A New Adaptive Distributed Space-Time-Coding Scheme for Cooperative Relaying

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Abstract

A non-regenerative dual-hop wireless system based on a distributed space-time-coding strategy is considered. It is assumed that each relay station retransmits an appropriate scaled space-time coded version of the received signals. The main goal of this paper is to provide an analytical study of an optimum power allocation strategy in relay stations in order to satisfy the quality of service requirements. In the high signal-to-noise ratio (SNR) regime for the relay-destination link, it is shown that the optimum power allocation strategy in each relay that minimizes the outage probability is to remain silent, if its direct channel gain with the source is less than a prespecified threshold level. The Monte-Carlo simulations show that the optimum power allocation scheme in each relay in order to minimize the outage probability or the frame-error rate is the threshold-based on-off power scheme (i.e., the relay which its channel gain with the source is above a certain threshold transmits at full power). Also, the numerical results show a dramatical improvement in the system performance by using this scheme in comparison with the case that the relay stations forward their received signals with full power.

Index Terms

Cooperative relaying, space-time-block-code, outage probability, multi-hop wireless networks.

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I. Introduction

A. History

The main challenge in wireless networks is to mitigate multipath-induced channel fading that degrades the network performance. Exploiting diversity techniques such as time, frequency and in particular space diversity are the most effective methods in combating the channel fading. Since, it is difficult to install multiple antenna in the mobile station (due to size and cost limitations), using a collection of distributed antennas belonging to multiple users improve the network performance as well as the link reliability. This new diversity approach, referred to as the *cooperative relaying*, has attracted considerable attention in wireless networks in recent years [1]–[4]. In the cooperative relaying, multiple relay nodes collaborate with each other through exploiting different independent fading channels in order to achieve a better performance without sacrificing the bandwidth. Cooperative relaying has been addressed from various perspectives; including capacity [1], [2], [5], resource allocation (e.g., power assignments [6]), coding schemes [7], etc. Central to the study of the cooperative relaying is the problem of using *distributed space-time coding* (DSTC) techniques and the power allocation for regenerative and non-regenerative relay networks [5], [8]–[15].

The first study of using DSTC schemes in cooperative relaying was framed in [5]; several relay stations transmit jointly to the same receiver in order to achieve full spatial diversity in terms of the number of cooperation terminals. They also analyze the outage capacity in the high signal-to-noise ratio (SNR) regime. Nabar *et al.* [8] analyze the pairwise-error-probability (PEP) of a single-relay fading system that relies on a DSTC scheme operating in the amplify-and-forward (AF) mode. In [9], the authors investigate the high SNR bit-error-rate (BER) of a two-relay system with a Rayleigh fading model and by using a switching scheme. Scutari and Barbarossa [11] analyze the performance of a DSTC scheme in regenerative relay networks by exploiting alternative cooperation and decoding strategies. The average BER of single and dual-hop non-regenerative relay-fading systems with DSTC has also analyzed in [12]. They investigate different optimum transmissions policies in order to maximize the end-to-end SNR. The performance of regenerative relay networks by using the DSTC scheme and the optimum power allocation over non-identical Ricean channels is considered in [14]. Zhao *et al.* [6] introduce two optimum power allocation algorithms for AF relay networks in order to minimize the outage probability without using DSTC scheme.

B. Contributions and Relations to Previous Works

In the most of the previous works, the power allocation problem in the relay network model used in IEEE 802.16j has not been sufficiently investigated. In this work, we address the question: How do we deploy an efficient distributed space-time coding scheme and the optimum power control strategy among relays in order to satisfy the quality-of-service (QoS), e.g., minimizing the outage probability ? To motivate our proposed idea, we consider a two-hop wireless network consisting of the source, two parallel relay stations (RS) and the destination. In this setup, RSs forward the space-time-coded (STC) version of their received signals from the source to destination. However, if the instantaneous received SNR of the relays are unbalanced, the performance of the cooperative relaying degrades substantially. To overcome this problem, an adaptive distributed space-time-coding (ADSTC) method is proposed, in which instead of transmitting the noisy signal at each relay with full power, the RS retransmits an appropriately scaled STC version of the received signals.

The main goal of the proposed scheme is to optimize the scaling factor based on the channel-state information (CSI) provided at each RS in order to minimize the outage probability. Our scheme is different from the power allocation algorithms in [6]; primarily we utilize an ADSTC scheme based on the local information (i.e., the direct channel gains of the source-relay and the relay-destination links), while in the algorithm developed in [6] no DSTC scheme is used. Also, our scheme is different from the algorithms proposed in [5], as we focus on the optimum power allocation strategy at RSs through controlling the scaling factors.

In the high SNR regime for the relay-destination link, it is shown that the optimum power allocation strategy in each relay that minimizes the outage probability is to remain silent, if its direct channel gain with the source is less than a prespecified threshold level. The Monte-Carlo simulations show that the optimum power allocation scheme in each relay in order to minimize the outage probability or the frame-error rate (FER) is the threshold-based on-off power scheme (i.e., the relay which its channel gain with the source is above a certain threshold transmits at full power). Also, the numerical results show a significant improvement in the system performance by using this scheme in comparison with the case that the relays stations forward their received signals with full power. In addition, the frame-error rate of the proposed scheme by using convolutioanal and turbo codes is numerically analyzed. The advantage of using this protocol is that it dose not alter the Alamouti decoder at the destination and it slightly modifies the relay operation which is practically appealing. Also, through using the on-off power allocation strategy at RSs, we can save the energy in the silent mode. This results in maximizing the battery life that is crucial for battery-powered RSs.

The rest of the paper is organized as follows. In Section II, the system model and objectives are described. We analyze the performance of the system in Section III. In Section IV, the simulation and numerical results are presented. Finally, in Section V, an overview of the results and conclusions is presented.

C. Notations

Throughout this paper, we use boldface lower case letters to denote vectors, boldface capital letters to denote matrices. Also, $\| \mathbf{a} \|$ indicates the Euclidean norm of the vector **a** and $\mathbb{E}[\ .\ ]$ represents the expectation operator. Also, the conjugate, transposition and conjugated transposition of a complex matrix **A** are denoted by $\mathbf{A}^*$, $\mathbf{A}^T$ and $\mathbf{A}^\dagger$, respectively. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$. A circularly symmetric complex Gaussian random variable is a random variable $Z = X + jY \sim \mathcal{CN}(0, \sigma^2)$, where X and Y are independent identically distributed (i.i.d.) $\mathcal{N}(0, \frac{\sigma^2}{2})$. Also, $A \rightarrow B$ denotes the link from node A to node B.

II. System Model and Objectives

In this work, we consider a dual-hop wireless system depicted in Fig. 1. The model consists of the source (S), the destination (D) and two parallel relay stations denoted by RS_1 and RS_2 that randomly located within the region between S and D. We assume that all the nodes are equipped with a single antenna. The relay stations are assumed to be in the form of the non-regenerative, i.e., they perform some simple operations on received

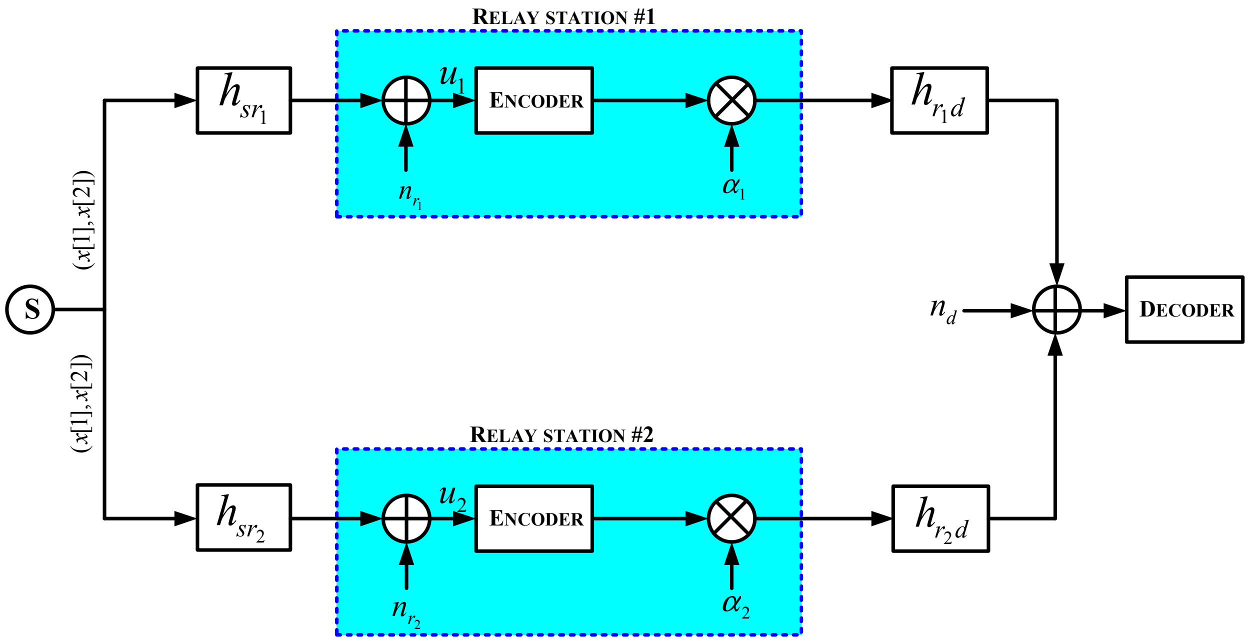

Fig. 1. A discrete-time baseband equivalent model of dual-hop wireless network.

signals and forward them to the destination node. Also, it is assumed that no information is exchanged between relays. The background noise at each receiver is assumed to be additive white Gaussian noise (AWGN).

The channel model considered in this paper is assumed to be frequency-flat block Rayleigh fading. The link S $\rightarrow$ RS$_i$ is represented by the channel gain $g_{sr_i} \triangleq |h_{sr_i}|^2$, where the complex random variables h_{sr_i}'s are the small scale fading channel coefficients. Also, the link RS$_i$ $\rightarrow$ D is represented by the channel gain $g_{r_id} \triangleq |h_{r_id}|^2$. Under the Rayleigh fading channel model, g_{sr_i}'s and g_{r_id}'s are exponentially distributed with unit mean and with the cumulative distribution function (CDF) of $F_Y(y) = 1 - e^{-y}$. We also assume that the channel-state information (CSI), i.e., the direct-link channel gains, is perfectly known at the receivers at the beginning of each block. Moreover, it is assumed that the destination node has perfect knowledge of the equivalent links S $\rightarrow$ RS$_i$ $\rightarrow$ D, $i = 1, 2$. For this model, the communication process between S and D is performed based on the following steps.

i) Data Transmission: The data transmission process between S and D is performed over two phases and through two hops. In the first phase, the source node broadcasts the

TABLE I

TRANSMISSION POLICY IN RELAY STATIONS

	RS_1 antenna	RS_2 antenna
First time slot	$\sqrt{\alpha_1}e^{-j\theta_{sr_1}}y_1[1]$	$\sqrt{\alpha_2}e^{-j\theta_{sr_2}}y_2[2]$
Second time slot	$-\sqrt{\alpha_1}\left(e^{-j\theta_{sr_1}}y_1[2]\right)^*$	$\sqrt{\alpha_2}\left(e^{-j\theta_{sr_2}}y_2[1]\right)^*$

symbols $x[1]$ and $x[2]$ to relay nodes during two consecutive time slots and over one frequency subchannel. Let us assume that the channel gains remain constant over two successive symbol transmissions. In this case, the received discrete-time baseband equivalent signals at RS_1 and RS_2 are given by

$$u_1[k] = h_{sr_1}x[k] + n_{r_1}[k], \quad k = 1, 2, \tag{1}$$

$$u_2[k] = h_{sr_2}x[k] + n_{r_2}[k], \quad k = 1, 2, \tag{2}$$

respectively, where the quantity k represents the transmission time index and $n_{r_i}[k] \sim \mathcal{CN}(0, \sigma_r^2)$, for $i = 1, 2$. In the above equations, we assume that the background noise and the interference from other transmitters at each relay are represented by $n_{r_i}[k]$ and with power σ_r^2. In this work, we assume that the source node uses the BPSK constellation[1], i.e., $(x[1], x[2]) \in \mathbb{S}^2$, where $\mathbb{S} = \{-1, 1\}$ is the constellation set. Letting $\mathbf{x} \triangleq \Big[x[1], x[2]\Big]^T$, we assume that the covariance matrix $\mathbf{R}_{\mathbf{xx}} \triangleq \mathbb{E}[\mathbf{xx}^\dagger] = \mathbf{I}_2$. Thus, the energy of the transmitted signals denoted by $E_b \triangleq \mathbb{E}\left[|x[k]|^2\right]$ are unity. In order to balance the energies of the received signals at relays, we normalize $u_i[k]$ with $\sqrt{\mathbb{E}\left[|u_i[k]|^2 \Big| h_{sr_i}\right]}$, $i = 1, 2$. Thus, (1) and (2) can be written as

$$y_1[k] = \frac{1}{\sqrt{g_{sr_1} + \sigma_r^2}}\left(h_{sr_1}x[k] + n_{r_1}[k]\right), \quad k = 1, 2, \tag{3}$$

$$y_2[k] = \frac{1}{\sqrt{g_{sr_2} + \sigma_r^2}}\left(h_{sr_2}x[k] + n_{r_2}[k]\right), \quad k = 1, 2. \tag{4}$$

In the second phase, the relay nodes cooperate with each other and forward the space-time-coded version of their received noisy signals to the destination over another frequency

[1]Using BPSK assumption is made only for simplifying the analysis. The extension to higher order constellations is straightforward.

subchannel[2] (Table I). The reason for using two orthogonal subchannels is that the relays can receive and transmit at the same time over single antenna. It should be noted that the channel phase of the link S $\rightarrow$ RS$_i$, denoted by θ_{sr_i}, is compensated at RS$_i$ through multiplying the received signal by the factor $e^{-j\theta_{sr_i}} = h^*_{sr_i}/|h_{sr_i}|$. Denoting the average signal-to-noise ratios of S $\rightarrow$ RS$_i$ and RS$_i$ $\rightarrow$ D by SNR$_{sr} \triangleq E_b/\sigma_r^2$ and SNR$_{rd} \triangleq E_b/\sigma_d^2$, respectively, RS$_i$ multiplies $y_i[.]$ by the scaling factor $\sqrt{\alpha_i}$, where $0 \leq \alpha_i \leq 1$ is a function of g_{sr_i}, SNR$_{sr}$ and SNR$_{rd}$. Here, we assume that SNR$_{sr}$ and SNR$_{rd}$ are known at RSs. Assuming fixed SNR$_{sr}$, the main goal of the proposed scheme is to choose the optimum factor α_i for every value of SNR$_{rd}$ such that the quality of service requirement like the minimum outage probability or minimum BER is satisfied.

In each time slot, the destination receives a superposition of the transmitted signals by RS's. To this end, the received signals at D in the first and the second time slots are given by

$$
\begin{aligned}
r_d[1] &= \sqrt{\alpha_1} e^{-j\theta_{sr_1}} y_1[1] h_{r_1 d} + \sqrt{\alpha_2} e^{-j\theta_{sr_2}} y_2[2] h_{r_2 d} + n_d[1], \\
r_d[2] &= -\sqrt{\alpha_1} \left(e^{-j\theta_{sr_1}} y_1[2] \right)^* h_{r_1 d} + \sqrt{\alpha_2} \left(e^{-j\theta_{sr_2}} y_2[1] \right)^* h_{r_2 d} + n_d[2],
\end{aligned}
$$

respectively, where $n_d[k] \sim \mathcal{CN}(0, \sigma_d^2)$ is the corresponding noise plus the interference from the other transmitters at destination. During the cooperation phase, we assume that the signals transmitted by relays in each time slot arrive at the same time at the destination node. The manner in which the relay stations are perfect synchronized is beyond the scope of this work. It should be noted that due to the long distance between S and D or due to the strong shadowing, we ignore the received signal of the direct link S $\rightarrow$ D. Substituting (3) and (4) in above equations yields

$$
r_d[1] = h_1^{'} x[1] + h_2^{'} x[2] + z_d[1], \tag{5}
$$

$$
r_d[2] = h_2^{'} x^*[1] - h_1^{'} x^*[2] + z_d^*[2], \tag{6}
$$

where

$$
h_i^{'} = h_{r_i d} \sqrt{\frac{\alpha_i g_{sr_i}}{g_{sr_i} + \sigma_r^2}} \quad ; \quad i = 1, 2, \tag{7}
$$

[2]It is assumed that the relay nodes have the same configurations and use the same protocols. Thus, the transmission strategy for relay nodes are agreed in advance.

and

$$z_d[1] \triangleq \frac{h_1' e^{-j\theta_{sr_1}}}{|h_{sr_1}|} n_{r_1}[1] + \frac{h_2' e^{-j\theta_{sr_2}}}{|h_{sr_2}|} n_{r_2}[2] + n_d[1], \tag{8}$$

$$z_d[2] \triangleq -\frac{h_1'^* e^{-j\theta_{sr_1}}}{|h_{sr_1}|} n_{r_1}[2] + \frac{h_2'^* e^{-j\theta_{sr_2}}}{|h_{sr_2}|} n_{r_2}[1] + n_d^*[2]. \tag{9}$$

It is observed that $\mathbb{E}\left[|z_d[1]|^2 \middle| \mathbf{h}\right] = \mathbb{E}\left[|z_d[2]|^2 \middle| \mathbf{h}\right] = \sigma^2$, where $\mathbf{h} \triangleq [h_{sr_1}, h_{sr_2}, h_{r_1 d}, h_{r_2 d}]$ and

$$\sigma^2 \triangleq \left(\frac{|h_1'|^2}{g_{sr_1}} + \frac{|h_2'|^2}{g_{sr_2}}\right) \sigma_r^2 + \sigma_d^2. \tag{10}$$

ii) Decoding Process: Amplify-and-forward systems require that full CSI, i.e., the fading coefficients vector $\mathbf{h}$, be available at the destination node to coherently decode the received signal. According to the Alamouti's scheme [16], the maximum likelihood (ML) decoding process is performed as follows:

$$\begin{bmatrix} r_d[1] \\ r_d^*[2] \end{bmatrix} = \begin{bmatrix} h_1' & h_2' \\ h_2'^* & -h_1'^* \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \end{bmatrix} + \begin{bmatrix} z_d[1] \\ z_d[2] \end{bmatrix},$$

or equivalently

$$\mathbf{r}_d = \mathbf{H}\mathbf{x} + \mathbf{z}_d. \tag{11}$$

It is worth mentioning that the channel matrix $\mathbf{H}$ preserves the orthogonal structure property, i.e., $\mathbf{H}^\dagger \mathbf{H} = \left(|h_1'|^2 + |h_2'|^2\right) \mathbf{I}_2$. To this end, the input of the ML decoder is

$$\begin{aligned} \tilde{\mathbf{r}}_d &= \mathbf{H}^\dagger \mathbf{r}_d \\ &= \mathbf{H}^\dagger \mathbf{H}\mathbf{x} + \mathbf{H}^\dagger \mathbf{z}_d \\ &= \Lambda \mathbf{x} + \tilde{\mathbf{z}}_d, \end{aligned} \tag{12}$$

where $\Lambda \triangleq |h_1'|^2 + |h_2'|^2$ and $\tilde{\mathbf{z}}_d \triangleq \mathbf{H}^\dagger \mathbf{z}_d$. In this case, the two-dimensional decision rule in the ML decoder will be as

$$\hat{\mathbf{x}} = \arg \min_{\hat{\mathbf{x}} \in \mathbb{S}^2} \|\tilde{\mathbf{r}}_d - \Lambda \hat{\mathbf{x}}\|^2. \tag{13}$$

Hence, the estimated symbols $\hat{x}[1]$ and $\hat{x}[2]$ are obtained as

$$\hat{x}[1] = \arg \min_{\hat{x}[1]} \left|\tilde{r}_d[1] - \Lambda \hat{x}[1]\right|^2, \tag{14}$$

$$\hat{x}[2] = \arg \min_{\hat{x}[2]} \left|\tilde{r}_d[2] - \Lambda \hat{x}[2]\right|^2. \tag{15}$$

III. Performance Analysis

In this section, we characterize the performance of the model described in Section II in terms of the outage probability and for the uncoded case. We derive an approximation formula for the outage probability based on the end-to-end SNR for the high SNR_{rd} regime. The considered optimization criterion is to minimize the end-to-end outage probability with respect to scaling factor α_i. To handle that, we first obtain the instantaneous end-to-end SNR of the proposed model.

Using the fact that the destination node has perfect knowledge of the links S $\rightarrow$ RS$_i$ $\rightarrow$ D, it is concluded that the matrix $\mathbf{H}|\mathbf{h}$ is deterministic. Noting that $\mathbb{E}\left[\mathbf{z}_d\mathbf{z}_d^{\dagger}|\mathbf{h}\right] = \sigma^2\mathbf{I}_2$ and $\mathbf{H}^{\dagger}\mathbf{H} = \Lambda\mathbf{I}_2$, it yields,

$$\mathbb{E}\left[\tilde{\mathbf{z}}_d\tilde{\mathbf{z}}_d^{\dagger}|\mathbf{h}\right] = \mathbb{E}\left[\mathbf{H}^{\dagger}\mathbf{z}_d\mathbf{z}_d^{\dagger}\mathbf{H}|\mathbf{h}\right] = \Lambda\sigma^2\mathbf{I}_2. \tag{16}$$

This implicitly indicates that the noise vector $\tilde{\mathbf{z}}_d|\mathbf{h}$ preserves the white Gaussian noise property. Since $\mathbb{E}\left[|x[i]|^2\right] = 1$, the instantaneous end-to-end SNR conditional upon the vector $\mathbf{h}$ is obtained as

$$\gamma_{\mathbf{h}} = \frac{\Lambda^2}{\Lambda\sigma^2} = \frac{|h_1^{'}|^2 + |h_2^{'}|^2}{\left(\dfrac{|h_1^{'}|^2}{g_{sr_1}} + \dfrac{|h_2^{'}|^2}{g_{sr_2}}\right)\sigma_r^2 + \sigma_d^2}. \tag{17}$$

Hence, the average end-to-end SNR, denoted by $\bar{\gamma}$, is now given as

$$\bar{\gamma} = \int_0^{\infty} \gamma_{\mathbf{h}} f_{\gamma}(\gamma) d\gamma, \tag{18}$$

where $f_{\gamma}(.)$ is the probability density function (pdf) of $\gamma_{\mathbf{h}}$.

Now we are ready to prove the main result of this paper. We assume that the QoS requested is provided when the end-to-end SNR exceeds a prescribed threshold γ_t. In the outage-based transmission framework, the outage event happens whenever $\gamma_{\mathbf{h}} < \gamma_t$. Hence, the outage probability is defined as

$$\text{P}_{out} \triangleq \text{Pr}\{\gamma_{\mathbf{h}} < \gamma_t\}. \tag{19}$$

Clearly, the outage probability can be reduced by allocating the optimum power in each relay (through controlling the scaling factors α_i). It is extremely difficult to directly compute the exact expression for the outage probability, if it is not impossible. However, the following theorems present the outage probability of the underlying system in the high SNR_{rd} regime.

Lemma 1 *Let X_1 and X_2 be independent exponential random variables with parameters λ_1 and λ_2, respectively. Then, the probability density function of $Z \triangleq \frac{X_1}{X_2}$ is obtained as*

$$f_Z(z) = \frac{\lambda_1 \lambda_2}{(\lambda_1 z + \lambda_2)^2} U(z). \tag{20}$$

Proof: See Appendix I. ■

Theorem 1 *Under the high SNR_{rd} regime, the optimum power allocation policy in each relay that minimizes (19) is obtained as*

$$\alpha_i = \begin{cases} 0, & \textit{if} \quad 0 < g_{sr_i} < \xi, \\ \mathcal{F}(g_{sr_i}), & \textit{Otherwise}, \end{cases} \tag{21}$$

where $\xi \triangleq \sigma_r^2 \gamma_t$ and $\mathcal{F}(g_{sr_i})$ is any arbitrary positive function of g_{sr_i}.

Proof: Using (17), the instantaneous end-to-end SNR conditioned upon $\mathbf{h}$ can be written as

$$\gamma_{\mathbf{h}} = \frac{X_1 g_{sr_1} + X_2 g_{sr_2}}{(X_1 + X_2)\sigma_r^2 + \sigma_d^2}, \tag{22}$$

where $X_i \triangleq \alpha_i^{'} g_{r_i d}, \; i = 1, 2$ and $\alpha_i^{'}$ is a function of g_{sr_i} defined as

$$\alpha_i^{'} \triangleq \frac{\alpha_i}{g_{sr_i} + \sigma_r^2}. \tag{23}$$

Under the Rayleigh fading channel model, $X_i | g_{sr_i}$ is exponentially distributed with parameter $\frac{1}{\alpha_i^{'}}$ and with the following pdf

$$f_{X_i|g_{sr_i}}(x_i | g_{sr_i}) = \frac{1}{\alpha_i^{'}} e^{-\frac{x_i}{\alpha_i^{'}}} U(x_i),$$

where $U(.)$ is the unit step function. In the high SNR_{rd} regime[3], i.e., $\sigma_d^2 \ll \sigma_r^2$, (22) can be simplified as follows:

$$\gamma_{\mathbf{h}} = \frac{X_1 g_{sr_1} + X_2 g_{sr_2}}{(X_1 + X_2)\sigma_r^2}.$$

[3]Physically, this may occur when the relay stations are located very close to the destination.

Using the fact that g_{sr_1} is independent of g_{sr_2},

$$f_{g_{sr_1}g_{sr_2}}(v_1, v_2) = f_{g_{sr_1}}(v_1)f_{g_{sr_2}}(v_2) = e^{-v_1}e^{-v_2}, \qquad v_1,\ v_2 > 0.$$

Thus, the outage probability is given by

$$\mathrm{P}_{out} = \mathbb{E}_{\mathbf{g}_s}[\Omega(\mathbf{g}_s)] = \int_0^{\infty}\int_0^{\infty} \Omega(\mathbf{g}_s)e^{-v_1}e^{-v_2}dv_1dv_2, \tag{24}$$

where $\mathbf{g}_s \triangleq [g_{sr_1} = v_1, g_{sr_2} = v_2]$ and $\Omega(\mathbf{g}_s) \triangleq \Pr\{\gamma_{\mathbf{h}} < \gamma_t|\mathbf{g}_s\}$. To find the outage probability, we first compute $\Omega(\mathbf{g}_s)$ as follows

$$\begin{aligned}\Omega(\mathbf{g}_s) &= \Pr\{X_1v_1 + X_2v_2 < (X_1 + X_2)\xi|\mathbf{g}_s\} \\ &= \Pr\{(v_1 - \xi)X_1 < (\xi - v_2)X_2|\mathbf{g}_s\},\end{aligned} \tag{25}$$

where $\xi \triangleq \sigma_r^2\gamma_t$. Depends on the values of v_1 and v_2 with respect to ξ, we have the following cases:

<u>**Case 1:**</u> $v_2 < \xi < v_1$

In this case, (25) can be written as

$$\begin{aligned}\Omega(\mathbf{g}_s) &= \Pr\left\{\frac{X_1}{X_2} < \frac{\xi - v_2}{v_1 - \xi}\middle|\mathbf{g}_s\right\} \\ &= \Pr\left\{Z < \phi|\mathbf{g}_s\right\},\end{aligned} \tag{26}$$

where $Z \triangleq \frac{X_1}{X_2}$ and $\phi \triangleq \frac{\xi - v_2}{v_1 - \xi}$. Using Lemma 1, the pdf of the random variable Z conditioned on the vector $\mathbf{g}_s$ is obtained as

$$f_{Z|\mathbf{g}_s}(z|\mathbf{g}_s) = \frac{\alpha_1^{'}\alpha_2^{'}}{(\alpha_2^{'}z + \alpha_1^{'})^2}U(z). \tag{27}$$

Thus, (26) can be written as

$$\begin{aligned}\Omega(\mathbf{g}_s) &= \int_0^{\phi} f_{Z|\mathbf{g}_s}(z|\mathbf{g}_s)dz \\ &= \frac{\alpha_2^{'}\phi}{\alpha_2^{'}\phi + \alpha_1^{'}}.\end{aligned} \tag{28}$$

<u>**Case 2:**</u> $\xi < v_1$ and $\xi < v_2$

In this case, the quantity $\phi = \frac{\xi - v_2}{v_1 - \xi}$ is negative. Thus, since X_1 and X_2 are non-negative, it is concluded that

$$\Omega(\mathbf{g}_s) = \Pr\left\{\frac{X_1}{X_2} < \phi\middle|\mathbf{g}_s\right\} = 0. \tag{29}$$

TABLE II

Case	$\Omega(\mathbf{g}_s)$	$\mathbf{g}_s$ and ξ
1	$\frac{\alpha_2^{'}\phi}{\alpha_2^{'}\phi+\alpha_1^{'}}$	$v_2 < \xi < v_1$
2	0	$\xi < v_1$ and $\xi < v_2$
3	1	$\xi > v_1$ and $\xi > v_2$
4	$\frac{\alpha_1^{'}}{\alpha_2^{'}\phi+\alpha_1^{'}}$	$v_1 < \xi < v_2$

Case 3: $\xi > v_1$ and $\xi > v_2$

From (25), we have

$$\begin{aligned}\Omega(\mathbf{g}_s) &= \Pr\left\{(v_1-\xi)X_1 < (\xi - v_2)X_2|\mathbf{g}_s\right\} \\ &= \Pr\{Z > \phi|\mathbf{g}_s\} \\ &= 1-\Pr\{Z \leq \phi|\mathbf{g}_s\} \\ &\overset{(a)}{=} 1, \end{aligned} \tag{30}$$

where (a) follows from the fact that for $\xi > v_1$ and $\xi > v_2$, ϕ is a negative value, and this results in $\Pr\{Z \leq \phi|\mathbf{g}_s\} = 0$.

Case 4: $v_1 < \xi < v_2$

In this case, (25) can be written as

$$\begin{aligned}\Omega(\mathbf{g}_s) &= \Pr\{Z > \phi|\mathbf{g}_s\} \\ &= 1-\Pr\{Z \leq \phi|\mathbf{g}_s\}, \end{aligned} \tag{31}$$

where ϕ is positive. Using (27) and (28), we come up with the following equality:

$$\begin{aligned}\Omega(\mathbf{g}_s) &= 1-\frac{\alpha_2^{'}\phi}{\alpha_2^{'}\phi+\alpha_1^{'}} \\ &= \frac{\alpha_1^{'}}{\alpha_2^{'}\phi+\alpha_1^{'}}. \end{aligned} \tag{32}$$

Now, we can use the results summarized in Table II in order to compute the outage probability obtained in (24). To this end,

$$
\begin{aligned}
\mathrm{P}_{out} &= \int_0^\infty \int_0^\infty \Omega(\mathbf{g}_s) e^{-v_1} e^{-v_2} dv_1 dv_2 \\
&= \int_0^\xi \int_0^\xi e^{-v_1} e^{-v_2} dv_1 dv_2 + \int_0^\xi \left[\int_\xi^\infty \frac{\alpha_1^{'} e^{-v_2}}{\alpha_2^{'}\phi + \alpha_1^{'}} dv_2 \right] e^{-v_1} dv_1 \\
&+ \int_\xi^\infty \left[\int_0^\xi \frac{\alpha_2^{'}\phi e^{-v_2}}{\alpha_2^{'}\phi + \alpha_1^{'}} dv_2 \right] e^{-v_1} dv_1 \\
&= (1 - e^{-\xi})^2 + \int_0^\xi \left[\int_\xi^\infty \frac{\alpha_1^{'} e^{-v_2}}{\alpha_2^{'}\phi + \alpha_1^{'}} dv_2 \right] e^{-v_1} dv_1 \\
&+ \int_\xi^\infty \left[\int_0^\xi \frac{\alpha_2^{'}\phi e^{-v_2}}{\alpha_2^{'}\phi + \alpha_1^{'}} dv_2 \right] e^{-v_1} dv_1. \qquad (33)
\end{aligned}
$$

Defining $\alpha_i^{'} = \dfrac{\alpha_i}{v_i + \sigma_r^2} \triangleq g(v_i)$ and $\varphi \triangleq \frac{1}{\phi} = \frac{\xi - v_1}{v_2 - \xi}$, (33) can be simplified as

$$
\begin{aligned}
\mathrm{P}_{out} &= (1 - e^{-\xi})^2 + \int_0^\xi \left[\int_\xi^\infty \frac{e^{-v_2}}{g(v_1) + g(v_2)\phi} dv_2 \right] g(v_1) e^{-v_1} dv_1 \\
&+ \int_0^\xi \left[\int_\xi^\infty \frac{e^{-v_1}}{g(v_2) + g(v_1)\varphi} dv_1 \right] g(v_2) e^{-v_2} dv_2 \qquad (34) \\
&\overset{(a)}{=} (1 - e^{-\xi})^2 + 2 \int_0^\xi \left[\int_\xi^\infty \frac{e^{-v_2}}{g(v_1) + g(v_2)\phi} dv_2 \right] g(v_1) e^{-v_1} dv_1, \qquad (35)
\end{aligned}
$$

where (a) follows from the fact that the second and the third terms of (34) are symmetric. Hence, the optimizationion problem is

$$
\alpha_i^* = \arg \min_{\alpha_i} \ \mathrm{P}_{out}, \quad i = 1, 2. \qquad (36)
$$

It can be easily shown that the optimum function $g(.)$ that minimizes (35) is obtained as

$$
g(v_i) = \begin{cases} 0, & 0 < v_i < \xi, \\ \mathcal{F}(v_i), & \text{otherwise}, \end{cases} \qquad (37)
$$

where $\mathcal{F}(v_i)$ is an arbitrary positive function of v_i. Thus, for high SNR_{rd} regime, the optimum scaling factor α_i that minimizes the outage probability is obtained as

$$
\alpha_i = \begin{cases} 0, & 0 < g_{sr_i} < \xi, \\ \mathcal{F}(g_{sr_i}), & \text{otherwise}. \end{cases} \qquad (38)
$$

In this case, the minimum outage probability is equal to $(1-e^{-\xi})^2$. ■

Remark 1: It is clear that under the high SNR$_{sr}$ regime, the optimum power allocation policy for relay stations that minimizes (19) is to transmit with the same powers.

IV. Numerical Results

In this section, we present some Monte-Carlo simulation results to evaluate the impact of the scaling factor α_i on the performance of the underlying system. First, we introduce some forms of the scaling factors α_i in terms of the direct channel gain g_{sr_i}.

- **Full Power Scheme:** In this approach and independent of the channel gain g_{sr_i}, each relay retransmits its received signal with full power, i.e., $\alpha_i = 1$.
- **On-Off Power Scheme:** In this scheme and based on the direct channel gain g_{sr_i}, each relay decides to transmit with full power or to remain at the silence mode, i.e.,

$$\alpha_i = \begin{cases} 1, & \text{if} \quad g_{sr_i} > \tau \\ 0, & \text{Otherwise}, \end{cases} \tag{39}$$

  where τ is a prescribed threshold level. Assuming a fixed value of SNR$_{sr}$, the main goal is to find the optimum τ such that the QoS requirement is satisfied.
- **Piecewise Linear Function Scheme:** In this setup, the scaling factor α_i is defined as a piecewise linear function with two parameters T_1 and T_2 as follows:

$$\alpha_i = \begin{cases} 0 & , \quad g_{sr_i} < T_1 \\ \dfrac{1}{T_2 - T_1} g_{sr_i} - \dfrac{T_1}{T_2 - T_1}, & T_1 \leq g_{sr_i} < T_2 \\ 1 & , \quad g_{sr_i} \geq T_2. \end{cases} \tag{40}$$

  In this scheme, the optimization problem is to find the optimum values of T_1 and T_2 such that the QoS is satisfied.
- **Sigmoid Function Scheme:** In this scheme, the scaling factor variations is defined according to the sigmoid function as follows:

$$\alpha_i = \frac{1}{1 + e^{-\epsilon_1 (g_{sr_i} - \epsilon_2)}}, \tag{41}$$

  where the slope of the function is determined by the parameter ϵ_1.

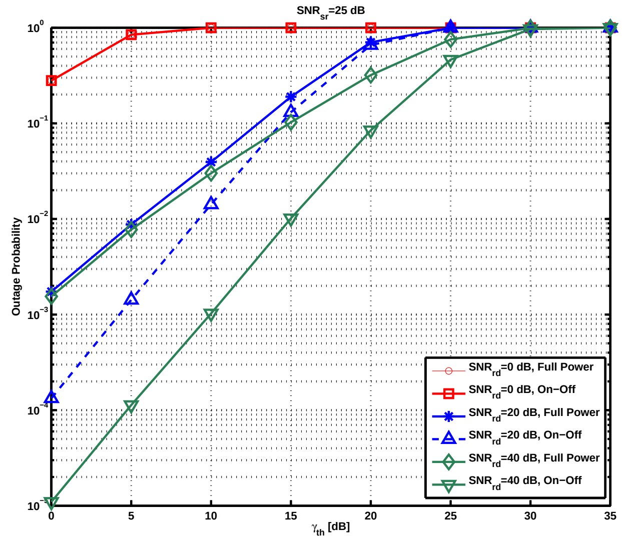

Fig. 2. Outage probability versus γ_{th} for full power and on-off power schemes.

Fig. 2 illustrates the outage probability versus γ_{th} for different values of SNR$_{rd}$, SNR$_{sr}$ $= 25$ dB, and for the full power and the on-off power schemes. The results are obtained by averaging over 10^6 channel realizations. To compare the results, we plot the outage probability of some different power schemes in Fig. 3 for high SNR$_{rd}$. We can observe that the performance of the underlying model is improved by using the on-off power scheme in comparison with the full power strategy. For example, we achieve 10 dB gain in using the on-off power strategy for SNR$_{rd}$ $= 40$ dB and P$_{out}$ $= 10^{-3}$ with respect to the full power scheme.

Fig. 4 plots the uncoded average BER versus SNR$_{rd}$ for different functions of α_i and for SNR$_{sr}$ $= 25$ dB. We observe an error flooring effect in Fig. 4 for high SNR$_{rd}$, which can be attributed to the amplified noise received at D through the relay-destination link. In fact, for sufficiently large SNR$_{rd}$, the performance is governed by SNR$_{sr}$. Thus, the BER floor can decrease, if the SNR$_{sr}$ at the relay increases.

Fig. 5-a and b illustrate the FER of the proposed model using the standard turbo code [17] with the rate of $1/3$ versus the SNR$_{rd}$ for different functions of α_i and for SNR$_{sr}$ $= 20,\ 25$ dB. In these figures, the frame length is 957 bits. Also, Fig. 5-c and d

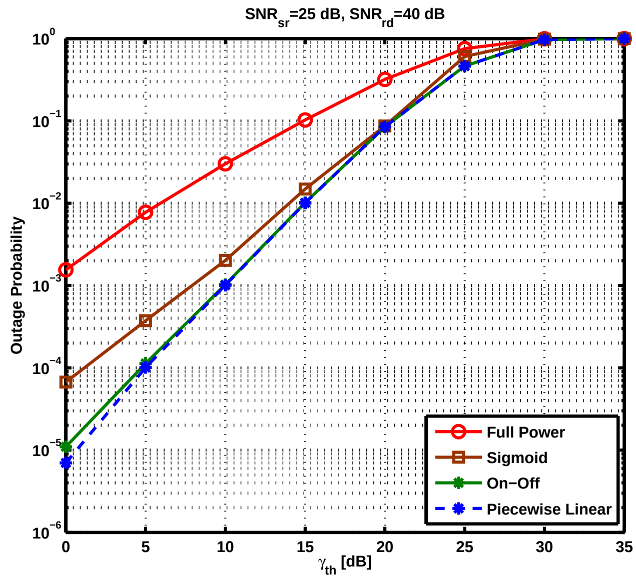

Fig. 3. Outage probability versus γ_{th} for different power schemes, SNR$_{sr}$ $= 25$ dB and for high SNR$_{rd}$.

illustrate the FER of the proposed model using the convolutional code (introduced in section $8.4.9.2.1$ of the IEEE 802.16 standard [18]) with the rate of 0.5 versus the SNR$_{rd}$. For this case, the frame length is 576 bits. Compared to the full power scheme, the simulation results show a significant improvement in the system performance with the on-off power scheme in RSs.

V. Conclusion

In this paper, a non-regenerative dual-hop wireless system based on a distributed space-time-coding strategy has been considered. It is assumed that each relay station retransmits an appropriate scaled space-time coded version of the received signals. In the high signal-to-noise ratio (SNR) regime for the relay-destination link, it has been shown that the optimum power allocation strategy in each relay that minimizes the outage probability is to remain silent, if its direct channel gain with the source is less than a prespecified threshold level. Also, under the high SNR regime for the source-relay link, the minimum outage probability is achieved when the relay stations transmit with the same powers. The simulations results show that the optimum power allocation scheme in each relay in order to minimize the

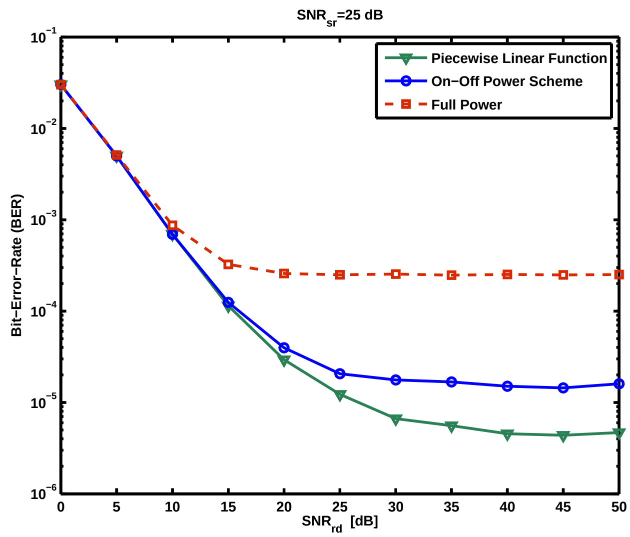

Fig. 4. Uncoded BER versus SNR$_{rd}$ for different functions of α_i and for SNR$_{sr} = 25$ dB,

outage probability or the frame-error rate is the threshold-based on-off power scheme. Also, the numerical results show a dramatical improvement in the system performance by using this scheme in comparison with the case that the relays stations forward their received signals with full power. The advantage of using this protocol is that it does not alter the Alamouti decoder at the destination and it slightly modifies the relay operation which is practically appealing. Also, through using the on-off power allocation strategy at RSs, we can save the energy in the silent mode. This results in maximizing the battery life that is crucial for battery-powered RSs.

Appendix I
Proof of Lemma 1

We shall find the probability density function of the random variable $Z = \frac{X_1}{X_2}$ using the auxiliary variable $W = X_2$. The system $z = \dfrac{x_1}{x_2}$ and $w = x_2$ has a single solution $x_1 = zw$ and $x_2 = w$. Hence, the joint pdf of (Z, W) can be obtained as

$$f_{ZW}(z, w) = \frac{f_{X_1 X_2}(x_1, x_2)}{\mathbf{J}(x_1, x_2)},$$

where $\mathbf{J}(x_1,x_2)$ is the *Jacobian determinant* of the transformation $z=\dfrac{x_1}{x_2}$ and $w=x_2$, and is defined as [19]

$$\mathbf{J}(x_1,x_2)=\begin{vmatrix}\dfrac{\partial z}{\partial x_1} & \dfrac{\partial z}{\partial x_2}\\ \\ \dfrac{\partial w}{\partial x_1} & \dfrac{\partial w}{\partial x_2}\end{vmatrix}=\frac{1}{w}.$$

Noting that $w=x_2>0$, the Jacobian determinant $\mathbf{J}(x_1,x_2)\neq 0$. To this end, the joint pdf of (Z,W) can be simplified as

$$f_{ZW}(z,w)=wf_{X_1X_2}(zw,w).$$

Since X_1 is independent of X_2, we have

$$\begin{aligned}f_{ZW}(z,w) &= wf_{X_1}(zw)f_{X_2}(w)\\ &= w\lambda_1 e^{-\lambda_1 zw}U(zw)\lambda_2 e^{-\lambda_2 w}U(w)\\ &= w\lambda_1\lambda_2 e^{-(\lambda_1 z+\lambda_2)w}U(z)U(w).\end{aligned}$$

Hence, the probability density function of the random variable $Z=\dfrac{X_1}{X_2}$ is given by

$$\begin{aligned}f_Z(z) &= \lambda_1\lambda_2\left[\int_0^\infty we^{-(\lambda_1 z+\lambda_2)w}dw\right]U(z)\\ &= \frac{\lambda_1\lambda_2}{(\lambda_1 z+\lambda_2)^2}U(z),\end{aligned}$$

and this completes the proof of the lemma.

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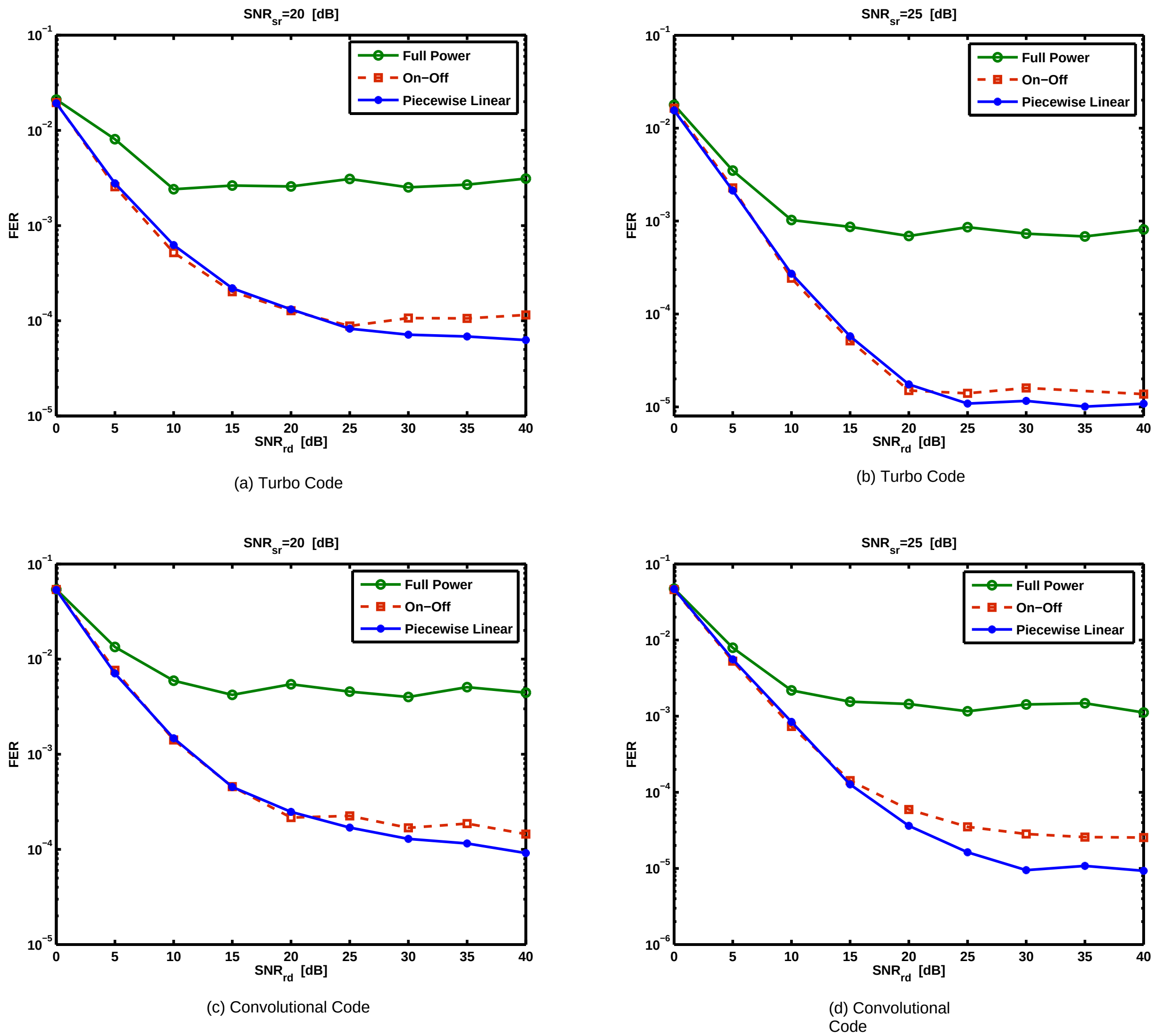

(a) Turbo Code
(b) Turbo Code
(c) Convolutional Code
(d) Convolutional Code

Fig. 5. FER versus SNR$_{rd}$ for different functions of α_i and for a) turbo code with the rate $1/3$ and SNR$_{sr} = 20$ dB, b) turbo code with the rate $1/3$ and SNR$_{sr} = 25$ dB, c) convolutional code with the rate $1/2$ and SNR$_{sr} = 20$ dB, d) convolutional code with the rate $1/2$ and SNR$_{sr} = 25$ dB.