Rate-Constrained Wireless Networks with Fading Channels: Interference-Limited and Noise-Limited Regimes

Masoud Ebrahimi and Amir Khandani

Electrical and Computer Engineering Department
University of Waterloo
Waterloo, Ontario, Canada

Technical Report UW-ECE #2007-34

October 2007

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Masoud Ebrahimi and Amir K. Khandani

Abstract

A network of n wireless communication links is considered in a Rayleigh fading environment. It is assumed that each link can be active and transmit with a constant power P or remain silent. The objective is to maximize the number of active links such that each active link can transmit with a constant rate λ. An upper bound is derived that shows the number of active links scales at most like $\frac{1}{\lambda}\log n$. To obtain a lower bound, a decentralized link activation strategy is described and analyzed. It is shown that for small values of λ, the number of supported links by this strategy meets the upper bound; however, as λ grows, this number becomes far below the upper bound. To shrink the gap between the upper bound and the achievability result, a modified link activation strategy is proposed and analyzed based on some results from random graph theory. It is shown that this modified strategy performs very close to the optimum. Specifically, this strategy is *asymptotically almost surely* optimum when λ approaches ∞ or 0. It turns out the optimality results are obtained in an interference-limited regime. It is demonstrated that, by proper selection of the algorithm parameters, the proposed scheme also allows the network to operate in a noise-limited regime in which the transmission rates can be adjusted by the transmission powers. The price for this flexibility is a decrease in the throughput scaling law by a multiplicative factor of $\log\log n$.

This work is financially supported by Nortel Networks and by matching funds from the federal government of Canada (NSERC) and province of Ontario (OCE).

The authors are affiliated with the Coding and Signal Transmission Laboratory, Electrical and Computer Engineering Department, University of Waterloo, Waterloo, ON, N2L 3G1, Canada, Tel: 519-885-1211 ext. 35324, Fax: 519-888-4338, Emails: {masoud, khandani}@cst.uwaterloo.ca.

This material was presented in part at the 10th Canadian Workshop on Information Theory (CWIT), Edmonton, AB, Canada, June 2007.

Index Terms

Wireless networks, fading channel, throughput, scaling law, random graph.

I. INTRODUCTION

Wireless networks consist of a number of nodes communicating over a shared wireless channel. The design and analysis of such configurations, even in their simplest forms, are among the most difficult problems in information theory. However, as the number of nodes becomes large, wireless networks become more tractable, where scaling laws for network parameters, such as throughput, can be derived.

Most of the works dealing with the throughput of large wireless networks consider a channel model in which the signal power decays according to a distance-based attenuation law [1]–[8]. However, in a wireless environment, the presence of obstacles and scatterers adds some randomness to the received signal. This random behavior of the channel, known as fading, can drastically change the scaling laws of a network in both multihop [9]–[12] and single-hop scenarios [13, Chapter 8], [14]–[18]. In this paper, we follow the model of [9], [13], [19], where fading is assumed to be the dominant factor affecting the strength of the channels between nodes.

In this work, we consider a single-hop network, i.e., a network in which data is transmitted directly from sources to their corresponding receivers without utilizing any other nodes as routers. Each communication link can be active and transmit with a constant power P or remain silent. Throughput and rate-per-link are the network parameters which are of concern to us. Despite the randomness of the channel, we are only interested in events that occur with high probability, i.e., with probability tending to one as $n \to \infty$. This deterministic approach to random wireless networks has been also deployed in [2], [8], [19].

In a previous work by the authors [19], the throughput maximization of a single-hop wireless network in a Rayleigh fading environment has been investigated without any rate constraints. It is shown that the maximum throughput scales like $\log n$. Also, a decentralized link activation strategy, called the threshold-based link activation strategy (TBLAS), is proposed that achieves this scaling law. The throughput maximization using TBLAS yields an average rate per active link that approaches zero as $n \to \infty$. The same phenomenon has been observed in [1], [2], [9], [11]. Since most of the existing efficient channel codes are designed for moderate rates, it is a drawback for a system to have zero-approaching rates. Thus, from a practical point of view, it is appealing to assign constant rates to active communication links. In [7], it is shown that a nondecreasing rate-per-node is achievable when nodes are mobile.

In this paper, we consider the problem of rate-constrained throughput maximization in a Rayleigh fading environment. More specifically, the objective is to maximize the number of active links such that each active link can transmit with a constant rate λ. We derive an upper bound that shows the number of active links scales at most like $\frac{1}{\lambda}\log n$. To obtain a lower bound, first, we examine the simple TBLAS of [19] and show that it is capable of guaranteeing rate-per-links equal to λ. The number of active links provided by this method scales like $\Theta(\log n)$. The scaling factor is close to the optimum when λ is small. However, as λ grows large, the scaling factor decays exponentially with λ, making it far below the upper bound $\frac{1}{\lambda}$. This inspires developing an improved link activation strategy that works well for large values of desired rates, as well. To this end, we propose a double-threshold-based link activation strategy (DTBLAS).

DTBLAS is attained by adding an interference management phase to TBLAS. This is done by choosing from good enough links only those with small enough mutual interference. The analysis of DTBLAS is more complicated than that of TBLAS. However, it can be carried out using some results from the random graph theory. It is shown that DTBLAS performs very close to the optimum. Indeed, its performance reaches the upper bound when the demanded rate approaches ∞ or 0. This shows the asymptotic optimality of DTBLAS for the rate-constrained throughput maximization problem.

In all scenarios described above, the interference power is much larger than the noise power and the rates become independent of *signal-to-noise ratio* (*SNR*). In other words, the network performs in an interference-limited regime. A natural question is whether it is possible to have rate-per-links which depend on the *SNR*. The importance of this scenario, which is called the noise-limited regime, is that the transmission rate λ can be adjusted by adjusting the transmission power P. We show that the answer to the above question is affirmative and the noise-limited regime can be realized by using DTBLAS. However, the throughput achieved by this method scales like $\frac{\log n}{\log\log n}$, which is by a multiplicative factor of $\log\log n$ less than what is achievable in an interference-limited regime.

It is worth mentioning that link activation strategies studied in this paper can be considered as special power allocation schemes. The problem of throughput maximization via power allocation is a challenging problem for which only suboptimum solutions have been reported [20]–[22]. However, variations of this problem have been extensively studied in the literature, where the on-off scheme has frequently appeared. Recently, for a decentralized utility-based network[1], it is shown that the optimum power allocation follows an on-off paradigm when the number of links is large [23]. The on-off power allocation has been also used in [16], [17] for a cellular network in which the number of cell (links) are limited, but in each cell there are infinite number of users to choose from. For cellular systems, a distributed joint power allocation and scheduling has been proposed in [24], in which again an on-off strategy is followed.

The rest of the paper is organized as follows: In Section II, network model and problem formulation are presented. An upper bound on the throughput is derived in Section III. In Sections IV and V, achievability results via decentralized and centralized schemes are presented. Some optimality results are provided in Section VI. The operation of the network in a noise-limited regime is investigated in Section VII. Finally, the paper is concluded in Section VIII.

Notation: $\mathcal{N}_n$ represents the set of natural numbers less than or equal to n; $\log(\cdot)$ is the natural logarithm function; $\lfloor x \rfloor$ denotes the largest integer less than or equal to x; $\chi^2(M)$ represents the chi-squared distribution with M degrees of freedom; $\mathrm{P}(A)$ denotes the probability of event A; $\mathrm{E}(x)$ and $\mathrm{Var}(x)$ represent the expected value and the variance of the random variable x, respectively; $\approx$ means approximate equality; for any functions $f(n)$ and $h(n)$, $h(n) = O(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| < \infty$, $h(n) = o(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = 0$, $h(n) = \omega(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = \infty$, $h(n) = \Theta(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = c$, where $0 < c < \infty$, and $h(n) \sim f(n)$ is equivalent to $\lim_{n\to\infty} h(n)/f(n) = 1$; an event A_n holds *asymptotically almost surely* (a.a.s) if $\mathrm{P}(A_n) \to 1$ as $n \to \infty$.

II. Network Model and Problem Formulation

The network model is the same as in [19]; however, we repeat it here for completeness. We consider a wireless communication network with n pairs of transmitters and receivers. These n communication links are indexed by the elements of $\mathcal{N}_n$. Each transmitter aims to send data to its corresponding receiver in a single-hop fashion. The transmit power of link i is denoted

[1]Each node maximizes a locally computed network average throughput conditioned on its own channel gain.

by p_i. It is assumed that the links follow an on-off paradigm, i.e., $p_i \in \{0, P\}$, where P is a constant. Hence, any power allocation scheme translates to a *link activation strategy (LAS)*. Any LAS yields a set of active links $\mathcal{A}$, which describes the transmission powers as

$$p_i = \begin{cases} P & \text{if} \quad i \in \mathcal{A} \\ 0 & \text{if} \quad i \notin \mathcal{A} \end{cases}. \tag{1}$$

The channel between transmitter j and receiver i is characterized by the coefficient g_{ji}. This means the received power from transmitter j at the receiver i equals $g_{ji}p_j$. We assume that the channel coefficients are *independent identically distributed* (i.i.d.) random variables drawn from an exponential pdf, i.e., $f(x) = e^{-x}$, with mean $\mu = 1$ and variance $\sigma^2 = 1$. This channel model corresponds to a Rayleigh fading environment. We refer to the coefficients g_{ii} and g_{ji} ($j \neq i$) as *direct channel coefficients* and *cross channel coefficients*, respectively.

We consider an additive white Gaussian noise (AWGN) with limited variance η at the receivers. The transmit *SNR* of the network is defined as

$$\rho = \frac{P}{\eta}. \tag{2}$$

The receivers are conventional linear receivers, i.e., without multiuser detection. Since the transmissions occur simultaneously within the same environment, the signal from each transmitter acts as interference for other links. Assuming Gaussian signal transmission from all links, the distribution of the interference will be Gaussian as well. Thus, according to the Shannon capacity formula [25], the maximum supportable rate of link $i \in \mathcal{A}$ is obtained as

$$r_i(\mathcal{A}) = \log\left(1 + \gamma_i(\mathcal{A})\right) \text{ nats/channel use}, \tag{3}$$

where

$$\gamma_i(\mathcal{A}) = \frac{g_{ii}}{1/\rho + \sum_{\substack{j \in \mathcal{A} \\ j \neq i}} g_{ji}} \tag{4}$$

is the *signal-to-interference-plus-noise ratio* (*SINR*) of link i.

As a measure of performance, in this paper we consider the throughput of the network, which is defined as

$$T(\mathcal{A}) = \sum_{i \in \mathcal{A}} r_i(\mathcal{A}). \tag{5}$$

Also, the *average rate per active link* is defined as

$$\bar{r}(\mathcal{A}) = \frac{T(\mathcal{A})}{|\mathcal{A}|}. \tag{6}$$

In this paper, wherever there is no ambiguity, we drop the functionality of $\mathcal{A}$ from the network parameters and simply refer to them as r_i, γ_i, T, or $\bar{r}$.

Throughout the paper, we assume all active links transmit with a same constant rate λ. In this case, the throughput becomes proportional to the number of active links, i.e., $T(\mathcal{A}) = |\mathcal{A}|\lambda$. Hence, the problem of throughput maximization becomes equivalent to maximizing the number of active links subject to a constraint on the rate of active links, i.e.,

$$\begin{array}{cc} \max\limits_{\mathcal{A}\subseteq\mathcal{N}_n} & |\mathcal{A}| \\ \text{s.t.} & r_i(\mathcal{A}) \geq \lambda, \quad \forall i \in \mathcal{A} \end{array}. \tag{7}$$

This problem is referred to as the *rate-constrained throughput maximization*. We denote the throughput corresponding to the maximum value of this problem by T_c^*.

Due to the nonconvex and integral nature of the throughput maximization problem, its solution is computationally intensive. However, in this paper we propose and analyze LASs which lead to efficient solutions for the above problem. Indeed, we first show that the decentralized method of [19] is a.a.s. optimum when λ is vanishingly small. Then, we propose a new LAS which is asymptotically optimum for large values as well as small values of λ. Also, for moderate values of λ, there is a small gap between the performance of the proposed LAS and a derived upper bound. This shows the closeness of its performance to the optimum.

For simplicity of notation, we denote the number of active links by k instead of $|\mathcal{A}|$. Motivated by the result of [19] that shows the maximum throughput scales like $\log n$, we introduce the following definitions. The scaling factor of the throughput is defined as

$$\tau = \lim_{n\to\infty} \frac{T}{\log n}, \tag{8}$$

Similarly, the scaling factor of the number of active links is defined as

$$\kappa = \lim_{n\to\infty} \frac{k}{\log n}. \tag{9}$$

III. Upper Bound

In this section, we obtain an upper bound on the optimum solution of (7). This upper bound can be either presented as an upper bound on the throughput or as an upper bound on the number of active links.

Theorem 1: Assume $\mathcal{A}_c^*$ is the solution to the rate-constrained throughput maximization (7) and $k_c^* = |\mathcal{A}_c^*|$. Then, the associated throughput and the scaling factor of k_c^* a.a.s. satisfy

$$T_c^* < \log n - \log\log n + c, \tag{10}$$

$$\kappa_c^* < \frac{1}{\lambda}, \tag{11}$$

for some constant c.

Proof: For a randomly selected set of active links $\mathcal{A}$ with $|\mathcal{A}| = k$, the interference term $I_i = \sum_{\substack{j\in\mathcal{A}\\ j\neq i}} g_{ji}$ in the denominator of (4) has $\chi^2(2k-2)$ distribution. Hence, we have

$$\begin{aligned}
\mathrm{P}(\gamma_i > x) &= \int_0^\infty \mathrm{P}\left(\gamma_i > x | I_i = z\right) f_{I_i}(z) dz \\
&= \int_0^\infty e^{-x(1/\rho+z)} \frac{z^{k-2}e^{-z}}{(k-2)!} dz \\
&= \frac{e^{-x/\rho}}{(1+x)^{k-1}}.
\end{aligned} \tag{12}$$

Assume $\mathcal{L}_1$ is the event that there exists at least one set $\mathcal{A} \subseteq \mathcal{N}_n$ with $|\mathcal{A}| = k$ such that the constraints in (7) are satisfied. Also, assume γ_0 is a quantity that satisfies $\lambda = \log(1+\gamma_0)$. We have

$$\mathrm{P}(\mathcal{L}_1) \overset{(a)}{\leq} \binom{n}{k} \left(\mathrm{P}(r_i \geq \lambda)\right)^k \tag{13}$$

$$= \binom{n}{k} \left(\mathrm{P}(\gamma_i \geq \gamma_0)\right)^k \tag{14}$$

$$\overset{(12)}{=} \binom{n}{k} \frac{e^{-\gamma_0 k/\rho}}{(1+\gamma_0)^{k(k-1)}} \tag{15}$$

$$\overset{(b)}{\leq} \left(\frac{ne}{k}\right)^k \frac{e^{-\gamma_0 k/\rho}}{(1+\gamma_0)^{k(k-1)}} \tag{16}$$

$$= e^{k(\log n - \log k - \lambda k + \lambda + 1 - \gamma_0/\rho)}, \tag{17}$$

where (a) is due to the union bound and (b) is the result of applying the Stirling's approximation for the factorial. It can be verified that there exists a constant c such that if $k\lambda = \log n - \log\log n + c$, then, the above upper bound approaches zero for $n \to \infty$. Hence, for the event $\mathcal{L}_1$ to have non-zero probability, we should a.a.s. have

$$k\lambda < \log n - \log\log n + c. \tag{18}$$

This inequality holds for any feasible number of active links. By choosing $k = k_c^*$, the upper bounds in the lemma are proved. ■

IV. Lower Bound: A Decentralized Approach

To derive a lower bound, in this section, we consider the threshold-based link activation strategy (TBLAS) originally proposed in [19].

TBLAS: For a threshold Δ, choose the set of active links according to the following rule

$$i \in \mathcal{A} \quad \text{iff} \quad g_{ii} > \Delta. \tag{19}$$

As it is seen, in TBLAS each link only needs to know its own direct channel gain. If a direct channel gain is above the threshold Δ, the corresponding link is active; otherwise, it remains silent. The value of Δ determines the achievable throughput. We show that by proper choose of the threshold, TBLAS provides a solution for the rate-constrained throughput maximization. The importance of TBLAS is that it can be implemented in a decentralized fashion.

Let us denote the achieved throughput of TBLAS by $T_{_{TBLAS}}$. The following results are proven for TBLAS in [19]:

$$T_{_{TBLAS}} \sim ne^{-\Delta} \log\left(1 + \frac{\Delta}{ne^{-\Delta}}\right), \tag{20}$$

$$k_{_{TBLAS}} \sim ne^{-\Delta}, \tag{21}$$

$$|k_{_{TBLAS}} - ne^{-\Delta}| < \xi\sqrt{ne^{-\Delta}}, \quad a.a.s. \tag{22}$$

where the last inequality holds for any $\xi = \omega(1)$.

A necessary condition for the rate of active links being equal to λ is $\bar{r}_{_{TBLAS}} = \lambda$, where $\bar{r}_{_{TBLAS}}$ is the average rate per active link achieved by TBLAS. Hence, we should choose Δ such that the throughput and the number of active links both become proportional to $\log n$. The following lemma shows how to realize such a scenario.

Lemma 2: Assume the activation threshold for TBLAS is chosen to be $\Delta = \log n - \log\log n - \log\alpha$ for some $\alpha > 0$. Then, a.a.s. we have

$$\tau_{_{TBLAS}} = \alpha \log\left(1 + \frac{1}{\alpha}\right) \tag{23}$$

$$\kappa_{_{TBLAS}} = \alpha \tag{24}$$

$$\bar{r}_{_{TBLAS}} = \log\left(1 + \frac{1}{\alpha}\right) + o(1). \tag{25}$$

Proof: With the specified value of Δ, we have $ne^{-\Delta} = \alpha \log n$. The values of $\tau_{_{TBLAS}}$ and $\kappa_{_{TBLAS}}$ are readily obtained by substituting this value in (20) and (21) and using the definitions (8) and (9), respectively. The value of $\bar{r}_{_{TBLAS}}$ is obtained by using the definition (6). ∎

Lemma 2 indicates that by a proper choose of α, an average rate per active link equal to λ is achievable; however, it does not guarantee that all active links can support this rate. In other words, one may ask whether TBLAS is capable of satisfying the constraints in problem (7). The following lemma addresses this issue and shows that a.a.s. the rate of all active links are highly concentrated around the average rate per active link.

Lemma 3: Assume the activation threshold for TBLAS is chosen to be $\Delta = \log n - \log\log n - \log\alpha$ for some $\alpha > 0$. Then, a.a.s. we have

$$|r_i - \bar{r}| < 2\sqrt{\frac{\log\log n}{\alpha^3 \log n}}(1 + o(1)), \quad \forall i \in \mathcal{A}, \tag{26}$$

where $\bar{r} = \log\left(1 + \frac{1}{\alpha}\right)$.

To prove the lemma, we need the following result about the *central limit theorem* (*CLT*) for large deviations.

Theorem 4 ([26]): Let $\{Y_m\}$ be a sequence of i.i.d. random variables. Suppose that Y_1 has zero mean and finite positive variance ν and satisfies Cramér's condition[2]. For $Z_m = \frac{1}{\sqrt{m\nu}} \sum_{j=1}^{m} Y_j$, define $F_m(y) = \mathrm{P}(Z_m < y)$. If $y \geq 0$, $y = O(m^{1/6})$, then

$$1 - F_m(y) = [1 - \Phi(y)] \exp\left(\frac{\theta_3 y^3}{6\sqrt{m\nu^3}}\right) + O\left(\frac{e^{-y^2/2}}{\sqrt{m}}\right), \tag{27}$$

where $\Phi(y)$ is the cdf of normal distribution and $\theta_3 = \mathrm{E}(Y_1^3)$.

Proof of Lemma 3: From the definition of r_i and the concavity of the $\log(\cdot)$ function, we have

$$|r_i - \bar{r}| = \left|\log(1+\gamma_i) - \log\left(1 + \frac{1}{\alpha}\right)\right| \tag{28}$$

$$\leq \left|\gamma_i - \frac{1}{\alpha}\right|. \tag{29}$$

Thus, to prove the lemma, it is sufficient to prove that a.a.s.

$$\left|\gamma_i - \frac{1}{\alpha}\right| < 2\sqrt{\frac{\log\log n}{\alpha^3 \log n}}(1 + o(1)), \quad \forall i \in \mathcal{A}, \tag{30}$$

or equivalently

$$x_- < \gamma_i < x_+, \tag{31}$$

[2]A random variable Y satisfies the Cramér's condition if its moment-generating function exists in some interval with the center at the origin.

where

$$x_{\pm} = \frac{1}{\alpha}\left(1 \pm 2\sqrt{\frac{\log\log n}{\alpha \log n}}(1+o(1))\right). \tag{32}$$

Here, we just prove the left-side inequality in (31). The other side can be proved in a similar manner.

Let $\mathcal{L}_2$ denote the event that

$$\gamma_i > x_-, \quad \forall i \in \mathcal{A}. \tag{33}$$

In the following, we show that $\mathrm{P}(\mathcal{L}_2) \to 1$ as $n \to \infty$.

Denoting the cdf of γ_i conditioned on $|\mathcal{A}| = k$ by $F_\gamma(x,k)$, the probability of the event $\mathcal{L}_2$ is obtained as

$$\begin{aligned}
\mathrm{P}(\mathcal{L}_2) &= \sum_{k=0}^{n} \mathrm{P}(|\mathcal{A}| = k)\mathrm{P}(\mathcal{L}_2 || \mathcal{A}| = k) &(34)\\
&\stackrel{(a)}{=} \sum_{k=0}^{n} \mathrm{P}(|\mathcal{A}| = k)\left(1 - F_\gamma(x_-,k)\right)^k &(35)\\
&\stackrel{(b)}{\geq} \sum_{k=k_-}^{k_+} \mathrm{P}(|\mathcal{A}| = k)\left(1 - F_\gamma(x_-,k)\right)^k &(36)\\
&\stackrel{(c)}{>} \left(1 - F_\gamma(x_-,k_+)\right)^{k_+} \sum_{k=k_-}^{k_+} \mathrm{P}(|\mathcal{A}| = k) &(37)\\
&= \left(1 - F_\gamma(x_-,k_+)\right)^{k_+} \mathrm{P}(k_- \leq |\mathcal{A}| \leq k_+), &(38)
\end{aligned}$$

where (a) is because the channel gains are independent, (b) is valid for any $0 \leq k_- \leq k_+ \leq n$ and (c) is due to the fact that $\left(1 - F_\gamma(x,k)\right)^k$ is a decreasing function of k. According to (22), by choosing

$$\begin{aligned}
k_{\pm} &= ne^{-\Delta} \pm \xi\sqrt{ne^{-\Delta}} &(39)\\
&= \alpha \log n \pm \xi\sqrt{\alpha \log n}, &(40)
\end{aligned}$$

for some $\xi \to \infty$, we have $\mathrm{P}(k_- \leq |\mathcal{A}| \leq k_+) \to 1$. Hence, to prove $\mathrm{P}(\mathcal{L}_2) \to 1$, it is enough to show that $\left(1 - F_\gamma(x_-,k_+)\right)^{k_+} \to 1$. However, due to the inequality

$$\left(1 - F_\gamma(x_-,k_+)\right)^{k_+} \geq 1 - k_+ F_\gamma(x_-,k_+), \tag{41}$$

it is enough to show that

$$k_+ F_\gamma(x_-,k_+) \to 0. \tag{42}$$

To prove (42), we provide an upper bound on $k_+ F_\gamma(x_-, k_+)$ and show that it approaches zero as $n \to \infty$. We have

$$
\begin{aligned}
F_\gamma(x_-, k_+) &= \mathrm{P}(\gamma_i \le x_- \| \mathcal{A}| = k_+) \\
&\overset{(a)}{=} \mathrm{P}\left(\frac{g_{ii}}{1/\rho + \sum_{\substack{j=1 \\ j\neq i}}^{k_+} g_{ji}} \le x_- \right) \\
&= \mathrm{P}\left(\sum_{\substack{j=1 \\ j\neq i}}^{k_+} g_{ji} \ge \frac{g_{ii}}{x_-} - \frac{1}{\rho} \right) \\
&\overset{(b)}{<} \mathrm{P}\left(\sum_{\substack{j=1 \\ j\neq i}}^{k_+} g_{ji} \ge \frac{\Delta}{x_-} - \frac{1}{\rho} \right),
\end{aligned} \tag{43}
$$

where (a) is based on $\mathcal{A} = \{1, \cdots, k_+\}$, which has been assumed for simplicity of notation, and (b) is due to the fact that, in TBLAS, $g_{ii} > \Delta$ for any $i \in \mathcal{A}$. Let us define $Y_j = g_{ji} - 1$, which has the variance $\nu = 1$. Thus, the right-hand-side (RHS) of (43) translates to the complementary cdf of $Z = \frac{1}{\sqrt{k_+ - 1}} \sum_{\substack{j=1 \\ j\neq i}}^{k_+} Y_j$, i.e. (43) can be rewritten as

$$
F_\gamma(x_-, k_+) \;<\; 1 - \mathrm{P}(Z \le y), \tag{44}
$$

where

$$
y \;=\; \frac{\frac{\Delta}{x_-} - \frac{1}{\rho} - (k_+ - 1)}{\sqrt{k_+ - 1}}. \tag{45}
$$

By substituting $\Delta = \log n - \log\log n - \log\alpha$ and the value of x_- from (32) into (45), we obtain

$$
y = 2\sqrt{\log\log n}(1 + o(1)). \tag{46}
$$

Since Y_j is a shifted exponential random variable, its moment-generating function exists around zero and the Cramér's condition is satisfied. Also, by choosing $m = k_+ - 1$ we have $y = O(m^{1/6})$. Hence, the result of Theorem 4 can be applied to calculate the complementary cdf of Z. Consequently, by using (27) with $\theta_3 = \mathrm{E}(Y_j^3) = 2$, (44) can be rewritten as

$$
F_\gamma(x_-, k_+) < [1 - \Phi(y)] \exp\left(\frac{y^3}{3\sqrt{k_+ - 1}} \right) + O\left(\frac{e^{-y^2/2}}{\sqrt{k_+ - 1}} \right). \tag{47}
$$

Noting that $y^3 = o(\sqrt{k_+})$ and using the inequality $1 - \Phi(y) < \frac{e^{-y^2/2}}{y}$, from (47) and (46), we conclude that

$$
\begin{aligned}
k_+ F_\gamma(x_-, k_+) &< k_+ \frac{e^{-y^2/2}}{y} & (48)\\
&= \exp\left(-\log\log n(1+o(1))\right). & (49)
\end{aligned}
$$

It is clear that the above upper bound approaches zero as $n \to \infty$. Hence, $\mathrm{P}(\mathcal{L}_2) \to 1$ and the proof is complete. ■

Lemma 3 shows that with the specified threshold for TBLAS, all active links can transmit with rate $\lambda = \log(1+\frac{1}{\alpha})$. Hence, TBLAS provides a solution, albeit suboptimum, for the problem (7). Lemmas 2 and 3 reveal the following relation between the demanded rate λ and $\kappa_{_{TBLAS}}$ as well as $\tau_{_{TBLAS}}$

$$
\kappa_{_{TBLAS}} = \frac{1}{e^\lambda - 1}, \tag{50}
$$

$$
\tau_{_{TBLAS}} = \frac{\lambda}{e^\lambda - 1}. \tag{51}
$$

Noting that for small values of λ, the RHS of (50) can be approximated as $\frac{1}{\lambda}$ and using the upper bound in Theorem 1, it turns out that TBLAS is close to the optimum for small values of λ.

V. Lower Bound: A Centralized Approach

Although TBLAS enjoys the simplicity of decentralized implementation, its performance is far from the optimum. This can be seen by comparing the upper bound in Theorem 1 and the achievability result in (50). A reason for this suboptimality is that the mutual interference of the active links is not considered in choosing $\mathcal{A}$. In this section, we provide an LAS that performs close to the upper bound in Theorem 1 and turns out to be asymptotically optimum when λ is very large or very small. We name this method double-threshold-based LAS (DTBLAS).

DTBLAS: For the thresholds Δ and δ

i. Choose the largest set $\mathcal{A}_1 \subseteq \mathcal{N}_n$ such that $g_{ii} > \Delta$ for all $i \in \mathcal{A}_1$.
ii. Choose the largest set $\mathcal{A}_2 \subseteq \mathcal{A}_1$ such that $g_{ij} \leq \delta$ and $g_{ji} \leq \delta$ for all $i, j \in \mathcal{A}_2$.

The set of active links is $\mathcal{A} = \mathcal{A}_2$.

This strategy chooses the links to be active in a two-phase selection process; in the first phase, which is basically similar to TBLAS, a subset $\mathcal{A}_1$ of the links with good enough direct channel coefficients is chosen. In the second phase, which is the interference management phase, a subset of links in $\mathcal{A}_1$ is chosen such that their mutual interferences are small enough. Note that the second phase of the strategy requires full knowledge of the channel coefficients. Hence, this scheme should be implemented in a centralized fashion.

We aim to find Δ and δ such that the throughput is maximized subject to the rate constraints of the active links.

For simplicity, we use the notation $k_i = |\mathcal{A}_i|$ for $i = 1,\, 2$. Without loss of generality, assume $\mathcal{A}_i = \{1,\, \cdots,\, k_i\}$. By using (3), (4), and (5), and applying the Jensen's inequality, the throughput is lower bounded as

$$T \geq k_2 \log \left(1 + \frac{\Delta}{1/\rho + \frac{1}{k_2}\sum_{i=1}^{k_2} I_i} \right), \tag{52}$$

where $I_i = \sum_{\substack{j=1 \\ j\neq i}}^{k_2} g_{ji}$. Since $g_{ji} \leq \delta$, the mean and variance of I_i depend on δ. More precisely, we have

$$\begin{aligned} \mathrm{E}(I_i) &= (k_2 - 1)\hat{\mu}, & (53) \\ \mathrm{Var}(I_i) &= (k_2 - 1)\hat{\sigma}^2, & (54) \end{aligned}$$

where

$$\begin{aligned} \hat{\mu} &= \mathrm{E}\left\{ g_{ji} | g_{ji} \leq \delta \right\} = 1 - \frac{\delta e^{-\delta}}{1 - e^{-\delta}}, & (55) \\ \hat{\sigma}^2 &= \mathrm{Var}\left\{ g_{ji} | g_{ji} \leq \delta \right\} = 1 - \frac{\delta^2 e^{-\delta}}{(1 - e^{-\delta})^2}. & (56) \end{aligned}$$

Assume δ is a constant and $k_2 \to \infty$ as $n \to \infty$. To simplify the RHS of (52), we apply the Chebyshev inequality to obtain the upper bound

$$\frac{1}{k_2} \sum_{i=1}^{k_2} I_i < (k_2 - 1)\hat{\mu} + \psi, \tag{57}$$

which holds a.a.s. for any $\psi = \omega(1)$. Consequently, the lower bound (52) becomes

$$T \geq k_2 \log \left(1 + \frac{\Delta}{\hat{\mu} k_2 + \psi} \right) \qquad a.a.s. \tag{58}$$

Note that the constant $1/\rho - \hat{\mu}$ is absorbed in the function ψ. Since ψ can be chosen arbitrarily small, say with an order smaller than $\hat{\mu} k_2$, we can rewrite (58) as

$$T \geq T_{_{DTBLAS}}, \tag{59}$$

where

$$T_{_{DTBLAS}} = k_2 \left(\log\left(1 + \frac{\Delta}{\hat{\mu} k_2}\right) + o(1) \right) \qquad a.a.s. \tag{60}$$

denotes the throughput achievable by DTBLAS.

Since k_2 is a random variable, the right hand side of (60) is a random variable as well. However, the following discussion shows that k_2 is highly concentrated around a certain value. Hence, it can be treated as a deterministic value.

Construct an undirected graph $G(\mathcal{A}_1,\, \boldsymbol{E})$ with vertex set $\mathcal{A}_1$ and the adjacency matrix $\boldsymbol{E} = [e_{ij}]$ defined as

$$e_{ij} = \begin{cases} 1 & ; \quad g_{ij} \leq \delta \text{ and } g_{ji} \leq \delta \\ 0 & ; \quad \text{otherwise} \end{cases} .$$

The probability of having an edge between vertices i and j, when g_{ji} and g_{ij} have exponential distribution, equals

$$p = \left(1 - e^{-\delta}\right)^2 . \tag{61}$$

The definition of G implies that $G \in \mathcal{G}(k_1, p)$, where $\mathcal{G}(k_1, p)$, which is a well-studied object in the literature [27], is the family of k_1-vertex *random graphs* with edge probability p.

In the second phase of DTBLAS, we are interested to choose the maximum number of links whose cross channel coefficients are smaller than δ. This is equivalent to choosing the largest complete subgraph[3] of G. The size of the largest complete subgraph of G is called its clique number and denoted by $\mathrm{cl}(G)$. The above discussion yields

$$k_2 = \mathrm{cl}(G), \quad \text{for some} \quad G \in \mathcal{G}(k_1, p). \tag{62}$$

Although the clique number of a random graph G is a random variable, the following result from random graph theory states that it is concentrated in a certain interval.

Theorem 5: Let $0 < p < 1$ and $\epsilon > 0$ be fixed. The clique number $\mathrm{cl}(G)$ of $G \in \mathcal{G}(m, p)$, for large values of m, a.a.s. satisfies $s_1 \leq \mathrm{cl}(G) \leq s_2$ where

$$s_i = \lfloor 2\log_b m - 2\log_b \log_b m(1-p) + 2\log_b(e/2) + 1 + (-1)^i \epsilon/p \rfloor, \quad i = 1,\, 2, \tag{63}$$

$b = 1/p$.

[3]A complete graph is a graph in which every pair of vertices are connected by an edge.

Proof: The theorem is a direct result of Theorem 7.1 in [28], which states a similar result for the stability number of random graphs. Using the fact that the stability number of a random graph $\mathcal{G}(m, p)$ is the same as the clique number of a random graph $\mathcal{G}(m, 1-p)$, the theorem is proved. ■

Corollary 6: Consider DTBLAS with parameters Δ and δ. The number of active links, $k_{_{DTBLAS}} = k_2$, a.a.s. satisfies $k'_- \leq k_{_{DTBLAS}} \leq k'_+$, where

$$k'_\pm = \lfloor 2\log_b ne^{-\Delta} - 2\log_b \log_b ne^{-\Delta}(1-\frac{1}{b}) + 2\log_b(e/2) + 1 \pm \epsilon/p + o(1)\rfloor \tag{64}$$

and $b = (1-e^{-\delta})^{-2}$.

Proof: According to (22), a.a.s. we have $k_1 = ne^{-\Delta} + O(\xi\sqrt{ne^{-\Delta}})$. Assuming $\xi = o(\sqrt{ne^{-\Delta}})$, and by substituting this value of k_1 into (62) and using Theorem 5, the corollary is proved. ■

The next lemma indicates how to choose the thresholds Δ and δ such that the throughput and the number of active links both become proportional to $\log n$. As a result, a constant average rate per active link is achieved.

Lemma 7: Assume the threshold Δ for DTBLAS is chosen to be

$$\Delta = (1-\alpha')\log n(1+o(1)), \tag{65}$$

for some $\alpha' > 0$ and δ is a constant. Then, a.a.s. we have

$$\kappa_{_{DTBLAS}} = \frac{-\alpha'}{\log\left(1-e^{-\delta}\right)}, \tag{66}$$

$$\tau_{_{DTBLAS}} = \frac{-\alpha'}{\log\left(1-e^{-\delta}\right)} \log\left(1 - \frac{(1-\alpha')\log\left(1-e^{-\delta}\right)}{\alpha'\left(1-\dfrac{\delta e^{-\delta}}{1-e^{-\delta}}\right)}\right), \tag{67}$$

$$\bar{r}_{_{DTBLAS}} = \log\left(1 - \frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}\right) + o(1). \tag{68}$$

Proof: For the number of active links, we have

$$k_{_{DTBLAS}} \overset{(a)}{\sim} 2\log_b ne^{-\Delta} \tag{69}$$

$$\overset{(b)}{=} \frac{-\alpha'(1+o(1))}{\log\left(1-e^{-\delta}\right)}\log n, \tag{70}$$

where (a) is based on Corollary 6 and (b) is obtained by using (65). From (70), and by using the definition (9), $\kappa_{_{DTBLAS}}$ is obtained as given in (66).

The number of active links in (70) can be used along with the value of Δ in (65) to rewrite (60) as

$$T_{DTBLAS} = \left[\frac{-\alpha'}{\log\left(1-e^{-\delta}\right)} \log\left(1 - \frac{(1-\alpha')\log\left(1-e^{-\delta}\right)}{\alpha'\hat{\mu}}\right) + o(1)\right] \log n. \tag{71}$$

The scaling factor τ_{DTBLAS}, as given in the Lemma, is obtained by using the value of $\hat{\mu}$ from (55) and applying the definition (8). The value of $\bar{r}_{DTBLAS}$ is obtained by using the definition (6). This completes the proof. ■

According to this lemma, by proper choose of the constants α' and δ, the average rate per active link $\bar{r}_{DTBLAS}$ can be adjusted to be equal to the required rate λ. A natural question is whether, under the specified conditions in DTBLAS, all active links can support the rate λ. The following lemma addresses this issue and shows that a.a.s. the rate of all active links are highly concentrated around the average value $\bar{r}_{DTBLAS}$.

Lemma 8: Consider DTBLAS with thresholds δ and $\Delta = (1-\alpha')\log n$ for some $\alpha' > 0$. Then, a.a.s. we have

$$|r_i - \bar{r}| < c\sqrt{\frac{\log\log n}{\log n}}(1+o(1)), \quad \forall i \in \mathcal{A}, \tag{72}$$

for some constant $c > 0$, where

$$\bar{r} = \log\left(1 - \frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}\right).$$

Proof: See Appendix A. ■

According to Lemmas 7 and 8, when maximizing the throughput of DTBLAS, δ should be a constant and Δ is obtained from another constant α'. Hence, the rate-constrained throughput maximization simplifies to an optimization problem with constant parameters α' and δ. Assume γ_0 is a quantity that satisfies $\lambda = \log(1+\gamma_0)$, i.e., γ_0 is the required *SINR* by the active links. Instead of the number of active links, we can maximize the scaling factor of the number of active links given in Lemma 7. Hence, the rate-constrained throughput maximization problem (7) is converted for DTBLAS to the following optimization problem

$$\max_{\alpha',\delta} \quad \frac{-\alpha'}{\log\left(1-e^{-\delta}\right)} \tag{73}$$

$$\text{s.t.} \quad -\frac{(1-\alpha')\log\left(1-e^{-\delta}\right)}{\alpha'\left(1 - \frac{\delta e^{-\delta}}{1-e^{-\delta}}\right)} = \gamma_0. \tag{74}$$

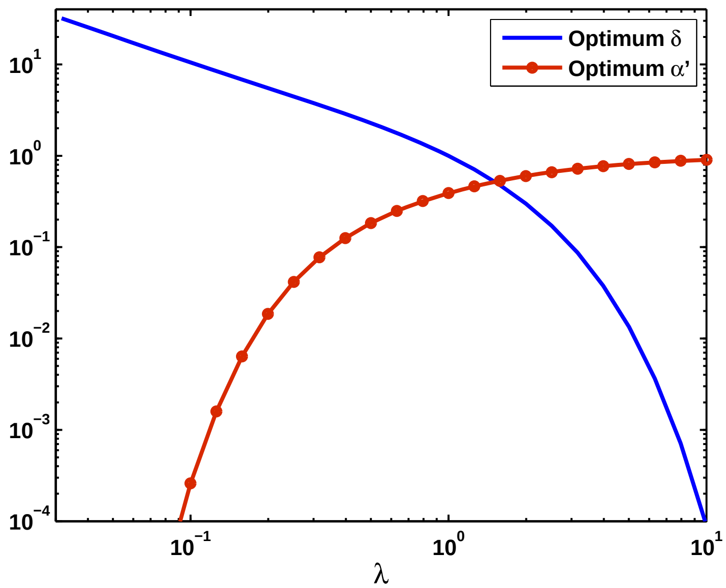

Fig. 1. Optimum of the threshold δ and the parameter α' vs. the demanded rate λ.

Note that in contrast to problem (7), there is only one constraint in this problem. However, according to Lemma 8, this single constraint guarantees the required rate for all active links. From the equality constraint (74), parameter α' can be found in terms of δ as

$$\alpha' = \frac{-\log\left(1-e^{-\delta}\right)}{\gamma_0\left(1-\frac{\delta e^{-\delta}}{1-e^{-\delta}}\right)-\log\left(1-e^{-\delta}\right)}. \tag{75}$$

By substituting this value in the objective function (73), we obtain the following equivalent unconstrained optimization problem

$$\min_{\delta} \gamma_0\left(1-\frac{\delta e^{-\delta}}{1-e^{-\delta}}\right)-\log\left(1-e^{-\delta}\right). \tag{76}$$

Consequently, (α'^*, δ^*), the solution of (73), can be obtained by first finding δ^* from (76) and then substituting it into (75) to obtain α'^*. Due to the complicated form of (76), it is not possible to find δ^* analytically and it should be found numerically.

Fig. 1 shows δ^* and α'^* versus λ. The values of δ^* and α'^* can be replaced in (67) and (66) to obtain the maximum throughput scaling factor (τ^*_{DTBLAS}) as well as the maximum scaling factor for the number of active links (κ^*_{DTBLAS}). The value τ^*_{DTBLAS} is shown in Fig. 2. Depicted in the figure is also the throughput scaling factor of TBLAS obtained from (51). As it is observed, for

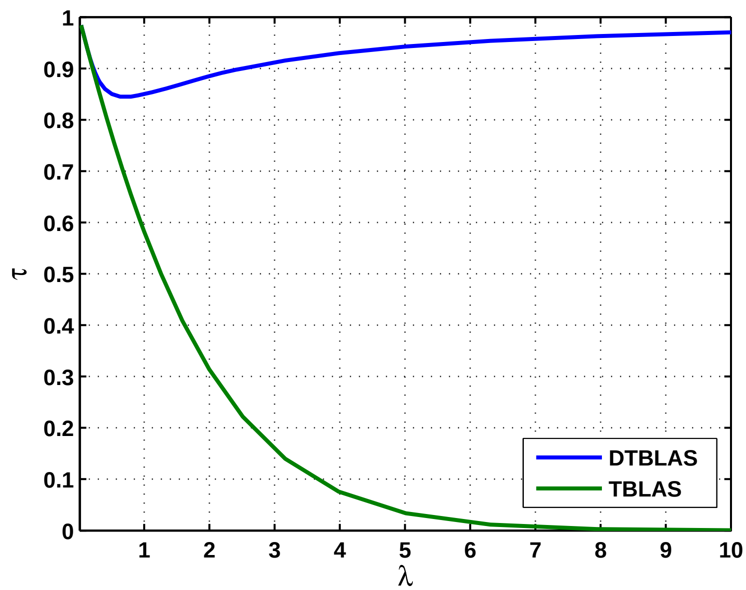

Fig. 2. Maximum throughput scaling factor vs. the demanded rate λ.

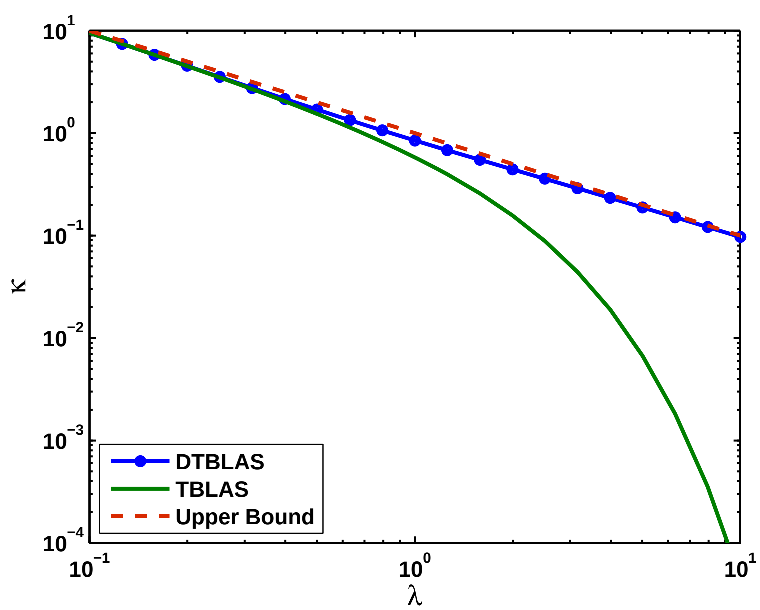

Fig. 3. Tradeoff between rate-per-link and the number of active links.

small values of λ, the performance of TBLAS and DTBLAS are almost the same. However, as λ grows larger, the scaling factor of TBLAS approaches zero, but the scaling factor of DTBLAS approaches 1. This shows some kind of optimality for DTBLAS which will be later proven formally. Figure 3 demonstrates the tradeoff between the number of supported links and the demanded rate-per-link for TBLAS and DTBLAS. The tradeoff curve for TBLAS is obtained from (50). The upper bound from Theorem 1 is also plotted for comparison. As observed, for a ceratin value of λ, DTBLAS can support larger number of users, especially for larger values of λ. Indeed, the tradeoff curve of DTBLAS is very close to the upper bound. Specifically, for large values of λ, these two curves coincide. This will be later proven formally.

VI. Optimality Results

Although the behaviour of DTBLAS is numerically described in Figs. 1, 2, and 3, it is possible and also insightful to obtain closed form expressions for δ^* and α'^* as well as κ^*_{DTBLAS} and τ^*_{DTBLAS} when λ is very small or very large. An important result of these extreme-case analyses is the asymptotic optimality of DTBLAS.

Setting the derivative of the objective function (76) equal to zero reveals that, at the optimum point, δ, satisfies

$$e^{\lambda}(1 - e^{-\delta} - \delta) + \delta = 0. \tag{77}$$

Two extreme cases of large λ and small λ are discussed separately in the following.

a) Large λ: In this case, solving (77) yields

$$\delta^* = 2e^{-\lambda} + O\left(e^{-2\lambda}\right). \tag{78}$$

Consequently, α'^*, τ^*, and κ^*_{DTBLAS} are obtained as

$$\alpha'^* = 1 - \frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right) \tag{79}$$

$$\tau^*_{DTBLAS} = 1 - \frac{\log(e/2)}{\lambda} + O\left(\frac{1}{\lambda^2}\right) \tag{80}$$

$$\kappa^*_{DTBLAS} = \frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right). \tag{81}$$

As it is seen from the above equations, for large values of λ, δ^* becomes very small and α'^* approaches one. This means, when large rate-per-links are demanded, it is more crucial to manage the interference than to choose links with high direct gain.

b) Small λ: In this case, solving (77) yields

$$\delta^* = \frac{1}{\lambda} + \frac{1}{2} + O(\lambda). \tag{82}$$

Consequently, α'^*, τ^*_{DTBLAS}, and κ^*_{DTBLAS} are obtained as

$$\alpha'^* = e^{-\frac{1}{\lambda}-\frac{1}{2}}\left(\frac{1}{\lambda} + \frac{1}{2} + O(\lambda)\right) \tag{83}$$

$$\tau^*_{DTBLAS} = 1 - \frac{\lambda}{2} + O\left(\lambda^2\right) \tag{84}$$

$$\kappa^*_{DTBLAS} = \frac{1}{\lambda} - \frac{1}{2} + O\left(\lambda\right). \tag{85}$$

The above equations show that for small values of λ, δ^* is very large and α'^* is very small. In other words, DTBLAS is converted to its special case, TBLAS.

The above discussion yields the following optimality result on DTBLAS.

Theorem 9: Consider the rate-constrained throughput maximization problem (7). Assume τ^*_c and κ^*_c are the maximum achievable scaling factors of the throughput and the number of supported links, respectively. Also, assume τ^*_{DTBLAS} and κ^*_{DTBLAS} are the maximum scaling factor of the throughput and the number of active links when DTBLAS is deployed. Then, a.a.s. we have

$$\lim_{\lambda\to\infty} \left(\tau^*_{DTBLAS} - \tau^*_c\right) = 0, \tag{86}$$

$$\lim_{\lambda\to\infty} \left(\kappa^*_{DTBLAS} - \kappa^*_c\right) = 0, \tag{87}$$

$$\lim_{\lambda\to 0} \left(\tau^*_{DTBLAS} - \tau^*_c\right) = 0, \tag{88}$$

$$\lim_{\lambda\to 0} \frac{\kappa^*_{DTBLAS}}{\kappa^*_c} = 1. \tag{89}$$

Proof: The proof of the theorem is straightforward by using the upper bounds provided in Theorem 1 and the asymptotic achievability results provided in this section. ■

VII. Noise-Limited Regime

In the previous sections, we considered an interference-limited regime in which the noise power is negligible in comparison with the interference power. In this case, the achievable throughput is not a function of the network *SNR*. In other words, changing the transmission powers does not affect the supportable rate of each link. However, in a practical scenario, it is appealing to have rates which scale by increasing ρ. This way, the transmission rates can be easily adjusted by changing the transmission powers. Specifically, it is desirable that the rate of active links a.a.s. scale as

$$r_i = \log\left(1 + \frac{g_{ii}}{1/\rho + \beta_i}\right), \quad \forall i \in \mathcal{A}, \tag{90}$$

for some $\beta_i = O(1)$, which are the design parameters. At the same time, we require the conditions of problem (7), i.e. $r_i \geq \lambda$, be satisfied. In this section, we show how to realize such a situation by using DTBLAS.

According to (90), we should a.a.s. have $I_i = \beta_i$, where I_i is the interference observed by active link i and is defined in (52). However, this requires that $\mathrm{E}(I_i) = \beta_i$. Noting that $\mathrm{E}(I_i) = (k_2 - 1)\hat{\mu}$ (see (53)), we conclude that all β_is should take a same value, say β. Hence, a necessary condition for being in the noise-limited regime is

$$(k_2 - 1)\hat{\mu} = \beta, \tag{91}$$

where $\beta = O(1)$ is a design parameters. Later, we show that (91) is also a sufficient condition for operating in a noise-limited regime.

Based on the above discussion, we propose the following scheme for choosing the parameters of DTBLAS for a noise-limited regime: For a given required rate $\lambda = \log(1 + \gamma_0)$ and the interference β,

i. choose Δ as

$$\Delta = \Delta_0 = \gamma_0(1/\rho + \beta). \tag{92}$$

ii. choose δ such that (91) is satisfied.

Note that the selection of Δ is such that the rate constraints $r_i \geq \lambda$ are satisfied. Also, as will be shown later, the selection of δ is such that operation in the noise-limited regime is guaranteed.

The next step is to solve (91) to obtain the value of δ and the corresponding number of active links k_2. By using (55), which gives the value of $\hat{\mu}$ in terms of δ, it is clear that (91) holds only if $\delta \to 0$ as $k_2 \to \infty$. In this case, (55) converts to $\hat{\mu} = \dfrac{\delta}{2} + O(\delta^2)$ and (91) simplifies to

$$k_2\delta = 2\beta \quad a.a.s \tag{93}$$

To solve (93) and obtain δ, we should first obtain the value of k_2 in terms of n and δ. From (22) and condition (92), the number of links chosen by phase (i) of DTBLAS is obtained as

$$k_1 = ne^{-\Delta_0} + O\left(\xi\sqrt{ne^{-\Delta_0}}\right). \tag{94}$$

Also, recall from (62) that k_2 is the clique number of a random graph $\mathcal{G}(k_1, p)$, where p is obtained from (61). Since $\delta \to 0$, (61) can be rewritten as

$$p = \delta^2 + O(\delta^3), \tag{95}$$

which approaches zero as well. Note that Theorem 5, which was adopted from [28], and a similar result that appears in [29], are valid only for a fixed value of p. A natural question is whether a similar concentration result on the clique number of random graphs holds when p approaches zero. In the following lemma, we address this issue and obtain a concentration result on the clique number for zero-approaching values of p.

Lemma 10: Let $p = p(m)$ be such that $p = o(1)$ and $p = \omega(m^{-a})$ for all $a > 0$. For fixed $\epsilon > 0$, the clique number $\mathrm{cl}(G)$ of $G \in \mathcal{G}(m, p)$ a.a.s. satisfies $\lfloor s \rfloor \leq \mathrm{cl}(G) \leq \lfloor s \rfloor + 1$, where

$$s = 2\log_b m - 2\log_b \log_b m + 1 - 4\log_b 2 - \frac{\epsilon}{\log b},$$

$b = 1/p$.

Proof: See the Appendix. ■

By using this lemma, (94), (95), and assuming $\xi = o(\sqrt{n})$, the number of active links a.a.s. becomes

$$k_2 = \left\lfloor \frac{\log n - \log\log n}{-\log\delta} \right\rfloor. \tag{96}$$

Thus, (93) can be rewritten as

$$\frac{\log n - \log\log n}{-\log\delta} \cdot \delta = 2\beta. \tag{97}$$

Assuming $|\log\beta| = o(\log\log n)$, it can be verified that the solution of (97) is

$$\delta = \frac{2\beta\log\log n}{\log n}(1 + o(1)). \tag{98}$$

With this value of δ, the number of active links is obtained from (96) as

$$k_2 = \left\lfloor \frac{\log n}{\log\log n}(1 + o(1)) \right\rfloor. \tag{99}$$

As mentioned before, we should show that the selected values of δ and Δ for DTBLAS, yields the network to operate in the noise-limited regime. The following theorem addresses this issue.

Theorem 11: For the values of Δ and δ given in (92) and (98), respectively, the interference of active links a.a.s. satisfy

$$|I_i - \beta| \to 0, \quad \forall i \in \mathcal{A}. \tag{100}$$

Proof: By using the central limit theorem it can be shown that

$$|I_i - \beta| < \frac{\beta \log \log n}{\sqrt{\log n}}, \quad \forall i \in \mathcal{A}, \tag{101}$$

which readily yields the desired result. Since the calculations are similar to those in the proof of Lemmas 3 and 8, we omit them for brevity. ■

Lemma 12: Let $T_{_{NL}}$ denote the throughput achieved by DTBLAS in the noise-limited regime described above. Then, almost surely we have

$$\log \left(1 + \frac{\Delta_0}{1/\rho + \beta}\right) \leq \lim_{n\to\infty} \frac{\log \log n}{\log n} T_{_{NL}} \leq \log \left(1 + \frac{\Delta_0 + 1}{1/\rho + \beta}\right). \tag{102}$$

Proof: According to Theorem 11, the throughput is obtained as

$$T_{_{NL}} = \sum_{i=1}^{k_2} \log \left(1 + \frac{g_{ii}}{1/\rho + \beta}\right). \tag{103}$$

Due to the fact that $g_{ii} > \Delta_0$, we have

$$T_{_{NL}} \geq k_2 \log \left(1 + \frac{\Delta_0}{1/\rho + \beta}\right). \tag{104}$$

The left-hand-side inequality in the lemma is readily obtained by using this inequality and the value of k_2 from (99). For the right-hand-side inequality, by utilizing the Jensen's inequality in (103), we obtain

$$T_{_{NL}} \leq k_2 \log \left(1 + \frac{\frac{1}{k_2}\sum_{i=1}^{k_2} g_{ii}}{1/\rho + \beta}\right). \tag{105}$$

According to the law of large numbers and due to the fact that $g_{ii} > \Delta_0$, we have

$$\frac{1}{k_2} \sum_{i=1}^{k_2} g_{ii} \to \mathrm{E}(g_{ii} | g_{ii} > \Delta_0) = 1 + \Delta_0. \tag{106}$$

The result is obtained by using (105), (106), and the value of k_2 from (99). ■

It is observed that the price for operating in the noise-limited regime is a decrease in the throughput by a multiplicative factor of $\log \log n$.

VIII. Conclusion

In this paper, wireless networks in Rayleigh fading environments are studied in terms of their achievable throughput. It is assumed that each link is either active and transmits with power P and rate λ, or remains silent. The objective is to maximize the network throughput or equivalently the number of active links. First, an upper bound is derived that shows the throughput and the number of active links scale at most like $\log n$ and $\frac{1}{\lambda}\log n$, respectively. To obtain lower bounds, we propose two LASs (TBLAS and DTBLAS) and prove that both of them a.a.s. yield feasible solutions for the throughput maximization problem. In TBLAS, the activeness of each link is solely determined by the quality of its direct channel. TBLAS, which can be implemented in a decentralized fashion, performs very close to the upper bound for small values of λ. However, its performance falls below the upper bound when λ grows large. In DTBLAS, the mutual interference of the links are also taken into account when choosing the active links. It is demonstrated that DTBLAS not only performs close to the upper bound for $\lambda \to 0$, but its performance meets the upper bound when $\lambda \to \infty$. The above discussions take place in an interference-limited regime in which the transmission power P does not affect the transmission rate λ. However, we show that by a proper choose of the DTBLAS parameters, the rate-constrained network can also operate in a noise-limited regime; this feature of the DTBLAS comes at the price of decreasing the network throughput by a multiplicative factor of $\log\log n$.

APPENDIX A
PROOF OF LEMMA 8

The proof is based on the same arguments as in the proof of Lemma 3. Thus, here we just highlight the differences.

Let us define $\bar{\gamma}$ as

$$\bar{\gamma} = -\frac{(1-\alpha')\log(1-e^{-\delta})}{\alpha'\hat{\mu}}. \tag{107}$$

Similar to the proof of Lemma 3, it is enough to show that a.a.s.

$$x'_- < \gamma_i < x'_+, \tag{108}$$

where

$$x'_\pm = \bar{\gamma}\left(1 \pm c'\sqrt{\frac{\log\log n}{\log n}}(1+o(1))\right), \tag{109}$$

with $c' = c/\bar{\gamma}$. We only prove the left side inequality in (108); the other inequality can be proved in a similar manner.

Let $\mathcal{L}_3$ denote the event that

$$\gamma_i > x'_-, \quad \forall i \in \mathcal{A}, \tag{110}$$

In the following, we show that $\mathrm{P}(\mathcal{L}_3) \to 1$ for some $c' > 0$.

Note that with $\Delta = (1-\alpha')\log n$, the parameter k'_+ in Corollary 6 is obtained as

$$\begin{aligned} k'_+ &= \kappa_{_{DTBLAS}} \log n - a \log\log n && (111)\\ &< \kappa_{_{DTBLAS}} \log n, && (112) \end{aligned}$$

where $\kappa_{_{DTBLAS}}$ is given in (66) and $a > 0$ is a constant. Denoting the cdf of γ_i conditioned on $|\mathcal{A}| = k$ by $F_\gamma(x,k)$, we have

$$\begin{aligned} \mathrm{P}(\mathcal{L}_3) &\overset{(a)}{>} \left(1 - F_\gamma(x'_-, k'_+)\right)^{k'_+} \mathrm{P}\left(k'_- \le |\mathcal{A}| \le k'_+\right) && (113)\\ &\overset{(b)}{\approx} \left(1 - F_\gamma(x'_-, k'_+)\right)^{k'_+} && (114)\\ &\overset{(c)}{>} \left(1 - F_\gamma(x'_-, \kappa_{_{DTBLAS}} \log n)\right)^{\kappa_{_{DTBLAS}} \log n}, && (115) \end{aligned}$$

where (a) is obtained in the same manner as (38), (b) results from Corollary 6, and (c) is due to (112) and the fact that $(1 - F_\gamma(x,k))^k$ is a decreasing function of k. To show that the RHS of (115) tends to one, we upper bound $\kappa_{_{DTBLAS}} \log n F_\gamma(x'_-, \kappa_{_{DTBLAS}} \log n)$ and show that it approaches zero.

Similar to the derivation of (43), it can be shown that

$$F_\gamma(x'_-, \kappa_{_{DTBLAS}} \log n) < \mathrm{P}\left(\sum_{\substack{j=1\\ j\neq i}}^{\kappa_{_{DTBLAS}} \log n} g_{ji} \ge \frac{\Delta}{x'_-} - \frac{1}{\rho} \right). \tag{116}$$

Let us define $Y_j = g_{ji} - \hat{\mu}$, where $\hat{\mu}$ is obtained from (55). Random variable Y_j has the variance $\nu = \hat{\sigma}^2$, where $\hat{\sigma}^2$ is given in (56). By defining $Z = \frac{1}{\sqrt{\nu(\kappa_{_{DTBLAS}} \log n - 1)}} \sum_{\substack{j=1\\ j\neq i}}^{\kappa_{_{DTBLAS}} \log n} Y_j$, (116) can be reformulated as

$$F_\gamma(x'_-, \kappa_{_{DTBLAS}} \log n) < 1 - \mathrm{P}(Z \le y), \tag{117}$$

where

$$y = \frac{\frac{\Delta}{x'_-} - \frac{1}{\rho} - (\kappa_{_{DTBLAS}} \log n - 1)\hat{\mu}}{\sqrt{(\kappa_{_{DTBLAS}} \log n - 1)\hat{\sigma}^2}}. \tag{118}$$

By substituting $\Delta = (1-\alpha')\log n$ and the value of x'_- from (109) into (118), we obtain

$$y = c' \sqrt{\frac{\kappa_{_{DTBLAS}} \hat{\mu}^2}{\hat{\sigma}^2}} \sqrt{\log\log n} \left(1 + O\left(\frac{1}{\sqrt{\log n \log\log n}}\right)\right). \tag{119}$$

It is straightforward to show that the moment-generating function of Y_j exists around zero. Hence, the Cramér's condition is satisfied. Also, by choosing $m = \kappa_{_{DTBLAS}} \log n - 1$, the condition $y = O(m^{1/6})$ is satisfied, as well. As a result, Theorem 4 can be utilized to calculate the RHS(117) as

$$1 - \mathrm{P}(Z \le y) = [1 - \Phi(y)] \exp\left(\frac{\theta_3 y^3}{6\sqrt{\nu^3 \kappa_{DTBLAS} \log n}}\right) + O\left(\frac{e^{-\frac{y^2}{2}}}{\sqrt{\kappa_{DTBLAS} \log n}}\right) \qquad (120)$$

By combining (117), (120), and (119), and noting that θ_3 is a constant, $y^3 = o(\sqrt{\kappa_{DTBLAS} \log n})$, and $1 - \Phi(y) < \frac{e^{-y^2/2}}{y}$, we conclude that

$$\begin{aligned} \kappa_{DTBLAS} \log n F_\gamma(x, \kappa_{DTBLAS} \log n) &< \kappa_{DTBLAS} \log n \frac{e^{-\frac{y^2}{2}}}{y} \qquad (121) \\ &= \exp\left((1 - \frac{c'^2 \kappa_{DTBLAS} \hat{\mu}^2}{2\hat{\sigma}^2}) \log\log n + O(\log\log\log n)\right) \end{aligned}$$

It is clear that if c' is chosen large enough, the above upper bound approaches zero as $n \to \infty$. This completes the proof.

Appendix B
Proof of Lemma 10

The proof is based on the standard second moment method.

A. Preliminary Calculations

Assume Y_s is the number of cliques of size s in G. Let us denote its mean and variance by μ_s and σ_s^2, respectively. According to [29], we have

$$\mu_s = \binom{m}{s} p^{\binom{s}{2}}, \qquad (122)$$

$$\frac{\sigma_s^2}{\mu_s^2} = \sum_{\ell=2}^{s} \frac{\binom{s}{\ell}\binom{m-s}{s-\ell}}{\binom{m}{s}} (b^{\binom{\ell}{2}} - 1), \qquad (123)$$

where $b = 1/p$. By applying the Stirling's approximation to (122), we obtain

$$\begin{aligned} \mu_s &= \frac{m^{m+\frac{1}{2}}}{\sqrt{2\pi} s^{s+\frac{1}{2}} (m-s)^{m-s+\frac{1}{2}}} p^{\frac{s(s-1)}{2}} \qquad (124) \\ &\le \frac{1}{\left(\frac{s}{m}\right)^s \left(1 - \frac{s}{m}\right)^m} p^{\frac{s(s-1)}{2}} \qquad (125) \end{aligned}$$

For any $\epsilon > 0$, the inequality $1 - x \ge e^{-(1+\epsilon)x}$ holds for sufficiently small values of x. Since we are interested in small values of s/m, from this inequality and (124), we obtain

$$\mu_s \le e^{s(\log m - \log s + (1+\epsilon) - \frac{s-1}{2} \log b)} \qquad (126)$$

Equation (123) is readily converted to the following inequality

$$\frac{\sigma_s^2}{\mu_s^2} \leq \sum_{\ell=2}^{s} F_\ell, \tag{127}$$

where

$$F_\ell = \frac{\binom{s}{\ell}\binom{m-s}{s-\ell}}{\binom{m}{s}} b^{\binom{\ell}{2}}. \tag{128}$$

By using the definition of the binomial coefficients, we obtain

$$\begin{aligned} F_\ell &\leq 2^s \cdot \frac{(m-s)!}{m!} \cdot \frac{(m-s)!}{(m-2s+\ell)!} \cdot \frac{s!}{(s-\ell)!} \cdot b^{\frac{\ell(\ell-1)}{2}} && (129) \\ &\leq \frac{2^s \cdot (m-s)^{s-\ell} \cdot s^\ell}{(m-s)^s} \cdot b^{\frac{\ell(\ell-1)}{2}} && (130) \\ &= 2^s \cdot \left(\frac{m}{s} - 1\right)^{-\ell} \cdot b^{\frac{\ell(\ell-1)}{2}} && (131) \end{aligned}$$

Noting that $\frac{m}{s} \gg 1$, the above inequality can be approximately written as

$$F_\ell \leq 2^s \cdot \left(\frac{s}{m}\right)^\ell \cdot b^{\frac{\ell(\ell-1)}{2}}. \tag{132}$$

Using (127) and (132), we obtain

$$\frac{\sigma_s^2}{\mu_s^2} \leq \sum_{\ell=2}^{s} e^{g(\ell)}, \tag{133}$$

where

$$g(\ell) = s\log 2 + \ell(\log s - \log m + \frac{\ell}{2}\log b - \frac{1}{2}\log b) \tag{134}$$

is a quadratic convex function with a minimum at $\ell_0 = \frac{\log m}{\log b} - \frac{\log s}{\log b} + \frac{1}{2}$. Define

$$s_0 = 2\log_b m - 2\log_b \log_b m - 2\log_b 2. \tag{135}$$

It is easy to show that if $s > s_0$, then $g(s) > g(2)$. Hence, (133) can be simplified as

$$\frac{\sigma_s^2}{\mu_s^2} \leq e^{\log s + g(s)}. \tag{136}$$

B. Proof

According to the Markov's inequality, we have

$$\mathrm{P}\left\{Y_s \geq 1\right\} \leq \mu_s. \tag{137}$$

For a fixed $\epsilon > 0$, define

$$s_1 = 2\log_b m - 2\log_b \log_b m + 1 + 2\log_b(e/2) + \frac{\epsilon}{\log b}. \tag{138}$$

Using (126), it is easy to verify that for $s \geq s_1$, we have $\mu_s \to 0$ as $m \to \infty$. Hence, from (137), we conclude that

$$\mathrm{P}\left\{Y_s \geq 1\right\} \to 0, \quad \text{for} \quad s \geq s_1 \tag{139}$$

as $m \to \infty$. This means a.a.s. the clique number of G is less than s_1, i.e., we have the following upper bound on cl(G)

$$\mathrm{cl}(G) < s_1 \quad a.a.s. \tag{140}$$

According to the Chebyshev's inequality, we have

$$\mathrm{P}\left\{Y_s = 0\right\} \leq \frac{\sigma_s^2}{\mu_s^2}. \tag{141}$$

For a fixed $\epsilon > 0$, define

$$s_2 = 2\log_b m - 2\log_b \log_b m + 1 - 4\log_b 2 - \frac{\epsilon}{\log b}. \tag{142}$$

Using (136), it is easy to verify that for $s \leq s_2$, we have $\sigma_s^2/\mu_s^2 \to 0$ as $m \to \infty$. Hence, from (141), we conclude that

$$\mathrm{P}\left\{Y_s = 0\right\} \to 0, \quad \text{for} \quad s \leq s_2 \tag{143}$$

as $m \to \infty$. This means a.a.s. the clique number of G is not less than $\lfloor s_2 \rfloor$, i.e., we have the following lower bound on cl(G)

$$\mathrm{cl}(G) \geq \lfloor s_2 \rfloor \quad a.a.s. \tag{144}$$

For sufficiently small ϵ, the difference between the upper bound s_1 and the lower bound s_2 is less than one. Hence, from (140) and (144) we can conclude that

$$\lfloor s_2 \rfloor \leq \mathrm{cl}(G) \leq \lfloor s_2 \rfloor + 1 \quad a.a.s. \tag{145}$$

ACKNOWLEDGMENT

The authors would like to thank T. Luczak and S. Oveis Gharan for helpful discussions on the proof of Lemma 10.

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