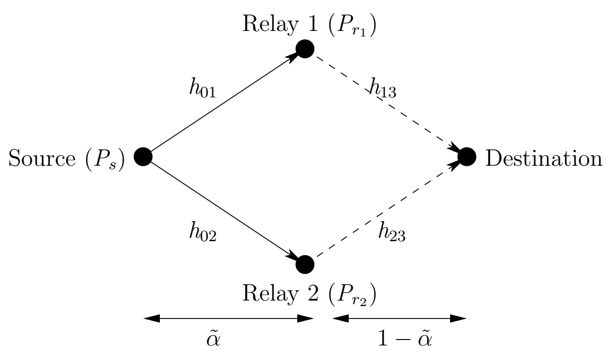

Fig. 10. Simultaneous relaying protocol for two relays.

following rate R is achievable

$$R = R_p + R_c, \tag{27}$$

$$R_p \leq \tilde{\alpha} C \left(\frac{h_{01}^2 P_{s-p}}{\tilde{\alpha}} \right), \tag{28}$$

$$R_c \leq \tilde{\alpha} C \left(\frac{h_{02}^2 P_{s-c}}{\tilde{\alpha} + h_{02}^2 P_{s-p}} \right), \tag{29}$$

$$R_p + R_c \leq (1-\tilde{\alpha}) C \left(\frac{h_{13}^2 P_{r_1-p} + (h_{13}\sqrt{P_{r1-c}} + h_{23}\sqrt{P_{r_2}})^2}{1-\tilde{\alpha}} \right), \tag{30}$$

$$R_p \leq (1-\tilde{\alpha}) C \left(\frac{h_{13}^2 P_{r_1-p}}{1-\tilde{\alpha}} \right). \tag{31}$$

Finally, we have also the following constraints on the powers available at the source and each relay.

$$\boldsymbol{P}_s = \boldsymbol{P}_{s-p} + \boldsymbol{P}_{s-c}, \tag{32}$$

$$\boldsymbol{P}_{r_1} = \boldsymbol{P}_{r_1-p} + \boldsymbol{P}_{r_1-c}, \tag{33}$$

$$\boldsymbol{P}_s \geq 0, \boldsymbol{P}_{r_1} \geq 0, \boldsymbol{P}_{r_2} \geq 0. \tag{34}$$

Now, we have the following *Proposition*.

Proposition 1 *The rate of DF scheme is achievable by successive decoding of the common and private messages at the receiver side.*

Proof: Consider the sum rate for both the common message and private message for the extended multiple access channel from relays to destination

$$R_p + R_c = (1-\tilde{\alpha}) C \left(\frac{h_{13}^2 P_{r_1-p} + (h_{13}\sqrt{P_{r_1-c}} + h_{23}\sqrt{P_{r_2}})^2}{1-\tilde{\alpha}} \right). \tag{35}$$

It can be readily verified that this sum rate is a decreasing and increasing functions of P_{r_1-p} and P_{r_1-c}, respectively. Now, let us equate R_p in (35) with the private rate $\acute{R}_p$ of another MAC which is achieved by successive decoding of common and private messages. Therefore, we have

$$R_p = \acute{R}_p = (1-\tilde{\alpha}) C \left(\frac{h_{13}^2 \acute{P}_{r_1-p}}{1-\tilde{\alpha}} \right). \tag{36}$$

As is indicated in fig. (11), we have

$$\acute{P}_{r_1-p} \leq P_{r_1-p} \implies \acute{P}_{r_1-c} \geq P_{r_1-c},$$
$$R_p + R_c \leq \acute{R}_p + \acute{R}_c,$$
$$R_c \leq \acute{R}_c.$$

Hence, successive decoding of common and private messages achieves the DF rate. ∎

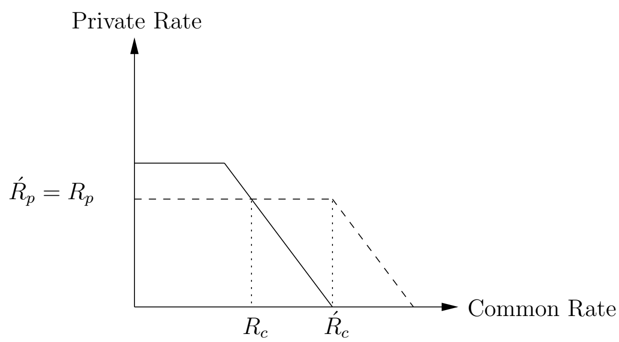

Fig. 11. The order of decoding *"Common"* and *"Private"* messages.

2) Proposed Scheme: Combined Amplify-and-Decode Forward: In this section, our proposed scheme for simultaneous relaying protocol is studied. For the sake of simplicity, we only consider the symmetric scenarios, in which $h_{01} = h_{02}$, and $h_{13} = h_{23}$. Generally speaking, gaussian parallel relay channel is a two-hop communication scenario in which the performance of the first hop limits the overall performance. This fact motivates us to propose some novel coding schemes to improve the performance of the first hop. Specifically, we want to benefit from utilizing a higher bandwidth in the first hop. As it was shown in the previous section, using DF scheme at each relay, this can be easily implemented. Source transmits a codeword of length $\tilde{\alpha}n$. After decoding the transmitted message at each relay, relays re-encode their decoded message by another codebook of length $(1-\tilde{\alpha})n$, and transmit the associated codeword all together coherently to the destination. This scheme is the first trivial scheme that can be obtained by simply modifying naive DF. The achievable rate of this Modified DF scheme was derived in the previous section. However, as will be shown in the numerical result section, DF schemes do not perform well with respect to AF. As a result, we are looking for a new scheme which benefits from both the advantages of AF and also reducing the effect of Gaussian noise by exploiting higher dimension at the first hop. In other words, we aim to answer this question: how can we convert a higher dimensional signal space to a lower one without decoding and re-encoding at the relays. Here we only consider the scenarios where the SNR of the first hop is less than or equal to the SNR of the second hop.

The combined AF-DF scheme is illustrated in figs. 12 and 13. In this scheme, we split our message to AF message and DF message. The AF message is transmitted in 2α dimension out of the total "1" dimension available: α dimension from the source to each relay, and α dimension from each relay to the destination. Basically, DF message is transmitted in all the available dimensions i.e. "1": from source to each relay, we put DF message in some extra β dimension and if the allocated power to this β portion exceeds the level ν, we equalize DF message power between the α and β dimensions that are available in the first hop, i.e. we do water-filling as follows and put some part of our DF message (DF message 1) over AF message in the first α dimension (Figure 13). If the power constraint at the source is P_s, and the assigned power to AF message, DF message 1, and DF message 2 are $P_{s\text{-}AF}$, P_{s-DF_1}, and P_{s-DF_2} respectively, from water-filling we have

$$P_{s\text{-}AF} + P_{s-DF_1} + P_{s-DF_2} = P_s, \tag{37}$$

$$\overbrace{(\nu - \frac{P_{s\text{-}AF}}{\alpha} - 1)\alpha}^{P_{s-DF_1}} + \overbrace{(\nu - 1)\beta}^{P_{s-DF_2}} = P_{s\text{-}DF}. \tag{38}$$

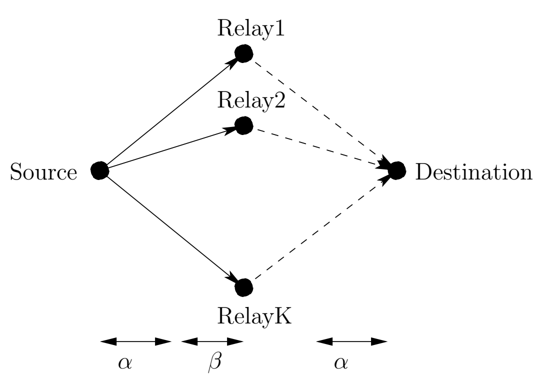

Fig. 12. Bandwidth Allocation in Gaussian Parallel Relay Channel (Proposed Scheme).

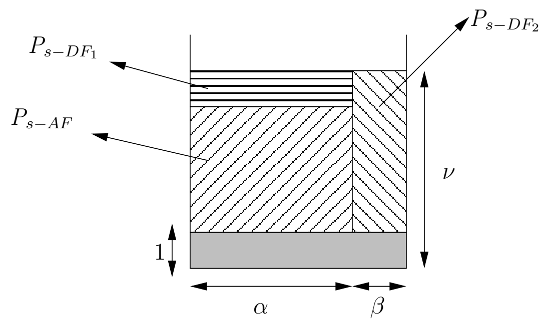

Fig. 13. Water-filling between DF and AF message.

Therefore

$$P_{s-DF_1} = \max\left(\frac{\alpha(P_s - P_{s\text{-}AF}) - \beta P_{s\text{-}AF}}{\alpha+\beta}, 0\right), \tag{39}$$

$$P_{s-DF_2} = \frac{\beta P_s}{\alpha+\beta}. \tag{40}$$

The relays decode the whole DF message, as described above and re-encode it and send the re-encoded version along with the AF message to the destination. Indeed, by decoding DF message 2 in that extra β dimension at each relay, we are exploiting some extra dimensions in the first hop to decrease the noise effect.

Furthermore, from the AF and DF model described in the previous section, P_{DF}, P_{AF}, and the effective noise power per dimension N_{AF} can be calculated as follows. If we split the total available power at each relay, namely P_r, into $P_{r\text{-}AF}$ and $P_{r\text{-}DF}$, we have

$$P_{r\text{-}AF} + P_{r\text{-}DF} = P_r, \tag{41}$$

$$P_{DF} = K^2 P_{r\text{-}DF}, \tag{42}$$

$$P_{AF} = K^2 \frac{P_{r\text{-}AF} P_{s\text{-}AF}}{P_{s\text{-}AF} + \alpha}, \tag{43}$$

$$N_{AF} = K \frac{P_{r\text{-}AF}}{P_{s\text{-}AF} + \alpha} + 1. \tag{44}$$

From the multiple access part of the relay channel, we have:

$$R \leq \alpha C\left(\frac{P_{AF} + P_{DF}}{\alpha N_{AF}}\right). \tag{45}$$

But, if the first hop limits the rate of DF message, from the above discussion, the rate from the source to the destination is

$$R \leq \alpha C\left(\frac{P_{AF}}{\alpha N_{AF}}\right) + \alpha C\left(\frac{P_{s-DF_1}}{P_{s\text{-}AF} + \alpha}\right) + \beta C\left(\frac{P_{s-DF_2}}{\beta}\right). \tag{46}$$

Now, we have the following theorem

Theorem 5 *For the half-duplex Gaussian parallel relay channel, assuming simultaneous relaying protocol with power constraint at source and each relay the following rate is achievable*

$$R = \min\left(\alpha C\left(\frac{P_{AF}+P_{DF}}{\alpha N_{AF}}\right), \alpha C\left(\frac{P_{AF}}{\alpha N_{AF}}\right) + \alpha C\left(\frac{P_{s-DF_1}}{P_{s-AF}+\alpha}\right) + \beta C\left(\frac{P_{s-DF_2}}{\beta}\right)\right), \tag{47}$$

$$\begin{aligned}
subject \quad to \quad & 2\alpha+\beta \leq 1 \\
P_{s-AF} \quad & +P_{s-DF_1} + P_{s-DF_2} = P_s, \\
P_{r-AF} \quad & +P_{r-DF} = P_r, \\
\alpha \quad & \geq 0, \\
\beta \quad & \geq 0, \\
P_{s-AF} \quad & \geq 0, \\
P_{s-DF_1} \quad & \geq 0, \\
P_{s-DF_2} \quad & \geq 0, \\
P_{r-AF} \quad & \geq 0, \\
P_{r-DF} \quad & \geq 0, \\
P_{DF} \quad & = K^2 P_{r-DF}, \\
P_{AF} \quad & = K^2 \frac{P_{r-AF}P_{s-AF}}{P_{s-AF}+\alpha}, \\
N_{AF} \quad & = K\frac{P_{r-AF}}{P_{s-AF}+\alpha} + 1.
\end{aligned}$$

In fact, the above optimization problem is a constrained non-convex optimization one.

Proof: See Appendix C. ■

Proposition 2 *The rate of the combined AF-DF scheme is achievable by successive decoding of the DF and AF messages at the receiver side.*

Proof: Substituting for P_{DF} and P_{AF}, from (41) into (45), we get

$$R \leq \alpha C\left(\frac{K^2 P_{s-AF}\left(P_{r-DF}+\eta\left(P_r-P_{r-DF}\right)\right)}{\alpha\left(K\eta\left(P_r-P_{r-DF}+P_{s-AF}\right)\right)}\right). \tag{48}$$

Where $0 \leq \eta = \frac{P_{s-AF}}{P_{s-AF}+\alpha} \leq 1$. It can be readily verified that (48) is an increasing and decreasing functions of P_{r-DF} and P_{r-AF}, respectively. Hence, from the same argument as in Proposition 1, the rate R is achievable by successive decoding of the DF and AF messages at the final destination. ■

By considering the appropriate order of decoding of DF message and AF message at the destination, the achievable rate can be simplified as

$$\begin{aligned} R &= \max\left(\alpha C\left(\frac{P_{AF}}{\alpha N_{AF}}\right)+\right. \\ &\left.\min\left(\alpha C\left(\frac{P_{DF}}{\alpha N_{AF}+P_{AF}}\right), \alpha C\left(\frac{P_{s-DF_1}}{P_{s-AF}+\alpha}\right)+\beta C\left(\frac{P_{s-DF_2}}{\beta}\right)\right)\right). \end{aligned} \tag{49}$$

B. Successive Relaying

In Gaussian case, assuming successive relaying with two relays, in the first time slot the source and the second relay transmit their zero mean gaussian codewords $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{x}}_2$ with variances P' and P'_2 to the first relay and final destination, respectively. Hence, the first relay and the final destination receive

$$\begin{aligned} \tilde{\mathbf{y}}_1 &= h_{01}\tilde{\mathbf{x}} + h_{21}\tilde{\mathbf{x}}_2 + \tilde{\mathbf{z}}_1, &(50)\\ \tilde{\mathbf{y}} &= h_{23}\tilde{\mathbf{x}}_2 + \tilde{\mathbf{z}}. &(51)\end{aligned}$$

Similarly, in the second time slot the source and the first relay transmit their zero mean gaussian codewords $\tilde{\tilde{\mathbf{x}}}$ and $\tilde{\tilde{\mathbf{x}}}_1$ with variances P'' and P''_1 to the second relay and final destination, respectively. Hence, the second relay and the final destination receive

$$\begin{aligned} \tilde{\tilde{\mathbf{y}}}_2 &= h_{02}\tilde{\tilde{\mathbf{x}}} + h_{12}\tilde{\tilde{\mathbf{x}}}_1 + \tilde{\tilde{\mathbf{z}}}_2, &(52)\\ \tilde{\tilde{\mathbf{y}}} &= h_{13}\tilde{\tilde{\mathbf{x}}}_1 + \tilde{\tilde{\mathbf{z}}}. &(53)\end{aligned}$$

where $\tilde{\mathbf{z}}_1$, $\tilde{\tilde{\mathbf{z}}}_2$, $\tilde{\mathbf{z}}$, and $\tilde{\tilde{\mathbf{z}}}$ are additive white gaussian noises with zero mean and variance "1" per dimension, and h_{01}, h_{02}, h_{12}, h_{21}, h_{13}, and h_{23} are channel coefficients from: the source to the first relay, the source to the second relay, the first relay to the second relay, the second relay to the first relay, the first relay to the destination, and the second relay to the destination, respectively. By channel receprosity assumption, $h_{12} = h_{21}$.

From Theorem 2 and 3, we have the following corollaries for the Gaussian case.

corollary 1 *For the half-duplex Gaussian parallel relay channel, assuming successive relaying protocol with power constraint at source and each relay Block Markov encoding achieves the following rate*

$$\begin{aligned} R_{BM} \leq \; & \min\left(R_{BM_1} + R_{BM_2},\right. &(54)\\ & \acute{t}_1 C\left(\frac{h_{01}^2 P' + h_{12}^2\theta_2 P_{r_2} + 2h_{01}h_{12}\sqrt{\bar{\alpha}_1\theta_2 P' P_{r_2}}}{\acute{t}_1}\right), \\ & \left.\acute{t}_2 C\left(\frac{h_{02}^2 P'' + h_{12}^2\theta_1 P_{r_1} + 2h_{02}h_{12}\sqrt{\bar{\alpha}_2\theta_1 P'' P_{r_1}}}{\acute{t}_2}\right)\right). \end{aligned}$$

$$
\begin{aligned}
&subject \quad to \\
&\qquad P' + P'' = P_s, \\
&\qquad \acute{t}_1 + \acute{t}_2 = 1, \\
&\qquad 0 \leq \alpha_1, \alpha_2 \leq 1, \\
&\qquad 0 \leq \theta_1, \theta_2 \leq 1.
\end{aligned}
$$

$$
\begin{aligned}
R_{BM_1} &= \min\left(\acute{t}_1 C\left(\frac{h_{01}^2\alpha_1 P'}{\acute{t}_1}\right), \acute{t}_1 C\left(\frac{h_{23}^2\bar{\theta}_2 P_{r2}}{\acute{t}_1}\right) + \acute{t}_2 C\left(\frac{h_{13}^2\theta_1 P_{r1}}{h_{13}^2\bar{\theta}_1 P_{r1} + \acute{t}_2}\right)\right), \\
R_{BM_2} &= \min\left(\acute{t}_2 C\left(\frac{h_{02}^2\alpha_2 P''}{\acute{t}_2}\right), \acute{t}_2 C\left(\frac{h_{13}^2\bar{\theta}_1 P_{r_1}}{\acute{t}_2}\right) + \acute{t}_1 C\left(\frac{h_{23}^2\theta_2 P_{r_2}}{h_{23}^2\bar{\theta}_2 P_{r_2} + \acute{t}_1}\right)\right).
\end{aligned}
$$

where $\bar{\theta}_i = 1 - \theta_i$*, and* $\bar{\alpha}_i = 1 - \alpha_i$*, for* $i = 1, 2$*.*

Proof: Let $\tilde{V}_T \sim \mathcal{N}(0, \alpha_1 \tilde{P}), \tilde{\tilde{V}}_T \sim \mathcal{N}(0, \alpha_2 \tilde{\tilde{P}}), \tilde{V} \sim \mathcal{N}(0, \theta_2 P_{r_2}), \tilde{\tilde{V}} \sim \mathcal{N}(0, \theta_1 P_{r_1}), \tilde{U} \sim \mathcal{N}(0, \bar{\theta}_2 P_{r_2})$ and $\tilde{\tilde{U}} \sim \mathcal{N}(0, \bar{\theta}_1 P_{r_1})$, which are independent of each other.

After letting $\tilde{X} = \tilde{V}_T + \sqrt{\frac{\bar{\alpha}_1 P}{\theta_2 P_{r_2}}}\tilde{V}, \tilde{\tilde{X}} = \tilde{\tilde{V}}_T + \sqrt{\frac{\bar{\alpha}_2 P}{\theta_1 P_{r_1}}}\tilde{\tilde{V}}, \tilde{X}_2 = \tilde{V} + \tilde{U}$ and $\tilde{\tilde{X}}_1 = \tilde{\tilde{V}} + \tilde{\tilde{U}}$ and applying them into the achievable rate formula of Theorem 3 and considering Proposition 1 we can get $R_{BM_{suc}}$ for the gaussian case as in [32]. However, here we show that backward decoding can give us a better rate in the gaussian case. This fact will be further depicted in the simulation results section. Let $\left(\tilde{R}_{bin}, \tilde{R}_{new}\right)$, and $\left(\tilde{\tilde{R}}_{bin}, \tilde{\tilde{R}}_{new}\right)$ denote the rate pair of the index of the bin and the rate of the new message which is received by the destination at time slot $\acute{t}_1$ and $\acute{t}_2$, respectively. Using backward decoding, we can think of two MAC with the following capacity regions

$$
\begin{aligned}
\psi_{\acute{t}_1}\left(\theta_2 P_{r_2}, \bar{\theta}_2 P_{r_2}\right) \quad &: \\
\tilde{R}_{bin} \quad &\leq \quad \acute{t}_1 C\left(\frac{h_{23}^2\bar{\theta}_2 P_{r_2}}{\acute{t}_1}\right), \\
\tilde{R}_{new} \quad &\leq \quad \acute{t}_1 C\left(\frac{h_{23}^2\theta_2 P_{r_2}}{\acute{t}_1}\right), \\
\tilde{R}_{bin} + \tilde{R}_{new} \quad &\leq \quad \acute{t}_1 C\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right).
\end{aligned} \tag{55}
$$

$$\begin{aligned}
\psi_{\acute{t}_2}\left(\theta_1 P_{r_1}, \bar{\theta}_1 P_{r_1}\right) & : \\
\tilde{\tilde{R}}_{bin} & \leq \acute{t}_2 C\left(\frac{h_{13}^2 \bar{\theta}_1 P_{r_1}}{\acute{t}_2}\right), \\
\tilde{\tilde{R}}_{new} & \leq \acute{t}_2 C\left(\frac{h_{13}^2 \theta_1 P_{r_1}}{\acute{t}_2}\right), \\
\tilde{\tilde{R}}_{bin} + \tilde{\tilde{R}}_{new} & \leq \acute{t}_2 C\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right).
\end{aligned} \tag{56}$$

Here, we claim that using backward decoding along with successive decoding of message and the bin gives better rate. From (55) and (56), the sum rates are not dependent on the powers $\theta_1 P_{r_1}$, $\bar{\theta}_1 P_{r_1}$, $\theta_2 P_{r_2}$, and $\bar{\theta}_2 P_{r_2}$. Now, by the same argument as in Proposition 1, let us equate the rate $\tilde{R}_{bin}$ in (55) which is achieved by joint decoding with R'_{bin} which is achieved by successive decoding of the new message and the index of the bin of the old message. Therefore, we have

$$R'_{bin} = \tilde{R}_{bin} = \acute{t}_1 C\left(\frac{h_{23}^2 \bar{\theta}'_2 P_{r_2}}{\acute{t}_1}\right), \tag{57}$$

$$\Rightarrow \bar{\theta}' \leq \bar{\theta}. \tag{58}$$

Since the sum rate $\tilde{R}_{bin} + \tilde{R}_{new}$ is independent of θ, $R'_{new} = \acute{t}_2 C\left(\frac{h_{13}^2 \theta_1 P_{r1}}{h_{13}^2 \bar{\theta}_1 P_{r1} + \acute{t}_2}\right) = \tilde{R}_{new}$ remains constant. Therefore, although R_{BM_1} remains constant since the second term in (54) is an increasing function of θ_2, this term increases. We can argue similarly for R_{BM_2} and the third term in (54). Hence, by backward decoding $R_{BM_1} + R_{BM_2}$ remains constant while the second and the third term in (54) increase. Hence, we can get corollary 1. ■

corollary 2 *For the half-duplex Gaussian parallel relay channel, assuming successive relaying protocol with power constraint at source and each relay, Dirty Paper Coding achieves the following rate*

$$R_{DPC} \;\; \leq \;\; \max\left(\tilde{R}_s + \tilde{\tilde{R}}_s\right).$$

$$subject \quad to$$

$$P' + P'' = P_s,$$

$$\acute{t}_1 + \acute{t}_2 = 1.$$

where

$$\begin{aligned}\tilde{R}_s &= \min\left(\acute{t}_1 C\left(\frac{h_{01}^2 P'}{\acute{t}_1}\right), \acute{t}_2 C\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right)\right),\\ \tilde{\tilde{R}}_s &= \min\left(\acute{t}_2 C\left(\frac{h_{02}^2 P''}{\acute{t}_2}\right), \acute{t}_1 C\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right)\right).\end{aligned}$$

Proof: From the Costa's Dirty Paper Coding result([29]) by having:

$$\tilde{U} = \tilde{X} + \frac{h_{01}h_{12}P'}{h_{01}^2 P' + \acute{t}_1}\tilde{X}_2, \tag{59}$$

$$\tilde{\tilde{U}} = \tilde{\tilde{X}} + \frac{h_{02}h_{12}P''}{h_{02}^2 P'' + \acute{t}_2}\tilde{\tilde{X}}_1. \tag{60}$$

where $\tilde{X} \sim \mathcal{N}(0, P')$, $\tilde{\tilde{X}} \sim \mathcal{N}(0, P'')$, $\tilde{X}_2 \sim \mathcal{N}(0, P_{r_2})$, and $\tilde{\tilde{X}}_1 \sim \mathcal{N}(0, P_{r_1})$, and applying them to Theorem 3, we get corollary 2. ■

C. Upper Bound

Assuming total power constraints P_s, P_{r_1}, and P_{r_2} for the signal transmitted by the source, relay 1, and relay 2, and by Theorem 4, we have the following upper bound for the maximum achievable rate for the Gaussian case

$$\begin{aligned}
C_{up} \quad \leq \min \Bigg(& t_1 C\left(\frac{(1-\tilde{\rho}^2)h_{01}^2\tilde{P}}{t_1}\right) + t_2 C\left(\frac{(1-\tilde{\tilde{\rho}}^2)h_{02}^2\tilde{\tilde{P}}}{t_2}\right) + t_3 C\left(\frac{(h_{01}^2+h_{02}^2)P}{t_3}\right), \\
& t_2 C\left(\frac{h_{02}^2\tilde{\tilde{P}}}{t_2} + \frac{(h_{12}^2+h_{13}^2)\tilde{\tilde{P}}_1}{t_2} + \frac{2\tilde{\tilde{\rho}}h_{02}h_{12}\sqrt{\tilde{\tilde{P}}\tilde{\tilde{P}}_1}}{t_2} + \frac{(1-\tilde{\tilde{\rho}}^2)h_{02}^2h_{13}^2\tilde{\tilde{P}}\tilde{\tilde{P}}_1}{t_2^2}\right) + \\
& t_3 C\left(\frac{h_{02}^2 P}{t_3}\right) + t_4 C\left(\frac{(1-\rho^2)h_{13}^2 P_1}{t_4}\right), \qquad (61)\\
& t_1 C\left(\frac{h_{01}^2\tilde{P}}{t_1} + \frac{(h_{12}^2+h_{23}^2)\tilde{P}_2}{t_1} + \frac{2\tilde{\rho}h_{01}h_{12}\sqrt{\tilde{P}\tilde{P}_2}}{t_1} + \frac{(1-\tilde{\rho}^2)h_{01}^2h_{23}^2\tilde{P}\tilde{P}_2}{t_1^2}\right) + \\
& t_3 C\left(\frac{h_{01}^2 P}{t_3}\right) + t_4 C\left(\frac{(1-\rho^2)h_{23}^2 P_2}{t_4}\right), \\
& t_1 C\left(\frac{h_{23}^2\tilde{P}_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2\tilde{\tilde{P}}_1}{t_2}\right) + t_4 C\left(\frac{h_{13}^2P_1 + h_{23}^2P_2 + 2\rho h_{13}h_{23}\sqrt{P_1P_2}}{t_4}\right)\Bigg).
\end{aligned}$$

$subject \quad to$

$$\begin{aligned}
& \tilde{P} + \tilde{\tilde{P}} + P = P_s, \\
& \tilde{\tilde{P}}_1 + P_1 = P_{r_1}, \\
& \tilde{P}_2 + P_2 = P_{r_2}, \\
& t_1 + t_2 + t_3 + t_4 = 1, \\
& 0 \leq \tilde{\tilde{\rho}} \leq 1, \\
& 0 \leq \tilde{\rho} \leq 1, \\
& 0 \leq \rho \leq 1,
\end{aligned}$$

where $\tilde{P}$, $\tilde{\tilde{P}}$, P, $\tilde{\tilde{P}}_1$, P_1, $\tilde{P}_2$, and P_2 are the powers associated with $\tilde{X}$, $\tilde{\tilde{X}}$, X, $\tilde{\tilde{X}}_1$, X_1, $\tilde{X}_2$, and X_2, respectively, and $\tilde{\tilde{\rho}}$ is the correlation coefficient between $\tilde{\tilde{X}}$ and $\tilde{\tilde{X}}_1$, $\tilde{\rho}$ is the correlation coefficient between $\tilde{X}$ and $\tilde{X}_2$, and ρ is the correlation coefficient between X_1 and X_2.

Now in high SNR scenarios we have the following theorem:

Theorem 6 *In high SNR scenarios, when power available for the source and each relay tends to infinity, time slots* t_3 *and* t_4 *in (61) tends to zero as* $\mathcal{O}\left(\frac{1}{\log P_s}\right)$. *Furthermore, the upper bound on the capacity of half-duplex parallel relay network in high SNR scenarios is:*

$$C_{up} = R_{DPC} + \mathcal{O}\left(\frac{1}{\log P_s}\right).$$

In other words, Dirty Paper Coding achieves the capacity of a half-duplex Gaussian parallel relay channel as SNR goes to infinity.

Proof: Throughout the proof, we assume the power of the relays goes to infinity as $P_{r_1} = \gamma_1 P_s,\ P_{r_2} = \gamma_2 P_s$ where γ_1, γ_2 are constants independent of the SNR. By setting $\tilde{\rho} = \tilde{\tilde{\rho}} = 0$, and $\rho = 1$ in equation (61), we can upper bound the upper bound on the capacity as follows:

$$\begin{aligned} C_{up} \leq \min \Bigg(& t_1 C\left(\frac{h_{01}^2 \tilde{P}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2 \tilde{\tilde{P}}}{t_2}\right) + t_3 C\left(\frac{(h_{01}^2 + h_{02}^2)P}{t_3}\right), \\ & t_2 C\left(\frac{h_{02}^2 \tilde{\tilde{P}}}{t_2} + \frac{(h_{12}^2 + h_{13}^2)\tilde{\tilde{P}}_1}{t_2} + \frac{2h_{02}h_{12}\sqrt{\tilde{\tilde{P}}\tilde{\tilde{P}}_1}}{t_2} + \frac{h_{02}^2 h_{13}^2 \tilde{\tilde{P}}\tilde{\tilde{P}}_1}{t_2^2}\right) + \\ & t_3 C\left(\frac{h_{02}^2 P}{t_3}\right) + t_4 C\left(\frac{h_{13}^2 P_1}{t_4}\right), \\ & t_1 C\left(\frac{h_{01}^2 \tilde{P}}{t_1} + \frac{(h_{12}^2 + h_{23}^2)\tilde{P}_2}{t_1} + \frac{2h_{01}h_{12}\sqrt{\tilde{P}\tilde{P}_2}}{t_1} + \frac{h_{01}^2 h_{23}^2 \tilde{P}\tilde{P}_2}{t_1^2}\right) + \\ & t_3 C\left(\frac{h_{01}^2 P}{t_3}\right) + t_4 C\left(\frac{h_{23}^2 P_2}{t_4}\right), \\ & t_1 C\left(\frac{h_{23}^2 \tilde{P}_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 \tilde{\tilde{P}}_1}{t_2}\right) + t_4 C\left(\frac{h_{13}^2 P_1 + h_{23}^2 P_2 + 2h_{13}h_{23}\sqrt{P_1 P_2}}{t_4}\right)\Bigg). \end{aligned} \tag{62}$$

$$subject \quad to$$

$$\tilde{P} + \tilde{\tilde{P}} + P = P_s,$$

$$\tilde{\tilde{P}}_1 + P_1 = P_{r_1},$$

$$\tilde{P}_2 + P_2 = P_{r_2},$$

$$t_1 + t_2 + t_3 + t_4 = 1.$$

Furthermore, from corollary 2, the achievable rate of the Dirty Paper Coding scheme can be formulated as:

$$
\begin{aligned}
R_{DPC} \leq \min \quad & \left(\acute{t}_1 C\left(\frac{h_{01}^2 P'}{\acute{t}_1}\right) + \acute{t}_2 C\left(\frac{h_{02}^2 P''}{\acute{t}_2}\right), \right. \\
& \acute{t}_2 C\left(\frac{h_{02}^2 P''}{\acute{t}_2}\right) + \acute{t}_2 C\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right), \\
& \acute{t}_1 C\left(\frac{h_{01}^2 P'}{\acute{t}_1}\right) + \acute{t}_1 C\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right), \\
& \left. \acute{t}_1 C\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right) + \acute{t}_2 C\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right) \right).
\end{aligned}
\tag{63}
$$

By setting $P' = P'' = \frac{P_s}{2}$, and $\acute{t}_1 = \acute{t}_2 = 0.5$ in equation (63), equation (63) can be simplified as:

$$R_{DPC} \geq \frac{1}{2} \ln P_s + c. \tag{64}$$

where c is some constant which depends on channel coefficients. Knowing that the term corresponding to each cut-set in (62) for the optimum values of $t_1, \cdots, t_4$ is indeed an upper-bound for R_{DPC}, and by setting $\tilde{P} = \tilde{\tilde{P}} = P_s$ in (62), we have the following inequality between R_{DPC} and the first cut of (62):

$$
\begin{aligned}
\frac{1}{2}\ln P_s + c \leq \quad & \frac{t_1}{2}\ln\left(\frac{h_{01}^2 P_s}{t_1}\right) + \frac{t_2}{2}\ln\left(\frac{h_{02}^2 P_s}{t_2}\right) + \frac{t_3}{2}\ln\left(\frac{(h_{01}^2 + h_{02}^2)P_s}{t_3}\right) + \\
& \frac{t_1^2}{2h_{01}^2 P_s} + \frac{t_2^2}{2h_{02}^2 P_s} + \frac{t_3^2}{2(h_{01}^2 + h_{02}^2)P_s} \\
& = (1 - t_4)\ln P_s + t_1 \ln h_{01}^2 + t_2 \ln h_{02}^2 + t_3 \ln\left(h_{01}^2 + h_{02}^2\right) \\
& - t_1 \ln t_1 - t_2 \ln t_2 - t_3 \ln t_3 + \frac{t_1^2}{h_{01}^2 P_s} + \frac{t_2^2}{h_{02}^2 P_s} + \frac{t_2^2}{\left(h_{01}^2 + h_{02}^2\right) P_s}.
\end{aligned}
$$

Note that in deriving (64) and (65), the following inequality is applied to lower/upper-bound the corresponding terms.

$$\ln(x) \leq \ln(1 + x) \leq \ln(x) + \frac{1}{x}, \forall x > 0. \tag{65}$$

Consequently, we have

$$
\begin{aligned}
t_4 \quad \leq \quad & \frac{1}{\ln P_s}\left(c + t_1 \ln h_{01}^2 + t_2 \ln h_{02}^2 + t_3 \ln\left(h_{01}^2 + h_{02}^2\right) - t_1 \ln t_1 - t_2 \ln t_2 - t_3 \ln t_3\right) \\
+ \quad & \frac{1}{\ln P_s}\left(\frac{t_1^2}{h_{01}^2 P_s} + \frac{t_2^2}{h_{02}^2 P_s} + \frac{t_2^2}{\left(h_{01}^2 + h_{02}^2\right) P_s}\right).
\end{aligned}
$$

Hence, we can bound the optimum value of t_4 in (62) as

$$0 \leq t_4 \leq \mathcal{O}\left(\frac{1}{\log P_s}\right). \tag{66}$$

Similarly, by considering the fourth cut in (62), we can give another bound on the optimum value of t_3

$$0 \leq t_3 \leq \mathcal{O}\left(\frac{1}{\log P_s}\right). \tag{67}$$

Now, applying the inequality between R_{DPC} and the term corresponding to the second cut in (62), knowing the fact that $t_3 \leq \frac{c_3}{\ln P_s}$, and $t_4 \leq \frac{c_4}{\ln P_s}$ (where c_3 and c_4 are constants), and using inequalities (65), and

$$\ln(1+x) \leq x, \forall x \geq 0. \tag{68}$$

,we have

$$\begin{aligned}
&\frac{1}{2}\ln P_s + c \leq \\
&\frac{t_2}{2}\ln\left(\frac{h_{02}^2 h_{13}^2 \gamma_1 P_s^2}{t_2^2}\left(1 + \frac{t_2}{\gamma_1 h_{13}^2 P_s} + \frac{t_2\left(h_{12}^2 + h_{13}^2\right)}{h_{02}^2 h_{13}^2 P_s} + \frac{t_2 h_{12}}{h_{13}^2 h_{02}\sqrt{\gamma_1} P_s}\right)\right) + \\
&\frac{t_3}{2}\ln\left(\frac{h_{02}^2 P_s}{t_3}\right) + \frac{t_4}{2}\ln\left(\frac{h_{13}^2 \gamma_1 P_s}{t_4}\right) + \\
&\frac{t_2^3}{2\left(t_2 h_{02}^2 P_s + t_2\gamma_1\left(h_{12}^2 + h_{13}^2\right)P_s + 2t_2 h_{02} h_{12}\sqrt{\gamma_1} P_s + h_{02}^2 h_{13}^2 \gamma_1 P_s^2\right)} + \\
&\frac{t_3^2}{2h_{02}^2 P_s} + \frac{t_4^2}{2\gamma_1 h_{13}^2 P_s} \\
&\leq t_2 \ln P_s + \frac{t_2}{2}\ln\left(\frac{h_{02}^2 h_{13}^2 \gamma_1}{t_2^2}\right) + \frac{c_3}{2\ln P_s}\ln h_{02}^2 - \frac{c_3}{2\ln P_s}\ln t_3 + \frac{c_3}{2} + \\
&\frac{c_4}{2\ln P_s}\ln \gamma_1 h_{13}^2 - \frac{c_4}{2\ln P_s}\ln t_4 + \frac{c_4}{2} \\
&\frac{t_3}{2}\ln\left(\frac{h_{02}^2 P_s}{t_3}\right) + \frac{t_4}{2}\ln\left(\frac{h_{13}^2 \gamma_1 P_s}{t_4}\right) + \\
&\frac{t_2^3}{2\left(t_2 h_{02}^2 P_s + t_2\gamma_1\left(h_{12}^2 + h_{13}^2\right)P_s + 2t_2 h_{02} h_{12}\sqrt{\gamma_1} P_s + h_{02}^2 h_{13}^2 \gamma_1 P_s^2\right)} + \\
&\frac{t_3^2}{2h_{02}^2 P_s} + \frac{t_4^2}{2\gamma_1 h_{13}^2 P_s}
\end{aligned}$$

Therefore, we have:

$$\begin{aligned}
\frac{1}{2}\ln P_s + c &\leq t_2 \ln P_s + \acute{c} \\
&+ \mathcal{O}\left(\frac{1}{\ln P_s}\right) + \mathcal{O}\left(\frac{1}{P_s}\right).
\end{aligned}$$

Hence:

$$\frac{1}{2} - \frac{c_2}{\log P_s} \leq t_2. \tag{69}$$

Similarly, from the third cut of (62), for t_1 we have:

$$\frac{1}{2} - \frac{c_1}{\log P_s} \leq t_1. \tag{70}$$

From equations (69), and (70), and also the fact that $t_1 + t_2 + t_3 + t_4 = 1$, we have:

$$\frac{1}{2} - \frac{c_2}{\log P_s} \leq t_2 \leq \frac{1}{2} + \frac{c_1}{\log P_s}, \tag{71}$$

$$\frac{1}{2} - \frac{c_1}{\log P_s} \leq t_1 \leq \frac{1}{2} + \frac{c_2}{\log P_s}. \tag{72}$$

Hence, as $P_s \to \infty$, t_3, $t_4 \to 0$, and t_1, $t_2 \to 0.5$. This proves the first part of the Theorem.

Similarly, considering the inequality between the first cut of R_{DPC} and (62) and knowing the fact that t_1, t_2 are strictly above zero (approaching 0.5), we observe that the optimum value of $\tilde{P}, \tilde{\tilde{P}}$ are

$$\tilde{P}, \tilde{\tilde{P}} \sim \Theta\left(P_s\right). \tag{73}$$

Now, we prove that the Dirty Paper Coding scheme with the parameters $\acute{t}1 = t_1 + \frac{t_3+t_4}{2}$, $\acute{t}2 = t_2 + \frac{t_3+t_4}{2}$, $P' = \tilde{P}$, and $P'' = \tilde{\tilde{P}}$, where $t_1, \cdots, t_4, \tilde{P}, \tilde{\tilde{P}}$ are the parameters corresponding to the minimum value of (62), achieves the capacity with a gap no more than $\mathcal{O}\left(\frac{1}{\log P_s}\right)$. To prove this, we show that each of the four terms in (63) is no more than $O\left(\frac{1}{\log P_s}\right)$ below the corresponding term (from the same cut) in (62). To show this, for the first cut we have

$$\begin{aligned}
&t_1 C\left(\frac{h_{01}^2\tilde{P}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2\tilde{\tilde{P}}}{t_2}\right) + t_3 C\left(\frac{(h_{01}^2+h_{02}^2)P}{t_3}\right) - \acute{t}_1 C\left(\frac{h_{01}^2 P'}{\acute{t}_1}\right) - \acute{t}_2 C\left(\frac{h_{02}^2 P''}{\acute{t}_2}\right) \overset{(a)}{\leq} \\
&t_1 \ln\left(\frac{h_{01}^2\tilde{P}}{t_1}\right) + t_2 \ln\left(\frac{h_{02}^2\tilde{\tilde{P}}}{t_2}\right) + t_3 \ln\left(\frac{(h_{01}^2+h_{02}^2)P_s}{t_3}\right) - \left(t_1 + \frac{t_3+t_4}{2}\right)\ln\left(\frac{h_{01}^2\tilde{P}}{\acute{t}_1}\right) \\
&-\left(t_2 + \frac{t_3+t_4}{2}\right)\ln\left(\frac{h_{02}^2\tilde{\tilde{P}}}{\acute{t}_2}\right) + \frac{t_1^2}{h_{01}^2\tilde{P}} + \frac{t_2^2}{h_{02}^2\tilde{\tilde{P}}} + \frac{t_3^2}{(h_{01}^2+h_{02}^2)P_s} \overset{(b)}{\lesssim} \\
&t_3 \ln\left(\frac{P_s}{\sqrt{\tilde{P}\tilde{\tilde{P}}}}\right) - \frac{t_4}{2}\ln\left(\tilde{P}\tilde{\tilde{P}}\right) + \frac{t_1^2}{h_{01}^2\tilde{P}} + \frac{t_2^2}{h_{02}^2\tilde{\tilde{P}}} + \mathcal{O}\left(\frac{1}{\log P_s}\right) \overset{(c)}{\lesssim} \\
&\mathcal{O}\left(\frac{1}{\log P_s}\right).
\end{aligned} \tag{74}$$

Here, (a) follows from (65), (b) follows from the fact that $t_3, t_4 \sim \mathcal{O}\left(\frac{1}{\log P_s}\right)$, and the fact that $\ln\left(\frac{\tilde{t}_1}{t_1}\right) \sim \mathcal{O}\left(\frac{1}{\log P_s}\right)$, and (c) results from the fact that $\tilde{P}, \tilde{\tilde{P}} \sim \Theta(P_s)$, and also the fact that $t_1, t_2 \sim 0.5 + \mathcal{O}\left(\frac{1}{\log P_s}\right)$.

Now, we bound the difference between the terms in the fourth cut of (62) and for the fourth term in R_{DPC}.

$$
\begin{aligned}
& t_1 C\left(\frac{h_{23}^2 \tilde{P}_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 \tilde{\tilde{P}}_1}{t_2}\right) + t_4 C\left(\frac{h_{13}^2 P_1 + h_{23}^2 P_2 + 2h_{13}h_{23}\sqrt{P_1 P_2}}{t_4}\right) \\
& - \acute{t}_1 C\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right) - \acute{t}_2 C\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right) \overset{(a)}{\lesssim} \\
& t_1 \ln\left(\frac{h_{23}^2 P_{r_2}}{t_1}\right) + t_2 \ln\left(\frac{h_{13}^2 P_{r_1}}{t_2}\right) + t_4 \ln\left(\frac{h_{13}^2 P_{r_1} + h_{23}^2 P_{r_2} + 2h_{13}h_{23}\sqrt{P_{r_1} P_{r_2}}}{t_4}\right) \\
& - \left(t_1 + \frac{t_3 + t_4}{2}\right) \ln\left(\frac{h_{23}^2 P_{r_2}}{\acute{t}_1}\right) - \left(t_2 + \frac{t_3 + t_4}{2}\right) \ln\left(\frac{h_{13}^2 P_{r_1}}{\acute{t}_2}\right) + \mathcal{O}\left(\frac{1}{P_s}\right) \overset{(b)}{\lesssim} \\
& t_4 \ln\left(2h_{13}h_{23} + h_{23}^2\sqrt{\frac{P_{r_2}}{P_{r_1}}} + h_{13}^2\sqrt{\frac{P_{r_1}}{P_{r_2}}}\right) - \frac{t_3}{2}\ln\left(P_{r_1} P_{r_2}\right) + \mathcal{O}\left(\frac{1}{\log P_s}\right) \overset{(c)}{\lesssim} \\
& \mathcal{O}\left(\frac{1}{\log P_s}\right).
\end{aligned} \tag{75}
$$

Here, (a) follows from (65) and upper-bounding $P_1, \tilde{\tilde{P}}_1 \le P_{r_1}, P_2, \tilde{P}_2 \le P_{r_2}$, and $(b), (c)$ follows from $t_3, t_4 \sim \mathcal{O}\left(\frac{1}{\log P_s}\right)$ and $t_1, t_2 \sim 0.5 + \mathcal{O}\left(\frac{1}{\log P_s}\right)$.

Next, we bound the difference between the terms in the second cut of (62) and for the second term in R_{DPC}.

$$
\begin{aligned}
&t_2 C\left(\frac{h_{02}^2\tilde{\tilde{P}}}{t_2}+\frac{(h_{12}^2+h_{13}^2)\tilde{\tilde{P}}_1}{t_2}+\frac{2h_{02}h_{12}\sqrt{\tilde{\tilde{P}}\tilde{\tilde{P}}_1}}{t_2}+\frac{h_{02}^2h_{13}^2\tilde{\tilde{P}}\tilde{\tilde{P}}_1}{t_2^2}\right)+t_3C\left(\frac{h_{02}^2P}{t_3}\right)+t_4C\left(\frac{h_{13}^2P_1}{t_4}\right)\\
&-\acute{t}_2C\left(\frac{h_{02}^2P''}{\acute{t}_2}\right)-\acute{t}_2C\left(\frac{h_{13}^2P_{r_1}}{\acute{t}_2}\right)\overset{(a)}{\lesssim}\\
&t_2\ln\left(\frac{h_{02}^2h_{13}^2\tilde{\tilde{P}}P_{r_1}}{t_2^2}\right)+t_3\ln\left(\frac{h_{02}^2P_s}{t_3}\right)+t_4C\left(\frac{h_{13}^2P_{r_1}}{t_4}\right)\\
&-\left(t_2+\frac{t_3+t_4}{2}\right)\ln\left(\frac{h_{02}^2\tilde{\tilde{P}}}{\acute{t}_2}\right)-\left(t_2+\frac{t_3+t_4}{2}\right)\ln\left(\frac{h_{13}^2P_{r_1}}{\acute{t}_2}\right)+\mathcal{O}\left(\frac{1}{P_s}\right)\overset{(b)}{\lesssim}\\
&\frac{t_3}{2}\ln\left(\frac{P_s^2}{\tilde{\tilde{P}}P_{r_1}}\right)+\frac{t_4}{2}\ln\left(\frac{P_{r_1}}{\tilde{\tilde{P}}}\right)+\mathcal{O}\left(\frac{1}{\log P_s}\right)\overset{(c)}{\lesssim}\\
&O\left(\frac{1}{\log P_s}\right).
\end{aligned}
\tag{76}
$$

Here, (a) follows from (65), the fact that $P''=\tilde{\tilde{P}}\sim\Theta(P_s)$, and upper-bounding $P\le P_s$, $P_1\le P_{r_1}$, (b) results from $t_3,t_4\sim\mathcal{O}\left(\frac{1}{\log P_s}\right),t_1,t_2\sim 0.5+\mathcal{O}\left(\frac{1}{\log P_s}\right)$, and finally, (c) results from the fact that $\tilde{\tilde{P}},P_{r_1}\sim\Theta(P_s)$, and also $t_3,t_4\sim\mathcal{O}\left(\frac{1}{\log P_s}\right)$.

Noting that the second and third cuts are the same, and using the same argument as (76), we can bound the difference between the terms in the third cut of (62) and the third term in R_{DPC} as

$$
\begin{aligned}
&t_1 C\left(\frac{h_{01}^2\tilde{P}}{t_1}+\frac{(h_{12}^2+h_{23}^2)\tilde{P}_2}{t_1}+\frac{2h_{01}h_{12}\sqrt{\tilde{P}\tilde{P}_2}}{t_1}+\frac{h_{01}^2h_{23}^2\tilde{P}\tilde{P}_2}{t_1^2}\right)+t_3C\left(\frac{h_{01}^2P}{t_3}\right)+t_4C\left(\frac{h_{23}^2P_2}{t_4}\right)\\
&-\acute{t}_1C\left(\frac{h_{01}^2P'}{\acute{t}_1}\right)-\acute{t}_1C\left(\frac{h_{23}^2P_{r_2}}{\acute{t}_1}\right)\le\mathcal{O}\left(\frac{1}{\log P_s}\right).
\end{aligned}
\tag{77}
$$

Observing (74), (75), (76), and (77), completes the proof of the Theorem. ■

VI. SIMULATION RESULTS

In this section the performance of our proposed combined AF-DF scheme for simultaneous relaying protocol and different successive relaying protocols such as Block Markov encoding and Dirty Paper Coding are investigated comprehensively.

First the achievable rate of the proposed combined AF-DF scheme for simultaneous relaying protocol with that of AF, DF, and Modified-DF are compared with each other in symmetric scenarios. As is shown in the sequel, combined AF-DF always performs better than the best known scheme. However, in some ranges of SNR like in low SNR and high SNR, combined AF-DF significantly outperforms the other schemes. First the symmetric case and then the asymmetric case are investigated. In fig. 14 the improvement percentage versus P_s and P_r is drawn when we have 4 relays. The improvement percentage is defined as:

$$\%\Delta Improv. = \frac{R - \max(R_{Modified-DF}, R_{DF}, R_{AF})}{R} \times 100. \tag{78}$$

Since the motivation for the proposed scheme is for scenarios when the SNR of the first hop is less than or equal to the SNR of the second hop (as indicated in fig. 14), the proposed scheme does not lead to any improvement in other ranges of SNR. As a matter of fact, by using extra β dimension in the first hop and successfully decoding the associated message at each relay, one can get rid of some part of the noise at the relays. It can be seen from fig. 14 that the significant performance improvement up to 70% is achieved at low SNR scenarios. Furthermore, in some part of high SNR ranges, our scheme still outperforms the other ones.

In the following subsections, each SNR regime is investigated comprehensively when there are 4 relays as a case study. For the sake of completeness, the simultaneous upper bound of the setting which is calculated as follows is included in the figures as well:

$$C_{up} \leq \max_{\tilde{\alpha}} \min(\tilde{\alpha} I(X; Y_1, \cdots, Y_K), (1-\tilde{\alpha}) I(X_1, \cdots, X_K; Y)). \tag{79}$$

And for the gaussian case with the power constraint P_s and P_r at the source and each relay, respectively, we have

$$C_{up} \leq \max_{\tilde{\alpha}} \min\left(\tilde{\alpha} C\left(\frac{KP_s}{\tilde{\alpha}}\right), (1-\tilde{\alpha})\, C\left(\frac{K^2 P_r}{1-\tilde{\alpha}}\right)\right). \tag{80}$$

In the above upper bound we assume complete cooperation between the relays.

A. Low SNR Regimes

In this subsection we consider the case when P_s and P_r are between -30 to 0 dB.

1) case1:$-30(dB) \leq \mathrm{P_r} \leq -10(dB)$*:* Fig. 15 shows the achievable rate for the range $-30(dB) \leq P_r \leq -10(dB)$ and when $P_s = P_r - 10(dB)$. As this figure shows, in the range of $-30(dB) \leq P_r \leq -14(dB)$, the achievable rate of the proposed scheme is significantly better than the known alternatives (in this case DF). The point $P_r = -14(dB)$ is the point that the

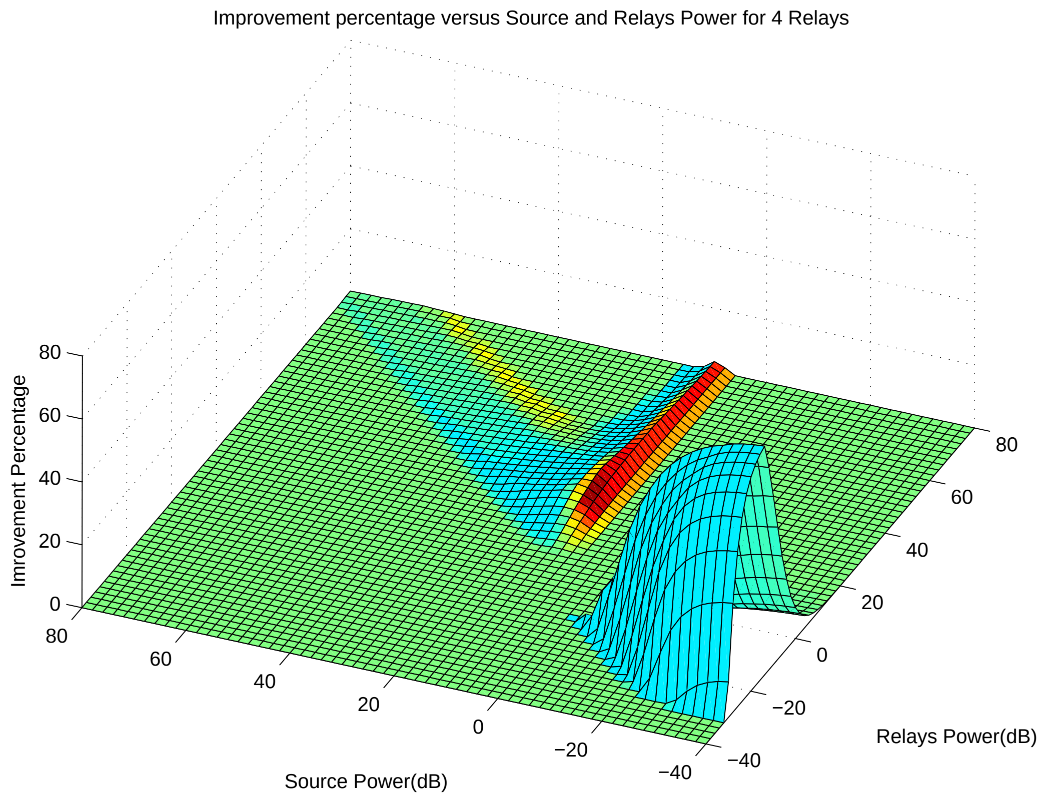

Fig. 14. Performance Improvement versus Source and Relay Power.

difference between our proposed scheme and other schemes is maximum. Indeed, that point is the point that the naive AF intersects Modified-DF and DF schemes and from that point on becomes closer and closer to our schemes.

2) case2:$-10(dB) \leq \mathrm{P_r} \leq 0(dB)$*:* Fig. 16 shows the achievable rate for the range $-10(dB) \leq P_r \leq 0(dB)$. At $P_r = -3(dB)$, AF scheme coincides with our proposed scheme. As shown above, the improvement we get from our scheme with respect to other schemes becomes significant in the range $-30(dB) \leq P_r \leq -10(dB)$. To see what happens in this range, let us have a closer look at the powers $P_{s\text{-}AF}$, $P_{s\text{-}DF1}$, and $P_{s\text{-}DF2}$, and also the way our available dimension is assigned to the two hops in different schemes, i.e. α, and $\tilde{\alpha}$ in Table I. In this table, $P_s = P_r - 10(dB)$. As Table I shows, in low SNR region, our scheme converts to AF. However, the total dimension should be much less than 1. This is interesting, because from this table it is clear that to obtain the maximum achievable rate with Modified DF scheme, one should use all the available dimension, and also assign more than 90% of that ($\tilde{\alpha}$) to the first hop, but by using AF scheme if one decrease

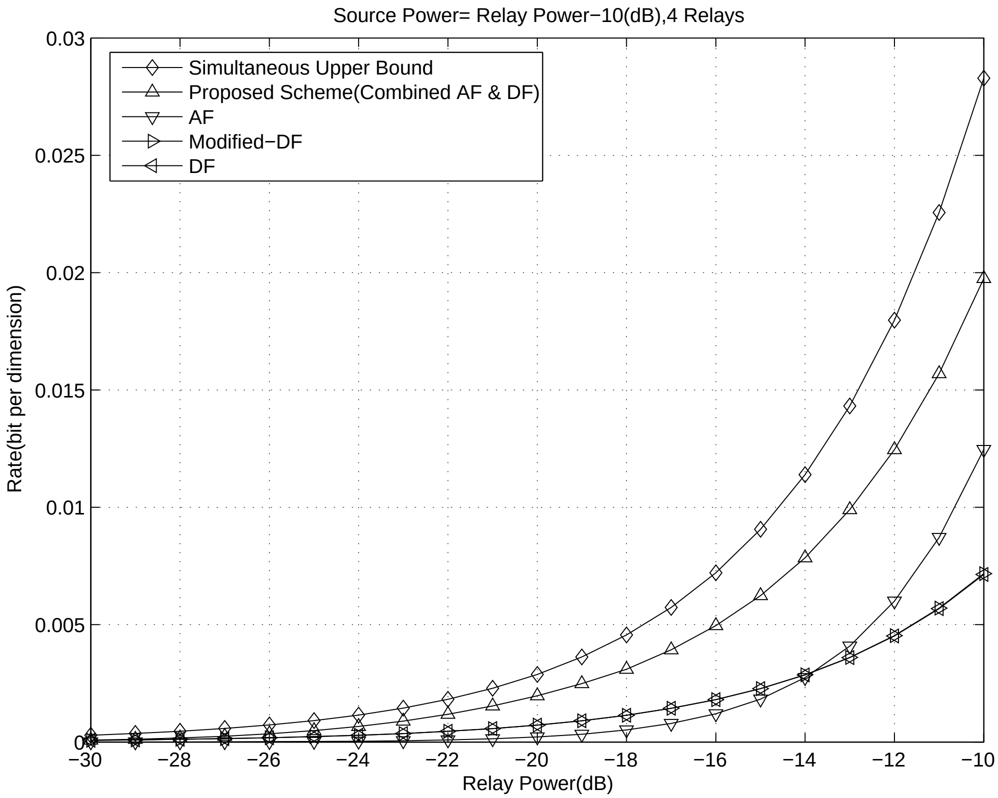

Fig. 15. Rate versus relay power($P_s = P_r - 10(dB)$)

TABLE I

PERFORMANCE IMPROVEMENT VERSUS SOURCE AND RELAY POWER($P_s = P_r - 10(dB)$).

	$P_{s\text{-}AF}$	$P_{s\text{-}DF1}$	$P_{s\text{-}DF2}$	α	$\tilde{\alpha}$
$P_r = -30(dB)$	0.0001	0	0	0.001	0.999
$P_r = -25(dB)$	0.000316	0	0	0.004	0.999
$P_r = -20(dB)$	0.001	0	0	0.01	0.999
$P_r = -15(dB)$	0.003162	0	0	0.028	0.9976
$P_r = -10(dB)$	0.01	0	0	0.085	0.9927
$P_r = -5(dB)$	0.031623	0	0	0.268	0.9781
$P_r = 0(dB)$	0.1	0	0	0.499	0.9404

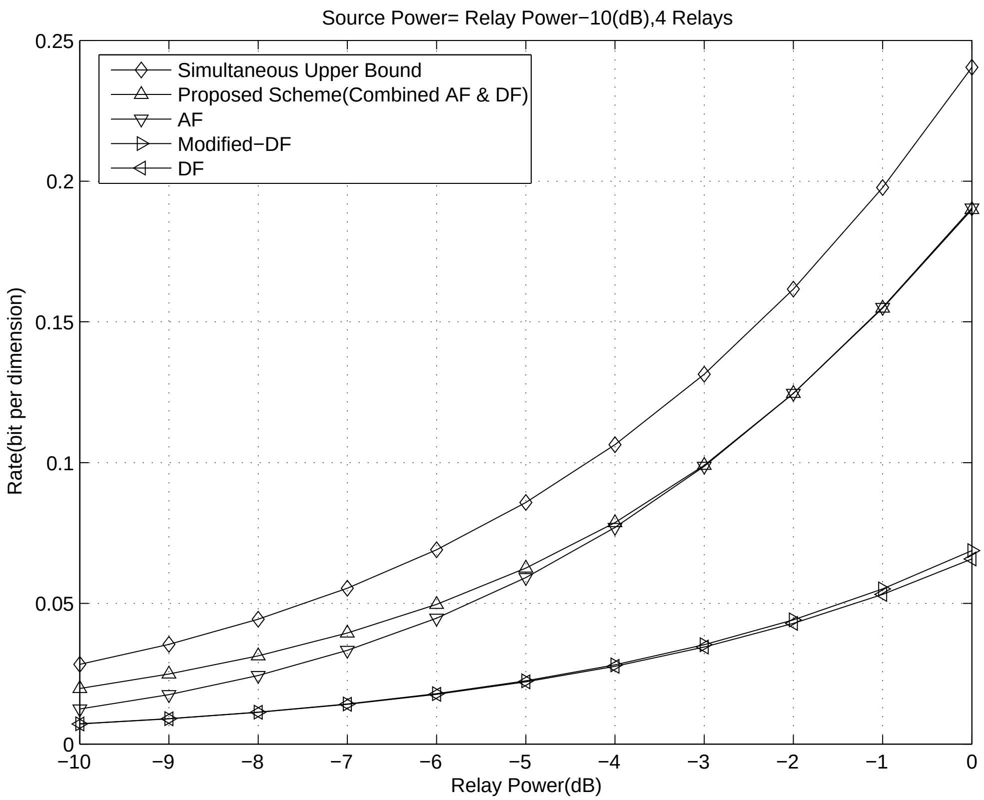

Fig. 16. Rate versus relay power($P_s = P_r - 10(dB)$)

the total available dimension to very little portion of it (near zero-i.e. 2α, where α is very close to zero), one can get the highest rate. This phenomenon is reasonable because in low SNR, by using AF without decreasing bandwidth, one devotes the available power in amplifying noise which at the end leads to deteriorating the performance. Indeed, this is the same result which was previously proved in [11] for full-duplex Gaussian parallel relay channel. Schein in [11] showed that bursting approach (in which the relays are silent most of the time but send with higher power at a small portion of time), in low SNR regimes gives the highest achievable rate and when the difference between source SNR and the relay SNR tends to infinity, it achieves the capacity. Table I shows that our combined AF and DF scheme is actually transformed to that bursting approach at low SNR.

B. Medium and High SNR Regimes

1) case1:$0(dB) \leq \mathrm{P_r} \leq 20(dB)$*:* For $0(dB) \leq P_r \leq 20(dB)$, the achievable rate of our scheme coincides with that of AF. In other words, our scheme converts to naive AF. At $P_r = 18(dB)$, again, our schemes starts to outperform AF (Fig. 17).

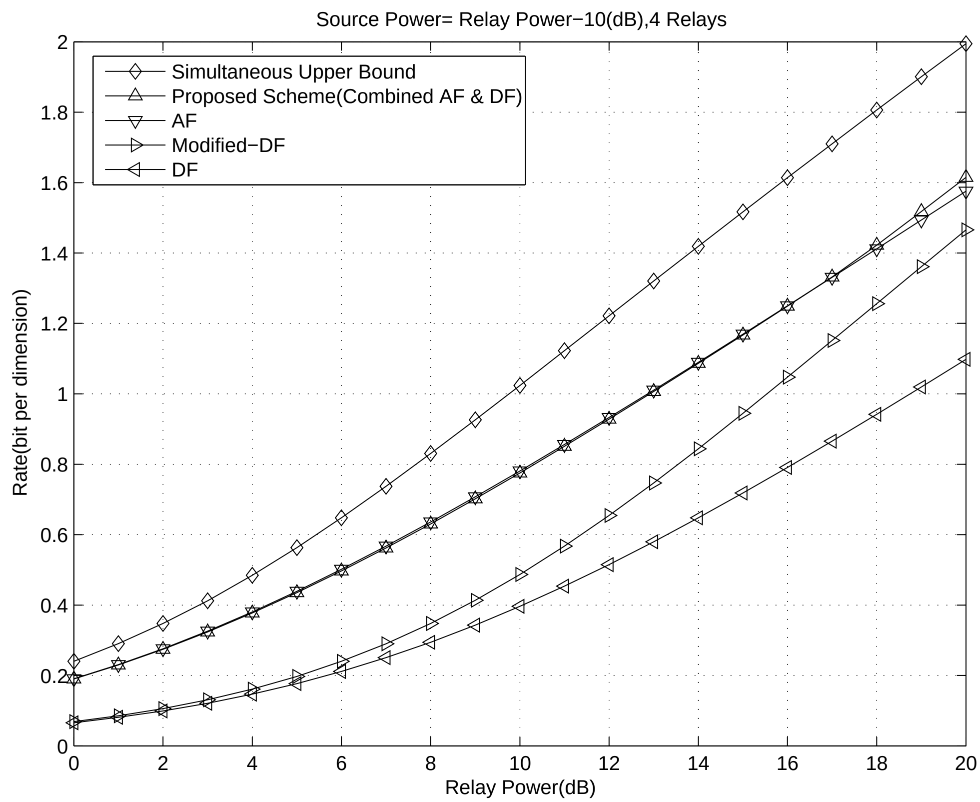

Fig. 17. Rate versus relay power($P_s = P_r - 10(dB)$).

2) case2:$20(dB) \leq \mathrm{P_r} \leq 40(dB)$ *and larger:* In this regime, our scheme outperforms other schemes. At $P_r \simeq 26dB$, Modified DF intersects AF and from that point on, it is the closest scheme to our scheme. As P_r becomes large, Modified DF gets closer and closer to our scheme and at very high SNR, they both coincide with each other (Fig. 18).

To show that it is indeed the combination of AF and DF which leads to better performance, as an example, we have brought $P_{s\text{-}AF}$, $P_{s\text{-}DF1}$, $P_{s\text{-}DF2}$, P_r, α, and $\tilde{\alpha}$ quantities in Table II for $20(dB) \leq P_r \leq 40(dB)$ and $P_s = P_r - 10(dB)$.

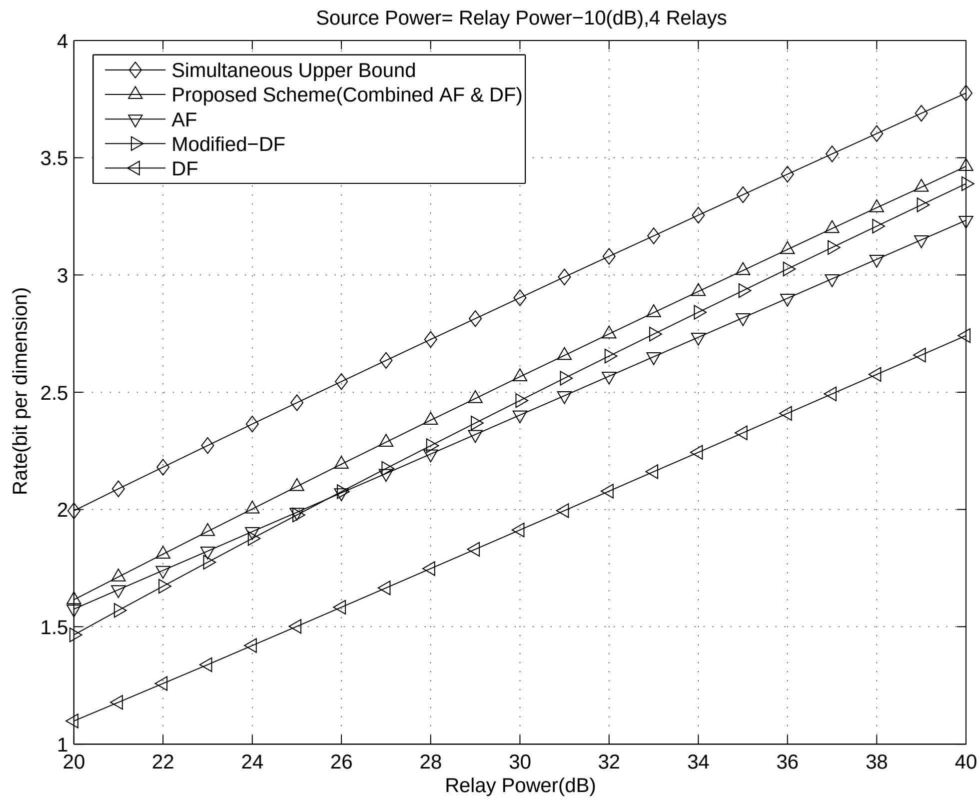

Fig. 18. Rate versus relay power($P_s = P_r - 10(dB)$).

TABLE II

PERFORMANCE IMPROVEMENT VERSUS SOURCE AND RELAY POWER($P_s = P_r - 10(dB)$).

	$P_{s\text{-}AF}$	$P_{s\text{-}DF1}$	$P_{s\text{-}DF2}$	α	$\tilde{\alpha}$
$P_r = 20(dB)$	5.7	0	4.3	0.361	0.6747
$P_r = 25(dB)$	16.760072	1.338499	13.524206	0.364	0.644
$P_r = 30(dB)$	47	14.030596	38.969404	0.379	0.6222
$P_r = 35(dB)$	161.276161	46.920229	108.031376	0.397	0.6061
$P_r = 40(dB)$	640	43.501684	316.498316	0.406	0.5937

Figs. 19 and 20 compare the achievable performance by successive relaying protocol based on Dirty Paper Coding with that of simultaneous relaying protocol based on our combined AF-DF scheme in symmetric scenarios. The general upper bound of half-duplex Gaussian parallel relay channel (61) is also included in the figures. It can be seen from fig. 19 that when $P_r \leq$

$12(dB)$, simultaneous relaying with combined AF-DF performs better than Dirty Paper Coding. However, fig. 20 shows that as source and relay powers become large, Dirty Paper Coding scheme outperforms combined AF-DF and as we proved in the previous section, it achieves the capacity of the half-duplex Gaussian parallel relay channel asymptotically. However, in high SNR regimes, due to the spectral defficiency of simultaneous relaying protocol, the best known schemes of this protocol, i.e. combined AF-DF goes far below the upper bound.

Fig. 21 compares the achievable performance of different successive schemes with each other and the successive upper bounds. It shows as the inter relay channel becomes stronger, Block Markov encoding scheme can achieve the successive capacity, while the achievable rate of the Dirty paper coding is independent of that channel. Furthermore, this figure shows Block Markov encoding with backward decoding gives better achievable rate with respect to Block Markov encoding with successive decoding which is proposed in [32].

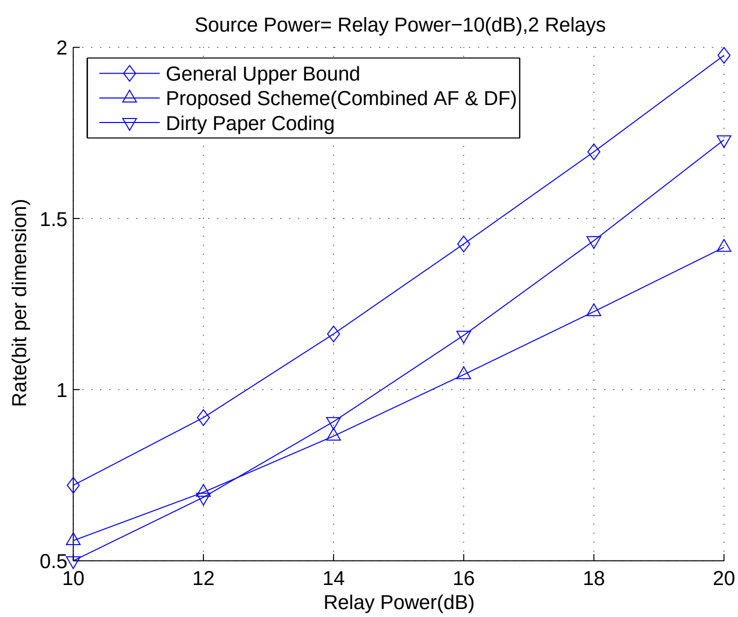

Fig. 19. Rate versus relay power($P_s = P_r - 10dB$)

VII. Conclusion

Here, *simultaneous* and *successive* relaying protocols for a half-duplex relay network were proposed. For simultaneous relaying and when the first hop (from the source to the relays) limits

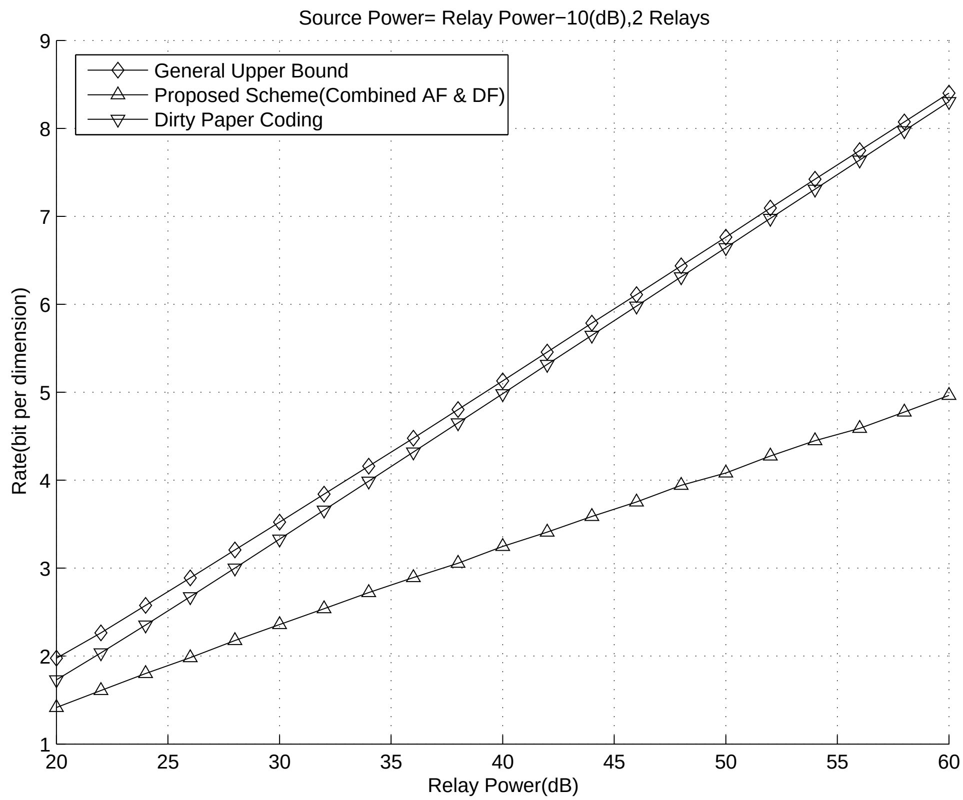

Fig. 20. Rate versus relay power($P_s = P_r - 10dB$)

the overall performance, we showed that our *combined AF-DF* scheme is a general scheme which can be converted to AF in very low SNR and DF in very high SNR regimes. However, in medium SNR scenarios and in a network with moderate number of relays combined AF-DF leads to a better achievable rate with respect to AF and DF schemes. Although we proposed combined AF-DF for symmetric scenarios, this scheme can be easily extended to asymmetric scenarios as well.

Furthermore, we proved that successive relaying protocol is optimum in high SNR Gaussian half-duplex paralle relay channel with two relays. Moreover, we proposed two cooperative strategies for successive relaying based on Block Markov encoding and Dirty Paper Coding. We showed that Dirty Paper Coding achieves the capacity of the Gaussian half-duplex parallel relay channel with two relays in high SNR scenarios.

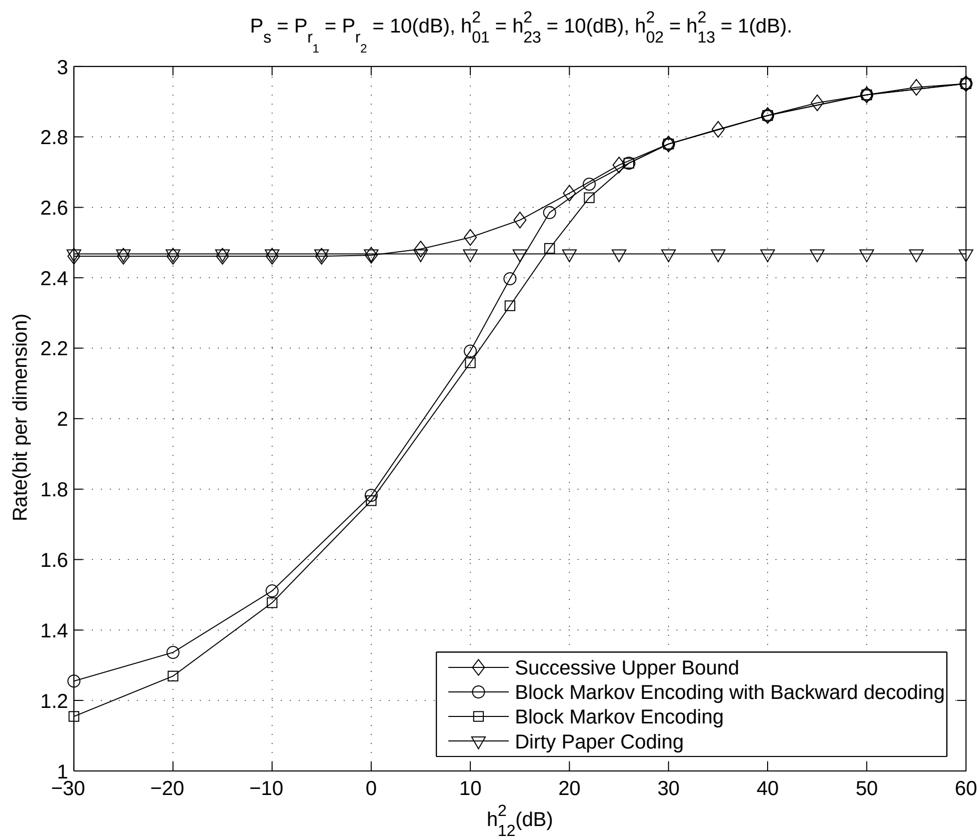

Fig. 21. Rate versus inter relay gain $P_s = P_{r1} = P_{r2} = 10(dB), h_{01}^2 = h_{23}^2 = 10(dB), h_{02}^2 = h_{13}^2 = 1(dB)$.

Appendix A

Proof of Theorem 1

Codebook Construction:

Let us divide block number *b*, $b = 1, 2, \cdots, B+2$ into odd and even numbers. The source generates two code-books $\tilde{\mathbf{x}}\left(w_b|w_{b-1}, s_1^{b-2}\right)$ and $\tilde{\tilde{\mathbf{x}}}\left(w_b|w_{b-1}, s_2^{b-2}\right)$ of size $2^{n\tilde{R}_s}$ and $2^{n\tilde{\tilde{R}}_s}$, respectively. The first code-book is generated according to the probabilities $p(\tilde{\mathbf{x}}|\tilde{\mathbf{x}}_2)p(\tilde{\mathbf{x}}_2) = \prod_{i=1}^{\acute{t}_1 n} p(\tilde{x}_i|\tilde{x}_{2i})p(\tilde{x}_{2i})$, and the second code-book is generated according to the probabilities

$$p(\tilde{\tilde{\mathbf{x}}}|\tilde{\tilde{\mathbf{x}}}_1)p(\tilde{\tilde{\mathbf{x}}}_1) = \prod_{i=1}^{\acute{t}_2 n} p(\tilde{\tilde{x}}_i|\tilde{\tilde{x}}_{1i})p(\tilde{\tilde{x}}_{1i}).$$

On the other hand, relay 2 generates 2^{nR_1} i.i.d codewords $\tilde{\mathbf{u}}$ and $2^{n\tilde{\tilde{R}}_s}$ i.i.d codewords $\tilde{\mathbf{x}}_2$ according to the probabilities $p(\tilde{\mathbf{u}}) = \prod_{i=1}^{\acute{t}_1 n} p(\tilde{u}_i)$ and $p(\tilde{\mathbf{x}}_2 \mid \tilde{\mathbf{u}}) = \prod_{i=1}^{\acute{t}_1 n} p(\tilde{x}_{2i} \mid \tilde{u}_i)$ at each odd interval and relay 1 generates 2^{nR_2} i.i.d codewords $\tilde{\tilde{\mathbf{u}}}$ and $2^{n\tilde{\tilde{R}}_s}$ i.i.d codewords $\tilde{\tilde{\mathbf{x}}}_1$ according to the probabilities $p(\tilde{\tilde{\mathbf{u}}}) = \prod_{i=1}^{\acute{t}_2 n} p(\tilde{\tilde{u}}_i)$ and $p(\tilde{\tilde{\mathbf{x}}}_1 \mid \tilde{\tilde{\mathbf{u}}}) = \prod_{i=1}^{\acute{t}_2 n} p(\tilde{\tilde{x}}_{1i} \mid \tilde{\tilde{u}}_i)$ at each even interval, respectively.

Encoding:

Encoding at the source:

Source encodes $w_b \in \{1, \cdots, 2^{n\tilde{R}_s}\}$ to $\tilde{\mathbf{x}}\left(w_b|w_{b-1}, s_1^{b-2}\right)$ and $w_b \in \{1, \cdots, 2^{n\tilde{\tilde{R}}_s}\}$ to $\tilde{\tilde{\mathbf{x}}}\left(w_b|w_{b-1}, s_2^{b-2}\right)$ and sends them in odd and even blocks, respectively.

Encoding at relay 1:

Relay 1 encodes the bin index s_2^{b-2} of the message w_{b-2} it received from relay 2 in the previous block to $\tilde{\tilde{\mathbf{u}}}\left(s_2^{b-2}\right)$. It also encodes w_{b-1} which was received from the source in block $b-1$ to $\tilde{\tilde{\mathbf{x}}}_1\left(w_{b-1}|s_2^{b-2}\right)$.

Encoding at relay 2:

Relay 2 encodes the bin index s_1^{b-2} of the message w_{b-2} it received from relay 1 in the previous block to $\tilde{\mathbf{u}}\left(s_1^{b-2}\right)$. It also encodes w_{b-1} which was received from the source in block $b-1$ to $\tilde{\mathbf{x}}_2\left(w_{b-1}|s_1^{b-2}\right)$.

Decoding:

Decoding at relay 1:

Knowing w_{b-2} and consequently s_1^{b-2}, at block b, relay 1 declares $(\hat{w}_{b-1}, \hat{w}_b) = (w_{b-1}, w_b)$ iff there exits a unique $(\hat{w}_{b-1}, \hat{w}_b)$ such that $\left(\tilde{\mathbf{x}}\left(\hat{w}_b|\hat{w}_{b-1}, s_1^{b-2}\right), \tilde{\mathbf{x}}_2\left(\hat{w}_{b-1}|s_1^{b-2}\right), \tilde{\mathbf{u}}(s_1^{b-2}), \tilde{\mathbf{y}}_1\right) \in A_\epsilon^{(n)}$, hence, we have:

$$
\begin{aligned}
\tilde{R}_s &\leq \acute{t}_1 I\left(\tilde{X}; \tilde{Y}_1 \mid \tilde{X}_2, \tilde{U}\right), \\
\tilde{R}_s + \tilde{\tilde{R}}_s &\leq \acute{t}_1 I(\tilde{X}, \tilde{X}_2; \tilde{Y}_1 \mid \tilde{U}).
\end{aligned} \tag{81}
$$

Decoding at relay 2:

Knowing w_{b-2} and consequently s_2^{b-2}, at block b, relay 2 declares $(\hat{w}_{b-1}, \hat{w}_b) = (w_{b-1}, w_b)$ iff there exits a unique $(\hat{w}_{b-1}, \hat{w}_b)$ such that $\left(\tilde{\tilde{\mathbf{x}}}\left(\hat{w}_b|\hat{w}_{b-1}, s_2^{b-2}\right), \tilde{\tilde{\mathbf{x}}}_1\left(\hat{w}_{b-1}|s_2^{b-2}\right), \tilde{\tilde{\mathbf{u}}}(s_2^{b-2}), \tilde{\tilde{\mathbf{y}}}_2\right) \in A_\epsilon^{(n)}$, hence, we have:

$$
\begin{aligned}
\tilde{\tilde{R}}_s &\leq \acute{t}_2 I(\tilde{\tilde{X}}; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1, \tilde{\tilde{U}}), \\
\tilde{R}_s + \tilde{\tilde{R}}_s &\leq \acute{t}_2 I(\tilde{\tilde{X}}, \tilde{\tilde{X}}_1; \tilde{\tilde{Y}}_2 \mid \tilde{\tilde{U}}).
\end{aligned} \tag{82}
$$

Decoding at the final destination:

Decoding at the final destination can be done either *Successively* or *Backwardly* as follows:

1) Successive Decoding:

At the end of block b, destination first declares the bin index $\hat{s}_1^{b-2} = s_1^{b-2}$ of the message w_{b-2} iff $(\tilde{\mathbf{u}}(\hat{s}_1^{b-2}), \tilde{\mathbf{y}}) \in A_\epsilon^{(n)}$, hence, we have:

$$R_1 \quad \leq \acute{t}_1 I(\tilde{U};\tilde{Y}). \tag{83}$$

Having decoded the bin index s_1^{b-2} of the message w_{b-2}, destination can resolve its uncertainty about the message w_{b-2} and declares $\hat{w}_{b-2} = w_{b-2}$ iff there exists a unique $\hat{w}_{b-2}$ such that $(\tilde{\tilde{\mathbf{x}}}_1(\hat{w}_{b-2}|s_2^{b-3}), \tilde{\tilde{\mathbf{u}}}(s_2^{b-3}), \tilde{\tilde{\mathbf{y}}}) \in A_\epsilon^{(n)}$, hence, we have:

$$\tilde{R}_s \quad -R_1 \leq \acute{t}_2 I(\tilde{\tilde{X}}_1;\tilde{\tilde{Y}} \mid \tilde{\tilde{U}}). \tag{84}$$

Using the same argument for the even block *b*, we have:

$$\begin{aligned} R_2 &\quad \leq \acute{t}_2 I(\tilde{\tilde{U}};\tilde{\tilde{Y}}), \\ \tilde{\tilde{R}}_s &\quad -R_2 \leq \acute{t}_1 I(\tilde{X}_2;\tilde{Y} \mid \tilde{U}). \end{aligned} \tag{85}$$

From (81), (82), (83), (84), and (85) we have:

$$\begin{aligned} \tilde{R}_s &\quad \leq \acute{t}_2 I\left(\tilde{\tilde{X}}_1;\tilde{\tilde{Y}} \mid \tilde{\tilde{U}}\right) + \acute{t}_1 I\left(\tilde{U};\tilde{Y}\right), \\ \tilde{\tilde{R}}_s &\quad \leq \acute{t}_1 I(\tilde{X}_2;\tilde{Y} \mid \tilde{U}) + \acute{t}_2 I(\tilde{\tilde{U}};\tilde{\tilde{Y}}). \end{aligned} \tag{86}$$

From the above argument and as figure 7 shows we have:

$$\begin{aligned} \tilde{R}_s &\quad \leq \min\left(\acute{t}_1 I\left(\tilde{X};\tilde{Y}_1 \mid \tilde{X}_2,\tilde{U}\right), \acute{t}_2 I\left(\tilde{\tilde{X}}_1;\tilde{\tilde{Y}} \mid \tilde{\tilde{U}}\right) + \acute{t}_1 I\left(\tilde{U};\tilde{Y}\right)\right), \\ \tilde{\tilde{R}}_s &\quad \leq \min\left(\acute{t}_1 I\left(\tilde{X}_2;\tilde{Y} \mid \tilde{U}\right) + \acute{t}_2 I\left(\tilde{\tilde{U}};\tilde{\tilde{Y}}\right), \acute{t}_2 I\left(\tilde{\tilde{X}};\tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1,\tilde{\tilde{U}}\right)\right). \end{aligned} \tag{87}$$

And from (81), (82), and (87), we have:

$$\begin{aligned} R_{BM} &\quad = \tilde{R}_s + \tilde{\tilde{R}}_s \leq \max_{0\leq \acute{t}_1,\acute{t}_2,\acute{t}_1+\acute{t}_2=1} \min\Big(\\ &\min\left(\acute{t}_1 I\left(\tilde{X};\tilde{Y}_1 \mid \tilde{X}_2,\tilde{U}\right), \acute{t}_2 I\left(\tilde{\tilde{X}}_1;\tilde{\tilde{Y}} \mid \tilde{\tilde{U}}\right) + \acute{t}_1 I\left(\tilde{U};\tilde{Y}\right)\right) + \\ &\min\left(\acute{t}_1 I\left(\tilde{X}_2;\tilde{Y} \mid \tilde{U}\right) + \acute{t}_2 I\left(\tilde{\tilde{U}};\tilde{\tilde{Y}}\right), \acute{t}_2 I\left(\tilde{\tilde{X}};\tilde{\tilde{Y}}_2 \mid \tilde{\tilde{X}}_1,\tilde{\tilde{U}}\right)\right), \\ &\acute{t}_1 I\left(\tilde{X},\tilde{X}_2;\tilde{Y}_1 \mid \tilde{U}\right), \acute{t}_2 I\left(\tilde{\tilde{X}},\tilde{\tilde{X}}_1;\tilde{\tilde{Y}}_2 \mid \tilde{\tilde{U}}\right)\Big). \end{aligned} \tag{88}$$

2) Backward Decoding: At the end of odd block b, destination declares $\left(\hat{w}_{b-1}, \hat{s}_1^{b-2}\right) = \left(w_{b-1}, s_1^{b-2}\right)$ iff there exists a unique pair $\left(\hat{w}_{b-1}, \hat{s}_1^{b-2}\right)$ such that $\left(\tilde{\mathbf{x}}\left(\hat{w}_{b-1}, \hat{s}_1^{b-2}\right), \tilde{\mathbf{u}}\left(\hat{s}_1^{b-2}\right), \tilde{\mathbf{y}}\right) \in A_\epsilon^{(n)}$. Similarly, at the end of even block b, destination declares $\left(\hat{w}_{b-1}, \hat{s}_2^{b-2}\right) = \left(w_{b-1}, s_2^{b-2}\right)$ iff there exists a unique pair $\left(\hat{w}_{b-1},\hat{s}_2^{b-2}\right)$ such that $\left(\tilde{\tilde{\mathbf{x}}}\left(\hat{w}_{b-1},\hat{s}_1^{b-2}\right),\tilde{\tilde{\mathbf{u}}}\left(\hat{s}_2^{b-2}\right),\tilde{\tilde{\mathbf{y}}}\right)\in A_\epsilon^{(n)}$. Hence, we have:

$$\tilde{R}_s+\tilde{\tilde{R}}_s\leq \acute{t}_1 I\left(\tilde{X}_2,\tilde{U};\tilde{Y}\right)+\acute{t}_2 I\left(\tilde{\tilde{X}}_1,\tilde{\tilde{U}};\tilde{\tilde{Y}}\right). \tag{89}$$

And finally:

$$\begin{aligned} R_{BM} &= \tilde{R}_s+\tilde{\tilde{R}}_s\leq \max_{0\leq\acute{t}_1,\acute{t}_2,\acute{t}_1+\acute{t}_2=1}\min\Big(\\ &\acute{t}_1 I\left(\tilde{X};\tilde{Y}_1\mid\tilde{X}_2,\tilde{U}\right)+\acute{t}_2 I\left(\tilde{\tilde{X}};\tilde{\tilde{Y}}_2\mid\tilde{\tilde{X}}_1,\tilde{\tilde{U}}\right), \\ &\acute{t}_1 I\left(\tilde{X}_2,\tilde{U};\tilde{Y}\right)+\acute{t}_2 I\left(\tilde{\tilde{X}}_1,\tilde{\tilde{U}};\tilde{\tilde{Y}}\right), \\ &\acute{t}_1 I\left(\tilde{X},\tilde{X}_2;\tilde{Y}_1\mid\tilde{U}\right),\acute{t}_2 I\left(\tilde{\tilde{X}},\tilde{\tilde{X}}_1;\tilde{\tilde{Y}}_2\mid\tilde{\tilde{U}}\right)\Big). \end{aligned} \tag{90}$$

APPENDIX B

Proof of Theorem 2

Codebook Construction:

Let us divide block number b, $b=1,2,\cdots,B+2$ into odd and even numbers. At odd and even blocks, source generates 2^{nR_1} and 2^{nR_2} $\tilde{\mathbf{u}}\left(q\right)$ and $\tilde{\tilde{\mathbf{u}}}\left(r\right)$ sequences according to $\prod_{i=1}^{\acute{t}_1 n}p(\tilde{u}_i)$ and $\prod_{i=1}^{\acute{t}_2 n}p(\tilde{\tilde{u}}_i)$, respectively. Then, source throws $\tilde{\mathbf{u}}$ and $\tilde{\tilde{\mathbf{u}}}$ sequences uniformly into $2^{n\tilde{R}_s}$ and $2^{n\tilde{\tilde{R}}_s}$ bins.

Relay 1 and relay 2 generate $2^{n\tilde{R}_s}$ and $2^{n\tilde{\tilde{R}}_s}$ i.i.d $\tilde{\tilde{\mathbf{x}}}_1$ and $\tilde{\mathbf{x}}_2$ sequences according to probabilities $\prod_{i=1}^{\acute{t}_2 n}p\left(\tilde{\tilde{x}}_{1i}\right)$ and $\prod_{i=1}^{\acute{t}_1 n}p\left(\tilde{x}_{2i}\right)$. Furthermore, for all q and r, the source generates double indexed code-books $\tilde{\mathbf{x}}\left(w_b|w_{b-1},q\right)$ and $\tilde{\tilde{\mathbf{x}}}\left(w_b|w_{b-1},r\right)$ according to $\prod_{i=1}^{\acute{t}_1 n}p(\tilde{x}_i\mid\tilde{x}_{2i},\tilde{u}_i)$ and $\prod_{i=1}^{\acute{t}_2 n}p(\tilde{\tilde{x}}_i\mid\tilde{\tilde{x}}_{1i},\tilde{\tilde{u}}_i)$, respectively.

Encoding:

Encoding at the source:

In odd blocks, since source knows what it has transmitted during the even block, from the desired bin $w_b\in\{1,\cdots,2^{n\tilde{R}_s}\}$, it can choose a codeword $\tilde{\mathbf{u}}\left(q\right)$ such that $q\in Bin\left(w_b\right)$ and $\left(\tilde{\mathbf{u}}\left(q\right),\tilde{\mathbf{x}}_2\left(w_{b-1}\right)\right)\in A_\epsilon^{(n)}$ if $R_1-\tilde{R}_s\geq\acute{t}_1 I\left(\tilde{U};\tilde{X}_2\right)$ and sends $\tilde{\mathbf{x}}(\tilde{\mathbf{u}},\tilde{\mathbf{x}}_2)$. Similarly, in even blocks, the source sends $\tilde{\tilde{\mathbf{x}}}(\tilde{\tilde{\mathbf{u}}},\tilde{\tilde{\mathbf{x}}}_1)$ if $R_2-\tilde{\tilde{R}}_s\geq\acute{t}_2 I\left(\tilde{\tilde{U}};\tilde{\tilde{X}}_1\right)$.

Encoding at relay 1:

Relay 1 encodes $w_b\in\{1,\cdots,2^{n\tilde{R}_s}\}$ to $\tilde{\tilde{\mathbf{x}}}_1\left(w_b\right)$ in even blocks.

Encoding at relay 2:

Relay 2 encodes $w_b \in \{1, \cdots, 2^{n\tilde{\tilde{R}}_s}\}$ to $\tilde{\mathbf{x}}_2(w_b)$ in odd blocks.

Decoding:

Decoding at relay 1:

In odd blocks, relay 1 declares $\hat{w}_b = Bin^{-1}(q)$ iff all the sequences $\tilde{\mathbf{u}}(q)$ which are jointly typical with $\tilde{\mathbf{y}}_1$ belong to a unique bin $\hat{w}_b$. Therefore, we should have have:

$$R_1 \leq \acute{t}_1 I\left(\tilde{U}; \tilde{Y}_1\right).$$

And consequently we have:

$$\tilde{R}_s \leq \acute{t}_1 \left(I(\tilde{U}; \tilde{Y}_1) - I(\tilde{U}; \tilde{X}_2)\right). \tag{91}$$

Decoding at relay 2:

In even blocks, relay 2 declares $\hat{w}_b = Bin^{-1}(r)$ iff all the sequences $\tilde{\tilde{\mathbf{u}}}(r)$ which are jointly typical with $\tilde{\tilde{\mathbf{y}}}_2$ belong to a unique bin $\hat{w}_b$. Therefore, we should have have:

$$R_2 \leq \acute{t}_2 I\left(\tilde{\tilde{U}}; \tilde{\tilde{Y}}_2\right).$$

And consequently we have:

$$\tilde{\tilde{R}}_s \leq \acute{t}_2 \left(I(\tilde{\tilde{U}}; \tilde{\tilde{Y}}_2) - I(\tilde{\tilde{U}}; \tilde{\tilde{X}}_1)\right). \tag{92}$$

Decoding at the final destination:

In odd blocks, destination declares $\hat{w}_b = w_b$ iff $(\tilde{\mathbf{x}}_2(\hat{w}_b), \tilde{\mathbf{y}}) \in A_\epsilon^{(n)}$. Hence, we have:

$$\tilde{R}_s \leq \acute{t}_1 I(\tilde{X}_2; \tilde{Y}). \tag{93}$$

Similarly in even blocks, we have:

$$\tilde{\tilde{R}}_s \leq \acute{t}_2 I(\tilde{\tilde{X}}_1; \tilde{\tilde{Y}}). \tag{94}$$

APPENDIX C

Proof of Theorem 4

Codebook Construction:

In the first α interval, i.e. "$t_{3\alpha}$", the source generates $2^{nR_{AF}}$, and $2^{nR_{DF}}$ gaussian sequences

$\mathbf{v}_{AF}(w_{AF})$, and $\mathbf{u}_{T,\alpha}(w_{DF})$, respectively. These sequences are generated according to the following probabilities:

$$\begin{aligned}\mathbf{v}_{AF}(w_{AF}) &\sim \prod_{i=1}^{\alpha n} p(v_{AF,i}) \sim \mathcal{N}(0, P_{s-AF}),\\ \mathbf{u}_{T,\alpha}(w_{DF}) &\sim \prod_{i=1}^{\alpha n} p(x_{T,\alpha i}|v_{AF,i}) \sim \mathcal{N}(0, P_{s-DF_1}).\end{aligned}$$

Hence, we have:

$$x_{T,\alpha i} = u_{T,\alpha i} + v_{AF,i}.$$

In the β interval, i.e. "$t_{3\beta}$", the source generates $2^{nR_{DF}}$ gaussian sequences $\mathbf{u}_{T,\beta}(w_{DF})$ according to $\prod_{i=1}^{\beta n} p(u_{T,\beta i}) \sim \mathcal{N}(0, P_{s-DF_2})$.

In the last α portion, i.e. $t_{4\alpha}$, each k^{th} relay, generates $2^{nR_{DF}}$ gaussian sequences $\mathbf{u}_{rk,\alpha}(w_{DF})$ according to $\prod_{i=1}^{\alpha n} p(u_{r1,\alpha i}) \sim \mathcal{N}(0, P_{r-DF})$.

Hence, at each "k^{th}" relay we have:

$$\sqrt{\frac{P_{r-AF}}{P_{s-AF}+\alpha}}(\mathbf{v}_{AF}+\mathbf{z}_k)+\mathbf{u}_{rk,\alpha}. \tag{95}$$

Encoding:

Encoding at the source:

In $t_{3\alpha}$ interval, the source encodes $w_{AF} \in \{1, \cdots, 2^{nR_{AF}}\}$, and $w_{DF} \in \{1, \cdots, 2^{nR_{DF}}\}$ into codewords $\mathbf{v}_{AF}(w_{AF})$, and $\mathbf{x}_{T,\alpha}(w_{DF}|w_{AF})$, respectively. Furthermore, in $t_{3\beta}$ interval it encodes $w_{DF} \in \{1, \cdots, 2^{nR_{DF}}\}$ into $\mathbf{u}_{T,\beta}(w_{DF})$.

Encoding at each relay:

In $t_{4\alpha}$ interval, each relay encodes $w_{DF} \in \{1, \cdots, 2^{nR_{DF}}\}$ into codewords $\mathbf{u}_{rk,\alpha}$.

Decoding:

Decoding at each relay:

Since we are considering symmetric scenarios, the first hop which is generally a broadcast channel simplifies to a single user channel. So at the end of $t_{3\alpha}+t_{3\beta}$ interval, each "k^{th}" relay declares $\hat{w}_{DF} = w_{DF}$ iff there exits unique $\mathbf{u}_{T,\alpha}(\hat{w}_{DF})$, and $\mathbf{u}_{T,\beta}(\hat{w}_{DF})$ sequences such that $(\mathbf{u}_{T,\alpha}(\hat{w}_{DF}), \mathbf{y}_{k,\alpha}) \in A_\epsilon^{(n)}$ and $(\mathbf{u}_{T,\beta}(\hat{w}_{DF}), \mathbf{y}_{k,\beta}) \in A_\epsilon^{(n)}$.

Hence, we have:

$$R_{DF} \le \alpha I(U_{T,\alpha}; Y_{1,\alpha}) + \beta I(U_{T,\beta}; Y_{1,\beta}). \tag{96}$$

Decoding at the final destination:

At the end of $t_{4\alpha}$, destination declares $(\hat{w}_{DF}, \hat{w}_{AF}) = (w_{DF}, w_{AF})$ iff

$$(\mathbf{v}_{AF}(\hat{w}_{AF}), \mathbf{u}_{r1,\alpha}(\hat{w}_{DF}), \mathbf{y}_{\alpha}) \in A_{\epsilon}^{(n)}.$$

Due to our modeling, we can think of a multiple access channel with independent AF and DF messages. Hence, from the capacity region of the multiple access channel, we have:

$$\begin{aligned} R_{AF} &\leq \alpha I(KV_{AF}; Y_{\alpha}|KU_{r1,\alpha}), \\ R_{DF} &\leq \alpha I(KU_{r1,\alpha}; Y_{\alpha}|KV_{AF}), \\ R_{AF} + R_{DF} &\leq \alpha I(KV_{AF}, KU_{r1,\alpha}; Y_{\alpha}). \end{aligned} \tag{97}$$

REFERENCES

[1] E. C. van-der Meulen, Three-terminal communication channels, *Adv. Appl. Prob.*, vol. 3, pp. 120-154, 1971.

[2] T. M. Cover and A. El Gamal, Capacity Theorems for the Relay Channel, *IEEE Transactions on Information Theory,* Vol. 25, No. 5, pp. 572-584, September 1979.

[3] M.A. Khojastepour, A. Sabharwal, B. Aazhang, On Capacity of Gaussian Cheap Relay Channel, IEEE 2003 Global Communications Conference (Globecom-2003), December 1-5, San Francisco, CA, 2003.

[4] M.A. Khojastepour, B. Aazhang, Cheap Relay Channels: A Unifying Approach to Time and Frequency Division Relaying, Forty-second Annual Allerton Conference on Communication, Control, and Computing, September 29-October 1, 2004.

[5] S. Zahedi, and A. El Gamal, Minimum energy communication over a relay channel, in *Proc. of International Symposium on Information Theory (ISIT03),* pp. 344, 29 June-4 July 2003.

[6] S. Zahedi, M. Mohseni, and A. El Gamal, On the capacity of AWGN relay channels with linear relaying functions, in *Proc. of International Symposium on Information Theory (ISIT04),* 27 June-2 July 2004.

[7] A. E. Gamal, and S. Zahedi, Capacity of a class of relay channels with orthogonal components, *IEEE Transactions on Information Theory,* Vol. 51, Issue 5, pp. 1815-1817, May 2005.

[8] Anders Host-Madsen, and Junshan Zhang, Capacity Bounds and Power Allocation for Wireless Relay Channels, *IEEE Transactions on Information Theory,* Vol. 51, Issue 6, pp. 2020-2040, June 2005.

[9] Y. Liang and V. V. Veeravalli, Gaussian orthogonal relay channels:Optimal resource allocation and capacity, *IEEE Transactions on Information Theory,* Vol. 51, No. 9, pp. 3284-3289, September 2005.

[10] B. Schein and R. G. Gallager, The Gaussian parallel relay network, *in Proc IEEE Int. Symp. Information Theory,* Sorrento, Italy, Jun. 2000, p. 22.

[11] B. E. Schein, Distributed coordination in network information theory, in *Ph.D thesis, Massachusetts Institute of Technology,* Sept. 2001.

[12] L. L. Xie and P. R. Kumar, A Network Information Theory for Wireless Communication: Scaling Laws and Optimal Operation, *IEEE Transactions on Information Theory,* Vol. IT-50, No. 5, pp. 748-767, May 2004.

[13] L. L. Xie and P. R. Kumar, An achievable rate for the multiple-level relay channel, *IEEE Transactions on Information Theory,* Vol. 51, No. 4, pp. 13481358, Apr. 2005.

[14] P. Gupta and P. R. Kumar, Toward an information theory of large networks: An achievable rate region, *IEEE Transactions on Information Theory,* Vol. 49, No. 8, pp. 18771894, Aug. 2003.

[15] M. Gastpar, To code or not to code, in *Ph.D. Thesis, Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland,* Nov. 2002.

[16] M. Gastpar and M. Vetterli, On asymptotic capacity of gaussian relay networks, in *Proc. of International Symposium on Information Theory(ISIT02),* June 2002.

[17] M. Gastpar, G. Kramer, and P. Gupta, The multiple-relay channel: Coding and antenna-clustering capacity, in *Proc. of International Symposium on Information Theory (ISIT02),* June 2002.

[18] G. Kramer, M. Gastpar, and P. Gupta, Cooperative Strategies and Capacity Theorems for Relay Networks, *IEEE Transactions on Information Theory,* Vol. 51, Issue 9, pp. 3037-3063, Sept. 2005.

[19] G. Kramer, M. Gastpar, and P. Gupta, Capacity theorems for wireless relay channels, in *Proc. of the Allerton Conference on Communications, Control and Computing, Monticello, IL,* Oct. 2003.

[20] A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, Communication via decentralized processing, in *Proc. of International Symposium on Information Theory (ISIT05),* Sept. 2005.

[21] Ivana Maric, Roy D. Yates, Forwarding Strategies for Gaussian Parallel-Relay Networks,in *Proc. 38th Annu. Conf. Information Sciences and Systems (CISS'04), Princeton, NJ,* Mar. 2004.

[22] P. Razaghi and W. Yu, Parity forwarding for multiple-relay networks, in *Proc. of International Symposium on Information Theory (ISIT06),* July 2006, pp. 16781682.

[23] T. Cover, and J. Thomas, Elements of Information Theory, Wiley, New York, first edition, 1991.

[24] J. Korner, and K. Marton, General broadcast channels with degraded message sets, *IEEE Transactions on Information Theory,* Vol. 23, Issue 1, pp. 60-64, Jan. 1977.

[25] D. Slepian and J. K. Wolf, A coding theorem for multiple access channels with correlated sources, Bell Syst. Tech. J., vol. 52, pp. 1037-1076, 1973.

[26] T. S. Han, The capacity region of a genera1 multiple-access channel with certain correlated sources, Inform. Contr., vol. 40, no. I, pp. 37-60, 1979.

[27] V. Prelov. Transmission over a multiple-access channel with a special source hierarchy, Problemy Peredachi Informatsii, 20:3-10, 1984. English translation pp. 233-9, 1985.

[28] Mohammad Ali Khojastepour, Ashutosh Sabharwal and Behnaam Aazhang Bounds on Achievable Rates for General Multi-terminal Networks with Practical Constraints, in the proceeding of *Information Processing in Sensor Networks: Second International Workshop*, IPSN 2003, Palo Alto, CA, USA, April 22-23, 2003.

[29] Max H. M. Costa, ”Writing on dirty paper,” IEEE Transactions on Information Theory, vol. 29, no. 3, pp. 439–441, May 1983.

[30] B. Rankov and A. Wittneben, Spectral efficient protocols for nonregenerative half-duplex relaying, in 43rd Allerton Conference on Communication, Control and Computing, Monticello (IL), USA, October 2005.

[31] B. Rankov and A. Wittneben, Spectral efficient signaling for half-duplex relay channels, in Proc. Asilomar Conf. Signals, syst., comput., Pacific Grove, CA, Nov.,2005.

[32] Woohyuk Chang, Sae-Young Chung, and Yong H. Lee, Capacity Bounds for Alternating Two-Path Relay Channels, in *Proc. of the Allerton Conference on Communications, Control and Computing, Monticello, IL,* Oct. 2007.

[33] Feng Xue, and Sumeet Sandhu, Cooperation in a Half-Duplex Gaussian Diamond Relay Channel, *IEEE Transactions on Information Theory,* Vol. 53, Issue 10, pp. 3806-3814, Oct. 2007.