On the Throughput Maximization in Dencentralized Wireless Networks

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On the Throughput Maximization in Dencentralized Wireless Networks

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Abstract

A distributed single-hop wireless network with K links is considered, where the links are partitioned into M clusters each operating in an orthogonal subchannel with bandwidth $\frac{W}{M}$. We consider a general shadow-fading model, described with parameters (α, ϖ), where α denotes the probability of shadowing and ϖ represents the average power of the cross-link channels. The main goal of this paper is to maximize the network throughput. This is achieved by: i) proposing a distributed and non-iterative power allocation strategy, where the objective function for each user is to maximize its best estimate (based on its local information, i.e., direct channel gain) of the average network throughput, and ii) choosing the optimum value for $1 \leq M \leq K$. We analyze the throughput of the network in terms of M and (α, ϖ) in the asymptotic case of $K \to \infty$. It is proved that when the number of links is large, the optimum power allocation strategy for each user is the threshold-based on-off power scheme (i.e., links with a direct channel gain above certain threshold transmit at full power and the rest remain silent). Under the on-off power scheme, it is demonstrated that the maximum achievable network throughput for every value of $1 \leq M \leq K$, $0 \leq \alpha < 1$ and $\varpi \leq 1$ is achieved at $M = 1$.

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Index Terms

Throughput maximization, distributed power allocation, shadow-fading, wireless network.

I. INTRODUCTION

A. History

The main challenge in wireless networks originates from using resources efficiently such that the network throughput is maximized. Throughput maximization in multi-user wireless networks has been addressed from various perspectives; resource allocation (e.g., power and bandwidth assignments [3]–[5]), scheduling (e.g., user selection [6]), routing by using relay nodes [7], mobility of the nodes [8] and exploiting channel characteristics (e.g., power decay-versus-distance law [9]–[11], geometric pathloss as well as fading [12], [13] and random connections [14]). Central to the study of the network throughput maximization is the problem of resource allocation.

Among different resource allocation schemes, power and spectrum assignments have long been regarded as efficient tools in order to mitigate the interference and improve the network throughput. In recent years, various power and spectrum allocation schemes have been extensively studied in cellular and multihop wireless networks [3], [4], [15]–[20]. In [19], the authors provide a comprehensive survey in the area of resource allocation, in particular in the context of spectrum assignment for different kinds of wireless networks. Much of these works rely on centralized and cooperative algorithms. Clearly, centralized resource allocation schemes provide a significant improvement in the network throughput over distributed (or decentralized) approaches. However, they require extensive knowledge of the network configuration. In particular, when the number of nodes is large, deploying centralized schemes becomes computationally intractable. In addition, in a cooperative wireless network, when the number of nodes becomes large, the network is overwhelmed by the amount of exchanged information. This is critical for time-varying networks, as the resource assignments can not perfectly trace the speed of channel variations. Thus, due to significant challenges in using centralized approaches, the attention of the researchers has drawn to the distributed resource allocation schemes [21]–[26].

The main goal of applying a decentralized mechanism is that operational decisions for network parameters (e.g., the transmission rate) are made solely by the individual users based on decision parameters locally available to each node. The local decision parameters that can be used for adjusting the rate are the Signal-to-Interference-plus-Noise Ratio (SINR) and the direct channel gains. Most of the works dealing with the throughput maximization target the SINR parameter by using iterative algorithms [23]–[25]. This leads to use of game theory concepts such as repeated game [27], in which the main challenge is the convergence issue. For instance, Etkin *et al.* [25] develop power and spectrum allocation strategies in multiple wireless systems by using game theory. Under the assumptions of the omniscient nodes and strong interference, they show that Frequency-Division Multiplexing (FDM) is the optimal scheme in the sense of the throughput maximization. They use an iterative algorithm that converges to the optimum powers. A more realistic approach in time-varying networks is to adjust the transmission rate based on the channel gains non-cooperatively and without utilizing iterative schemes.

Motivated by the above considerations, we study the throughput maximization of a distributed wireless network with K links, where all the links operating in bandwidth W are partitioned into M clusters. Each cluster operates in an orthogonal subchannel with bandwidth $\frac{W}{M}$. The throughput maximization of the underlying network is achieved by proposing a distributed and non-iterative power allocation strategy based on the direct channel gains, and then choosing the optimum value for $1 \leq M \leq K$.

B. Contributions and Relations to Previous Works

In this paper, we study the throughput maximization of a spatially distributed wireless network with K links, where the sources and their corresponding destinations communicate directly with each other without using other nodes. Wireless networks with unlicensed spectrum (e.g. Wi-Fi systems based on IEEE 802.11 standard) are among typical distributed networks. We develop a general model of the shadowing effect that is caused by obstacles. In this model, it is considered a Rayleigh fading model for the direct-link channels. Also, it is assumed a Rayleigh shadow-fading model for the cross-link channels, described with parameters (α, ϖ), where α denotes the probability of shadowing and ϖ represents the average power of the cross-link channels. It is assumed K links operating in bandwidth W are partitioned into M clusters. Each cluster operates in an orthogonal subchannel with bandwidth $\frac{W}{M}$ in order to mitigate the interference.

The above configuration differs from the geometric models proposed in [8]–[11], in which the signal power decays based on the distance between nodes. Contrary to [22]–[25], in which the proposed iterative algorithms rely on the SINR, we focus on a more practical case, where each transmitter adjusts its power based on the direct channel gain with its corresponding receiver. Clearly, if each user maximizes its rate selfishly, the optimum power allocation strategy for each user is to transmit with full power. This results in the network throughput degradation. To prevent the users from selfishly increasing their powers, it is desired to consider the negative impact of each user's power increment on the other links performance. While designing such an algorithm, a reasonable objective is to choose an optimum non-iterative power allocation strategy in order to maximize its best local estimate of the average network throughput. In this setup, the optimization problem is subject to the individual power constraint for each link, instead of a total power constraint. This assumption is more practical for decentralized wireless networks. Under the aforementioned objective function, it is demonstrated that when the number of links is large, the optimum power allocation strategy for each user is the *threshold-based on-off power scheme* (i.e., links with a direct channel gain above certain threshold transmit at full power and the rest remain silent). This result is different from the link activation strategy studied in [28], where they use the on-off power scheme for $M = 1$ and without mentioning about the optimality of this power allocation in the sense of the throughput maximization. Also, our work differs from the analysis developed in [14] and [29]; primarily we use a distributed power allocation scheme for a single-hop wireless network with M disjoint subchannels, while [14] and [29] present an ad-hoc network model with random connections for $M = 1$ and using relay nodes.

We optimize the maximum achievable throughput of the network in terms of the number of the clusters, M. It is proved that the maximum throughput of the network for every value of $1 \leq M \leq K$, $0 \leq \alpha < 1$ and $\varpi \leq 1$ is achieved at $M = 1$. In other words, splitting the bandwidth W into M orthogonal subchannels has no gain in terms of enhancing the throughput. The interesting point is that under the on-off power allocation strategy, the total network energy for $M = 1$ can be minimized in comparison with $M = K$ case, where all the users transmit with full power all the time.

The rest of the paper is organized as follows. In Section II, the network model and objectives are described. The distributed on-off power allocation strategy is presented in Section III. We analyze the network throughput in Section IV. Finally, in Section V, an overview of the results and conclusions is presented.

C. Notations

Knuth's order notation [30]: For any functions $f(n)$ and $g(n)$:

- $f(n) = O(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| < \infty$.
- $f(n) = o(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = 0$.
- $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = \infty$.
- $f(n) = \Omega(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| > 0$.
- $f(n) = \Theta(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = c$, where $0 < c < \infty$.
- $f(n) \sim g(n)$ means that $\lim_{n\to\infty} f(n)/g(n) = 1$.

Also, throughout the paper, we use $\log(.)$ as the natural logarithm function; boldface letters denote vectors and $\bar{R}$ means $\mathbb{E}[R]$, where $\mathbb{E}[\ .\ ]$ represents the expectation operator.

II. Network Model and Objectives

A. Network Model

In this work, we consider a single-hop wireless network consisting of K pairs of nodes[1], operating in bandwidth W. The links are assumed to be randomly distributed among M clusters denoted by $\mathbb{C}_j$, $j = 1, ..., M$, such that the number of links in all clusters are the same. Without loss of generality, we assume that $\mathbb{C}_j \triangleq \{(j-1)n + 1, ..., jn\}$,

[1]The term "pair" is used to describe the transmitter and the related receiver, and "user" only for the transmitter.

where $n \triangleq \frac{K}{M}$ denotes the cardinality of the set $\mathbb{C}_j$ and is assumed to be known to all users[2]. In order to eliminate the mutual interference among clusters, we assume an M-dimensional orthogonal coordinate system[3]. Without loss of generality, we consider an orthogonal frequency coordinates system, in which the bandwidth W is split into M disjoint subchannels each with bandwidth $\frac{W}{M}$. It is supposed that the links in $\mathbb{C}_j$ operate in subchannel j. In this work, we assume that the quantity M is in the range of 1 to K. Also, all the nodes in the network are assumed to have a single antenna. In addition, we assume that each receiver knows its direct channel gain with the corresponding transmitter, as well as the interference power imposed by other users. These information are fed back to the corresponding transmitter without any error. The power of additive white Gaussian noise (AWGN) at each receiver is assumed to be $\frac{N_0 W}{M}$, where N_0 is defined as the noise power spectral density over bandwidth W.

The channel model considered in this paper is assumed to be flat Rayleigh fading with the shadowing effect. The channel gain[4] between transmitter k and receiver i is represented by the random variable $\mathcal{L}_{ki}$, where $\mathcal{L}_{ki} = h_{ii}$ (for $k = i$) is referred as the *direct-link channel gain*, and for $k \neq i$, the *cross-link channel gains* are defined based on a general shadowing model as follows

$$\mathcal{L}_{ki} = \begin{cases} \beta_{ki} h_{ki}, & \text{with probability} \quad \alpha \\ 0, & \text{with probability} \quad 1-\alpha, \end{cases} \tag{1}$$

where h_{ki}'s are exponentially distributed with unit mean and unit variance, $0 \leq \alpha \leq 1$ is a fixed parameter, and β_{ki} is a random variable with the following conditions:

- $\mathbb{E}\left[\beta_{ki}\right] \triangleq \varpi \leq 1$,
- $\mathbb{E}\left[\beta_{ki}^2\right] \triangleq \kappa$, where κ is a finite positive number,
- The probability density function of β_{ki} is bounded at zero and is continuous within the neighborhood of zero.

[2]It is assumed that K is divisible by M, and hence, $n = \frac{K}{M}$ is an integer number.

[3]There are several ways of generating M orthogonal subspaces. For example, we can allocate M different frequency bands, or M different time slots, or we could carefully design M orthogonal codes such as Walsh-Hadamard codes and allocate each code to each cluster.

[4]In this paper, *channel gain* is defined as the square magnitude of the *channel coefficient*.

Note that the second and third conditions on β_{ki} are not restrictive and most of the known density functions satisfy these conditions. All the channels in the network are supposed to be quasi-static block fading, where the channel gains remain constant during transmitting one block and change independently from block to block.

We assume a homogeneous network in the sense that all the links have the same configurations and use the same protocols. Thus, the transmission strategy for all the users are agreed in advance. We denote the transmit power of user i by $p_i \in \mathbb{P}$, where $\mathbb{P} \triangleq [0, \mathrm{P}_{max}]$. Also, the non-negative vector $\mathbf{P}^{(j)} = (p_{(j-1)n+1}, ..., p_{jn})$ represents the vector of all the users' power in $\mathbb{C}_j$. The power vector $\mathbf{P}^{(j)}_{-i}$ describes the vector $\mathbf{P}^{(j)}$ except p_i. To simplify the notations, we assume the noise power $\frac{N_0 W}{M}$ is normalized by P_{max}. Without loss of generality, in the sequel, we assume $\mathrm{P}_{max} = 1$. Assuming the transmitted signal from each transmitter to be Gaussian-distributed, the interference term seen by a link $i \in \mathbb{C}_j$ will be Gaussian with power

$$I_i = \sum_{\substack{k \in \mathbb{C}_j \\ k \neq i}} \mathcal{L}_{ki} p_k.$$

Due to the orthogonality of the allocated subchannels, no interference is imposed from links in $\mathbb{C}_k$ on links in $\mathbb{C}_j$, $k \neq j$. Under these assumptions, the achievable data rate of each link $i \in \mathbb{C}_j$ is expressed as

$$R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)}) = \frac{W}{M} \log \left(1 + \frac{h_{ii} p_i}{I_i + \frac{N_0 W}{M}} \right), \tag{2}$$

where $\boldsymbol{\mathcal{L}}^{(j)} = (\mathcal{L}_{((j-1)n+1)i}, ..., \mathcal{L}_{(jn)i})$. In order to analyze the performance of the underlying network, we define the network throughput as the average sum-rate of all the links, i.e.,

$$\bar{R}_{sum} \triangleq \sum_{j=1}^{M} \sum_{i \in \mathbb{C}_j} \mathbb{E} \left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)}) \right], \tag{3}$$

where the expectation is computed with respect to $\boldsymbol{\mathcal{L}}^{(j)}$.

B. Objectives

The main goal of this paper is to maximize the network throughput. This is achieved by:

1- Proposing a distributed and non-iterative power allocation strategy, where the objective function for each user is to maximize its best estimate (based on its local information, i.e., direct channel gain) of the average network throughput.

2- Choosing the optimum value for $1 \leq M \leq K$.

We propose a simple distributed power allocation strategy through defining a utility function, where only the direct channel gains are used. Clearly, if each user maximizes its rate selfishly, the optimum power allocation strategy for each user is to transmit with full power. This leads to the network throughput degradation. To prevent the users from selfishly increasing their powers, the negative impact that each user imposes on the other users should be considered. In addition, this should aim to maximize each user's utility function while improving the network throughput. Since the transmission power p_i depends on the channel gain h_{ii}, in the sequel, we use $p_i = g(h_{ii})$. To this end, the utility function of link $i \in \mathbb{C}_j$ is a function of its direct channel gain and the power p_i, and is defined as

$$u_i(p_i, h_{ii}) \triangleq \mathbb{E}\left[\sum_{i \in \mathbb{C}_j} R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right], \quad i \in \mathbb{C}_j, \quad j = 1, ..., M, \tag{4}$$

where the expectation is computed with respect to $\mathbf{P}_{-i}^{(j)}$ and $\boldsymbol{\mathcal{L}}_{-i}^{(j)}$. It should be noted that h_{ii} and p_i are considered as the local information of link i. However, the local information of the other links (their powers and direct channel gains), and also the channel gains $\boldsymbol{\mathcal{L}}_{-i}^{(j)}$, are unknown to user i. Thus, user i considers these parameters as random variables and selects its power such that its utility function is maximized, i.e.,

$$\hat{p}_i = \arg \max_{p_i \in \mathbb{P}} \; u_i(p_i, h_{ii}), \quad i \in \mathbb{C}_j, \quad j = 1, ..., M. \tag{5}$$

It will be shown that when the number of links is large, the optimum power allocation strategy for the optimization problem in (5) is the on-off power scheme. Since the channel gains change independently from block to block, each user updates its power based on the direct channel gain in each block.

Given the optimum power vector $\hat{\mathbf{P}}^{(j)} = (\hat{p}_{(j-1)n+1}, ..., \hat{p}_{jn})$ obtained from (5), we then choose the optimum value of M such that the network throughput is maximized, i.e.,

$$\hat{M} = \arg \max_{1 \leq M \leq K} \sum_{j=1}^{M} \sum_{i \in \mathbb{C}_j} \mathbb{E}\left[R_i(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right]. \tag{6}$$

III. Optimum Distributed Power Allocation

In this section, we introduce a simple distributed power allocation scheme, in which each user selects the optimum power $p_i = g(h_{ii}) \in \mathbb{P}$ in order to maximize its utility. Using (4), we can express the utility function of each user as

$$u_i(p_i, h_{ii}) = \bar{R}_i(p_i, h_{ii}) + \sum_{\substack{l \in \mathbb{C}_j \\ l \neq i}} \bar{R}_l(p_i), \quad i \in \mathbb{C}_j, \quad j = 1, ..., M, \tag{7}$$

where

$$\bar{R}_i(p_i, h_{ii}) = \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ii} p_i}{I_i + \frac{N_0 W}{M}}\right)\right], \tag{8}$$

and the expectation is computed with respect to I_i, and

$$\begin{aligned} \bar{R}_l(p_i) &= \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ll} p_l}{I_l + \frac{N_0 W}{M}}\right)\right] & (9) \\ &= \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ll} p_l}{\mathcal{L}_{il} p_i + \sum_{k \neq l,i} \mathcal{L}_{kl} p_k + \frac{N_0 W}{M}}\right)\right], \quad l \neq i \in \mathbb{C}_j, & (10) \end{aligned}$$

where the expectation is computed with respect to $\mathbf{P}_{-i}^{(j)}$, h_{ll} and $\{\mathcal{L}_{kl}\}_{k \neq l}$. Using the fact that all users follow the same power allocation policy, and since the channel gains $\mathcal{L}_{kl}$ are random variables with the same distributions, $\bar{R}_l(p_i)$ becomes independent of l. Thus, by dropping index l from $\bar{R}_l(p_i)$, the utility function of link i can be simplified as

$$u_i(p_i, h_{ii}) = \bar{R}_i(p_i, h_{ii}) + (n-1)\bar{R}(p_i). \tag{11}$$

Lemma 1 *Let assume $0 < \alpha \leq 1$ is fixed and $\mathbb{E}[p_k] \triangleq q_n$ and Var$[p_k] \triangleq \sigma_n^2$. Then, with probability one (w. p. 1), we have*

$$I_i \ \sim \ \hat{\alpha}(n-1)q_n, \quad i \in \mathbb{C}_j,$$

as $n \to \infty$, where $\hat{\alpha} = \alpha\varpi$.

Proof: See Appendix I. ■

Theorem 1 *For sufficiently large n, the optimum power for (5) is $\hat{p}_i = g(h_{ii}) = U(h_{ii} - \tau_n)$, where $\tau_n > 0$ is a threshold level that is a function of n, and $U(.)$ is a step function.*

Proof: The steps of the proof are as follows: we first derive an upper bound for the utility function defined in (11). Then, we prove that the optimum power allocation strategy that maximizes this upper bound is $\hat{p}_i = g(h_{ii}) = U(h_{ii} - \tau_n)$. Based on this optimum power allocation policy, in Lemma 2, we derive the optimum threshold level τ_n that maximizes the average utility function. Finally, the theorem is proved by showing that the maximum value of the utility function in (11) is achievable and this value is the same as the maximum value of the upper bound obtained in the first step.

Step 1: Upper Bound on the Utility Function

Let us assume $\mathbb{E}[p_k] = q_n$ and Var$[p_k] = \sigma_n^2$. Using (8) and Lemma 1, $\bar{R}_i(p_i, h_{ii})$ in (11) can be expressed as

$$\bar{R}_i(p_i, h_{ii}) \approx \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}p_i}{\hat{\alpha}(n-1)q_n + \frac{N_0 W}{M}}\right)\right] \quad (12)$$

$$\overset{(a)}{=} \frac{W}{M}\log\left(1 + \frac{h_{ii}p_i}{\hat{\alpha}(n-1)q_n + \frac{N_0 W}{M}}\right) \quad (13)$$

$$\approx \frac{W}{M}\log\left(1 + \frac{h_{ii}p_i}{\lambda}\right), \quad (14)$$

as $n \to \infty$, where $\lambda \triangleq \hat{\alpha} n q_n + \frac{N_0 W}{M}$. In the above equations, (a) follows from the fact that h_{ii} is a known parameter for user i and p_i is the optimization parameter. With a similar argument, (10) can be simplified as

$$\begin{aligned}\bar{R}(p_i) &\approx \mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ll}p_l}{\mathcal{L}_{il}p_i+\hat{\alpha}(n-2)q_n+\frac{N_0W}{M}}\right)\right], \quad i\neq l \\ &= \alpha\mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i+\hat{\alpha}(n-2)q_n+\frac{N_0W}{M}}\right)\right] \\ &+ (1-\alpha)\mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ll}p_l}{\hat{\alpha}(n-2)q_n+\frac{N_0W}{M}}\right)\right] \qquad (15) \\ &\approx \frac{\alpha W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i+\lambda}\right)\right]+(1-\alpha)\frac{W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}p_l}{\lambda}\right)\right], \qquad (16)\end{aligned}$$

as $n\to\infty$, where the expectation is computed with respect to h_{ll}, h_{il}, p_l and β_{il}. Using (14) and (16), and the inequality $\log(1+x)\leq x$, an upper bound for the utility function in (11) is obtained as

$$u_i(p_i,h_{ii})\leq\frac{W}{M}\frac{h_{ii}}{\lambda}p_i+n\frac{\alpha W}{M}\mathbb{E}\left[\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i+\lambda}\right]+n(1-\alpha)\frac{W}{M\lambda}\mathbb{E}\left[h_{ll}p_l\right]. \qquad (17)$$

Using the fact that h_{ll} is independent of h_{il}, $i\neq l$, we have

$$\begin{aligned}\mathbb{E}\left[\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i+\lambda}\Big|\beta_{il}=t\right] &= \int_0^\infty\int_0^\infty\frac{xg(x)}{ytp_i+\lambda}e^{-x}e^{-y}dxdy \\ &= \int_0^\infty xg(x)e^{-x}dx\int_0^\infty\frac{e^{-y}}{ytp_i+\lambda}dy \\ &= -\frac{\mu}{tp_i}e^{\frac{\lambda}{tp_i}}\mathrm{Ei}\left(-\frac{\lambda}{tp_i}\right), \qquad (18)\end{aligned}$$

where $\mu\triangleq\mathbb{E}\left[h_{ll}p_l\right]=\int_0^\infty xg(x)e^{-x}dx$ is a constant value, and $\mathrm{Ei}(x)\triangleq-\int_{-x}^\infty\frac{e^{-s}}{s}ds$, $x<0$ is the *exponential-integral function* [31]. To this end, the right hand side of (17) is simplified as

$$u_i(p_i,h_{ii})\leq\frac{W}{M}\frac{h_{ii}}{\lambda}p_i-n\frac{\alpha\mu W}{M}\mathbb{E}\left[\frac{1}{tp_i}e^{\frac{\lambda}{tp_i}}\mathrm{Ei}\left(-\frac{\lambda}{tp_i}\right)\right]+n(1-\alpha)\frac{W}{M}\frac{\mu}{\lambda}, \qquad (19)$$

where the expectation is computed with respect to $\beta_{il}=t$. An asymptotic expansion of $\mathrm{Ei}(x)$ is as [31]

$$\mathrm{Ei}(x)=\frac{e^x}{x}\left[\sum_{k=0}^{L-1}\frac{k!}{x^k}+O(|x|^{-L})\right]; \quad L=1,2,..., \qquad (20)$$

as $x \to -\infty$. Setting $L = 3$, we can rewrite (19) as

$$\begin{aligned}
u_i(p_i, h_{ii}) &\leq \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + n\frac{\alpha W\mu}{M\lambda}\mathbb{E}\left[\left(1 - \frac{tp_i}{\lambda} + 2\left(\frac{tp_i}{\lambda}\right)^2\right) + O\left(\left|\frac{tp_i}{\lambda}\right|^3\right)\right] \\
&+ n(1-\alpha)\frac{W\mu}{M\lambda} \\
&\overset{(a)}{\approx} \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + n\frac{\alpha W\mu}{M\lambda}\left(1 - \frac{\varpi p_i}{\lambda} + 2\kappa\left(\frac{p_i}{\lambda}\right)^2\right) + n(1-\alpha)\frac{W\mu}{M\lambda} \qquad (21) \\
&\triangleq \Xi_i(p_i, h_i),
\end{aligned}$$

as $\lambda \to \infty$, where (a) follows from the fact that for large values of n, the term $\mathbb{E}\left[O\left(\left|\frac{tp_i}{\lambda}\right|^3\right)\right]$ can be ignored. It will be shown that the condition $\lambda \to \infty$ is valid automatically for the optimum value of q_n.

Step 2: Optimum Power Allocation Policy for $\Xi_i(p_i, h_{ii})$

Taking the first-order derivative of (21) in terms of p_i yields,

$$\frac{\partial \Xi_i(p_i, h_{ii})}{\partial p_i} = \frac{W}{M}\left\{\frac{1}{\lambda}\left(h_{ii} - \frac{\hat{\alpha} n\mu}{\lambda}\right) + \left(\frac{2}{\lambda}\right)^2 \frac{\alpha n\mu\kappa}{\lambda}p_i\right\}.$$

Also, the second-order derivative of (21), $\frac{\partial^2 \Xi_i(p_i, h_{ii})}{\partial p_i^2} = \frac{W}{M}\left(\frac{2}{\lambda}\right)^2 \frac{\alpha n\mu\kappa}{\lambda}$, is a positive value. Thus, (21) is a *convex* function of p_i. It is a known fact that a convex function attains its maximum at one of the extreme points[5] of its domain [32]. In the following, we show that $g(h_{ii})$ is a monotonically increasing function of h_{ii}. Suppose the optimum power that maximizes $\Xi_i(p_i, h_{ii})$ is $p_i = 1$. Also, let us define $h'_{ii} \triangleq h_{ii} + \delta$, where $\delta > 0$. From (21), we have

$$\Xi_i(p_i, h'_{ii}) = \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + \frac{W}{M}\frac{\delta}{\lambda}p_i + n\frac{\alpha W\mu}{M\lambda}\left(1 - \frac{\varpi p_i}{\lambda} + 2\kappa\left(\frac{p_i}{\lambda}\right)^2\right) + n(1-\alpha)\frac{W\mu}{M\lambda}. \qquad (22)$$

It is observed that

$$\Xi_i(p_i = 1, h'_{ii}) > \Xi_i(p_i = 1, h_{ii}), \qquad (23)$$

i.e., the optimum power that maximize $\Xi_i(p_i, h'_{ii})$ is consistently equal to 1. On the other hand, from the convexity of $\Xi_i(p_i, h_{ii})$, and since the optimum power is $p_i = 1$, it is

[5] In the power domain $\mathbb{P} = [0, 1]$, the extreme points are 0 and 1.

concluded that

$$\Xi_i(p_i = 1, h_{ii}) > \Xi_i(p_i = 0, h_{ii}). \tag{24}$$

Using the fact that $\Xi_i(p_i = 0, h_{ii}) = \Xi_i(p_i = 0, h_{ii}^{'})$, we arrive at the following inequality

$$\Xi_i(p_i = 1, h_{ii}^{'}) > \Xi_i(p_i = 0, h_{ii}^{'}). \tag{25}$$

From (23)-(25), it is concluded that $g(h_{ii})$ is a monotonically increasing function of h_{ii}. Consequently, the optimum power allocation strategy that maximizes $\Xi_i(p_i, h_{ii})$ is a unit step function as follows:

$$\hat{p}_i = \begin{cases} 1, & \text{if} \quad h_{ii} > \tau_n \\ 0, & \text{Otherwise}, \end{cases} \tag{26}$$

where τ_n is a prespecified threshold level. We called this power allocation scheme as the threshold-based on-off power scheme.

Step 3: Optimum Threshold Level τ_n

From the results in Step 2, it is concluded that the optimum power $\hat{p}_i$ is a Bernoulli random variable with parameter q_n, i.e.,

$$f(p_i) = \begin{cases} q_n, & p_i = 1, \\ 1 - q_n, & p_i = 0, \end{cases} \tag{27}$$

where $f(.)$ is the probability density function (pdf) of p_i. We define the probability of the link activation in each cluster as $q_n \triangleq \Pr\{h_{ii} > \tau_n\}$ which is a function of n. In the next lemma, we obtain the optimum τ_n that maximizes the average utility function denoted by

$$\bar{u}_i \triangleq \mathbb{E}\left[u_i(p_i, h_{ii})\right], \tag{28}$$

where the expectation is computed with respect to h_{ii}. Noting that $p_i = g(h_{ii})$, $\bar{u}_i$ is independent of p_i.

Lemma 2 *For large values of n and $0 < \alpha \leq 1$ is fixed, the optimum threshold level that maximizes $\bar{u}_i$ is obtained as*

$$\hat{\tau}_n = \log \hat{\alpha} n - 2 \log \log \hat{\alpha} n + O(1). \tag{29}$$

Also, the probability of the link activation in each cluster is given by

$$q_n = \delta \frac{\log^2 \hat{\alpha} n}{\hat{\alpha} n}, \tag{30}$$

where δ is a constant.

Proof: See Appendix II. ■

Step 4: Optimum Power Allocation Strategy that Maximize $u_i(p_i, h_{ii})$

Defining the event $\mathscr{E} \triangleq \{h_{ii} > h_{th}\}$, where $h_{th} \triangleq 2\log n$, we have

$$\Pr\{\mathscr{E}\} \leq \Pr\{h_{max} > h_{th}\}, \tag{31}$$

where $h_{max} \triangleq \max \ \{h_{ii}\}_{i \in \mathbb{C}_j}$. Note that

$$\begin{aligned} \Pr\{h_{max} > h_{th}\} &= 1 - \left(1 - e^{-h_{th}}\right)^n \\ &= 1 - \left(1 - \frac{1}{n^2}\right)^n \\ &\approx \frac{1}{n}, \end{aligned} \tag{32}$$

when n is large. Since $\lim_{n\to\infty} \Pr\{h_{max} > h_{th}\} = 0$, it is concluded that with probability one, $h_{ii} < h_{th}$. Also, from (30), we have

$$\lambda \triangleq \hat{\alpha} n q_n + \frac{N_0 W}{M} = \delta \log^2 \hat{\alpha} n + \frac{N_0 W}{M}. \tag{33}$$

Hence, $\lambda \sim \log^2 \hat{\alpha} n$ goes to infinity as $n \to \infty$. Also, using the fact that $\frac{h_{ii} p_i}{\lambda} \ll 1$, when n is large, we can use the approximation $\log(1+x) \approx x$ in order to simplify (14) and (16) as follows:

$$\bar{R}_i(p_i, h_{ii}) \approx \frac{W}{M} \frac{h_{ii}}{\lambda} p_i, \tag{34}$$

$$\bar{R}(p_i) \approx \frac{\alpha W}{M} \mathbb{E}\left[\frac{h_{ll} p_l}{\beta_{il} h_{il} p_i + \lambda}\right] + (1-\alpha) \frac{W}{M\lambda} \mathbb{E}\left[h_{ll} p_l\right]. \tag{35}$$

Consequently, the utility function $u_i(p_i, h_{ii})$ is the same as the upper bound $\Xi_i(p_i, h_{ii})$ obtained in (21). Thus, the optimum power allocation strategy that maximizes $\Xi_i(p_i, h_{ii})$ is the same as the optimum power allocation policy for (5) and this completes the proof of the theorem. ■

Motivated by Theorem 1, we describe the threshold-based on-off power allocation strategy or simply on-off power scheme for single-hop wireless networks. Based on this scheme, all users perform the following steps during each block:

1- Based on the direct channel gain, the transmission policy is

$$p_i = \begin{cases} 1, & \text{if} \quad h_{ii} > \tau_n \\ 0, & \text{Otherwise}, \end{cases}$$

where τ_n is a prespecified threshold level that is a function of n and also depends on the channel model.

2- After adjusting the powers, each active user in $\mathbb{C}_j$ transmits a pilot signal with full power. The corresponding receivers measure their direct channel gains and the interference powers, and compute the rate using (2). Then, each receiver feedbacks its computed rate to the corresponding transmitter.

3- All the active users transmit data with the computed rate and with full power.

Corollary 1 *In the proposed model, if m_j is the number of active links in $\mathbb{C}_j$, then* $\mathbb{E}[m_j] = nq_n \sim \Theta\left(\log^2 n\right)$.

The on-off power scheme has the advantage of not requiring a central controller and is simple for implementation in practical time-varying networks.

IV. Optimum Spectrum Allocation

Let us consider a network with a large number of links in the system. We are interested in how the throughput of the network model of interest scales with K. In this section, we derive the network throughput in terms of M and K, in the two cases of $M \sim o(K)$ and $M \sim \Theta(K)$. Note that these two cases cover the range of $1 \leq M \leq K$. We prove that the maximum throughput of the network for every value of M and $0 \leq \alpha < 1$ is obtained at $M = 1$.

Theorem 2 *Assuming $M \sim o(K)$ and $0 < \alpha \leq 1$ is fixed, the maximum achievable throughput of the network is given by*

$$\bar{R}_{sum} \approx \frac{W}{\hat{\alpha}}(-\log q_n + O(1)) \tag{36}$$

$$= \frac{W}{\hat{\alpha}}\left(\log\frac{K}{M} + o\left(\log\frac{K}{M}\right) + O(1)\right). \tag{37}$$

Proof: See Appendix III. ■

Theorem 2 states that the throughput of the network for $M \sim o(K)$ depends on the value of $\hat{\alpha}$ and scales as $\frac{W}{\hat{\alpha}}\log\frac{K}{M}$. Also for values of M such that $\log M \sim o(\log K)$, the throughput of the network scales as $\frac{W}{\hat{\alpha}}\log K$.

Theorem 3 *Assuming $0 \leq \alpha < 1$ is fixed, the maximum achievable throughput of the network is obtained at $M = 1$.*

Proof: We prove the theorem in the following cases:

Case 1: $M \sim o(K)$ and $0 < \alpha < 1$ is fixed:

Using (30) and noting that $n = \frac{K}{M}$, the probability of the link activation is obtained in terms of M as follows:

$$q_n = \frac{\delta M}{\hat{\alpha}K} \times \left(\log\frac{\hat{\alpha}K}{M}\right)^2. \tag{38}$$

Considering q_n is a continuous function of M and taking the first-order derivative of (36) with respect to M yields,

$$\frac{\partial \bar{R}_{sum}}{\partial M} = -\frac{W}{\hat{\alpha}}\frac{\partial q_n}{\partial M}\frac{1}{q_n}.$$

Since,

$$\frac{\partial q_n}{\partial M} = \frac{\delta}{\hat{\alpha}K}\log\frac{\hat{\alpha}K}{M} \times \left(\log\frac{\hat{\alpha}K}{M} - 2\right) > 0,$$

we can conclude that $\frac{\partial \bar{R}_{sum}}{\partial M} < 0$, i.e., (36) is a monotonically decreasing function of M. Thus, the maximum throughput of the network for $M \sim o(K)$ and $0 < \alpha < 1$ is obtained at $M = 1$.

Case 2: $M \sim \Theta(K)$ and $0 < \alpha < 1$ is fixed:

Let us define $\mathbb{A}_j$ as the set of active links in cluster j. Thus, the random variable m_j is the cardinality of the set $\mathbb{A}_j$. Noting that for $M \sim \Theta(K)$, $\lim_{K\to\infty} \frac{M}{K}$ is constant, it is concluded that n and $m_j \in [1, n]$ do not grow with K. To obtain the network throughput, we assume that among M clusters, Γ clusters have $m_j = 1$ and the rest have $m_j > 1$. We first obtain an upper bound of the throughput in each cluster when $m_j = 1$, $1 \le j \le M$. Clearly, since only one user in each cluster activates its transmitter, $I_i = 0$. Thus, by using (B-1), the maximum achievable throughput of cluster $\mathbb{C}_j$ is obtained as

$$\bar{R}^{(j)}_{sum} = \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{M}{N_0 W} h_{max}\right)\right], \tag{39}$$

where $h_{max} = \max\ \{h_{ii}\}_{i\in\mathbb{C}_j}$ is a random variable. Since $\log x$ is a concave function of x, an upper bound of (39) is obtained through *Jensen's inequality*, $\mathbb{E}[\log x] \le \log(\mathbb{E}[x])$, $x > 0$. Thus,

$$\bar{R}^{(j)}_{sum} \le \frac{W}{M}\log\left(1 + \frac{M}{N_0 W}\mathbb{E}[Y]\right), \tag{40}$$

where $Y \triangleq h_{max}$. Under a Rayleigh fading channel model and noting that $\{h_{ii}\}$ is a set of i.i.d. random variables over $i \in \mathbb{C}_j$, we have

$$\begin{aligned} F_Y(y) &= \Pr\{Y \le y\}, \quad y > 0 \\ &= \prod_{i\in\mathbb{C}_j} \Pr\{h_{ii} \le y\} \\ &= \left(1 - e^{-y}\right)^n, \end{aligned}$$

where $F_Y(.)$ is the cumulative distribution function (CDF) of Y. Hence,

$$\mathbb{E}[Y] = \int_0^\infty nye^{-y}\left(1 - e^{-y}\right)^{n-1} dy.$$

Since $(1 - e^{-y})^{n-1} \le 1$, we arrive at the following inequality

$$\mathbb{E}[Y] \le \int_0^\infty nye^{-y} dy = n. \tag{41}$$

Consequently, the upper bound of (40) can be simplified as

$$\bar{R}^{(j)}_{sum} \le \frac{W}{M}\log\left(1 + \frac{K}{N_0 W}\right). \tag{42}$$

For $m_j > 1$ and due to the shadowing effect with parameters (α, ϖ), the average sum-rate of cluster $\mathbb{C}_j$ can be written as

$$\bar{R}^{(j)}_{sum} = \sum_{i\in\mathbb{A}_j} \frac{W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\sum_{k\neq i}\upsilon_k\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right], \quad k\in\mathbb{A}_j, \tag{43}$$

where υ_k's are Bernoulli random variables with parameter α. Thus,

$$\begin{aligned}\bar{R}^{(j)}_{sum} &= \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=0}^{m_j}\binom{m_j}{l}\alpha^l(1-\alpha)^{m_j-l}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\Sigma_l+\frac{N_0W}{M}}\right)\right]\\ &= \frac{W}{M}\sum_{i\in\mathbb{A}_j}(1-\alpha)^{m_j}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\frac{N_0W}{M}}\right)\right]+\\ &\quad \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=1}^{m_j}\binom{m_j}{l}\alpha^l(1-\alpha)^{m_j-l}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\Sigma_l+\frac{N_0W}{M}}\right)\right],\end{aligned} \tag{44}$$

where Σ_l is the sum of l i.i.d random variables $\{Z_i\}_{i=1}^{l}$, where $Z_i \triangleq \beta_{ki}h_{ki}$, $k \neq i$. For $m_j > 1$, Σ_l is greater than the interference term caused by one interfering link. Thus, an upper bound for the throughput of cluster j is given by

$$\begin{aligned}\bar{R}^{(j)}_{sum} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]\\ &+ \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=1}^{m_j}\binom{m_j}{l}\alpha^l(1-\alpha)^{m_j-l}\mathbb{E}\left[\log\left(1+\frac{Y}{Z_i+\frac{N_0W}{M}}\right)\right],\end{aligned} \tag{45}$$

where $Y \triangleq h_{max} = \max\ \{h_{ii}\}_{i\in\mathbb{C}_j}$. According to binomial formula, we have

$$\sum_{l=1}^{m_j}\binom{m_j}{l}\alpha^l(1-\alpha)^{m_j-l} = 1-(1-\alpha)^{m_j}.$$

Thus,

$$\begin{aligned}\bar{R}^{(j)}_{sum} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]\\ &+ \frac{W}{M}m_j\left(1-(1-\alpha)^{m_j}\right)\mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right].\end{aligned} \tag{46}$$

Defining the event $\mathscr{C} \triangleq \{\beta_{ki} < \beta_{th}\}$, we have

$$\begin{aligned}
\mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right] &= \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\middle|\mathscr{C}\right]\Pr\{\mathscr{C}\}+\\
&\quad \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\middle|\mathscr{C}^C\right]\Pr\{\mathscr{C}^C\}\\
&\overset{(a)}{\leq} \mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\middle|\mathscr{C}\right]\Pr\{\mathscr{C}\}+\\
&\quad \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{th}h_{ki}+\frac{N_0W}{M}}\right)\middle|\mathscr{C}^C\right]\Pr\{\mathscr{C}^C\}\\
&\overset{(b)}{=} \mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]\Pr\{\mathscr{C}\}+\\
&\quad \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{th}h_{ki}+\frac{N_0W}{M}}\right)\right]\Pr\{\mathscr{C}^C\}\\
&\leq \mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]\Pr\{\mathscr{C}\}+\\
&\quad \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{th}h_{ki}}\right)\right]\Pr\{\mathscr{C}^C\}, \qquad (47)
\end{aligned}$$

where $\mathscr{C}^C$ denotes the complement of $\mathscr{C}$. In the above equation, (a) follows from the fact that conditioned on $\mathscr{C}^C$, i.e., $\beta_{ki} \geq \beta_{th}$, $\beta_{ki}h_{ki} \geq \beta_{th}h_{ki}$, and (b) results from the fact that Y and h_{ki} are both independent of $\mathscr{C}$. Defining $Z \triangleq \beta_{th}h_{ki}$ and $X \triangleq \frac{Y}{Z}$, the CDF of X can be evaluated as

$$\begin{aligned}
F_X(x) &= \Pr\{X \leq x\}, \quad x > 0\\
&= \Pr\{Y \leq Zx\}\\
&= \int_0^\infty \Pr\{Y \leq Zx | Z = z\} f_Z(z)dz\\
&= \int_0^\infty \left(1-e^{-zx}\right)^n \frac{1}{\beta_{th}} e^{-\frac{z}{\beta_{th}}}dz\\
&= \int_0^\infty \left(1-e^{-t\beta_{th}x}\right)^n e^{-t}dt. \qquad (48)
\end{aligned}$$

Thus, the probability density function of X can be written as

$$\begin{aligned} f_X(x) &= \frac{dF_X(x)}{dx} \\ &= \beta_{th} \int_0^\infty nte^{-t(1+\beta_{th}x)} \left(1-e^{-t\beta_{th}x}\right)^{n-1} dt \\ &\le \beta_{th} \int_0^\infty nte^{-t(1+\beta_{th}x)} dt \\ &= \frac{n\beta_{th}}{(1+\beta_{th}x)^2}. \end{aligned} \tag{49}$$

Using the above equation, the second expectation in the last line of (47) can be upper-bounded as

$$\begin{aligned} \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{th}h_{ki}}\right)\right] &= \int_0^\infty f_X(x)\log(1+x)dx \\ &\le n\beta_{th}\int_0^\infty \frac{\log(1+x)}{(1+\beta_{th}x)^2}dx \\ &= \frac{-n\log\beta_{th}}{1-\beta_{th}}. \end{aligned} \tag{50}$$

Also, using (39)-(42), the first expectation in the last line of (47) is upper-bounded as

$$\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right] \le \log\left(1+\frac{K}{N_0W}\right). \tag{51}$$

Selecting $\beta_{th} = \frac{1}{\log K}$, noting that as $\beta_{th} \to 0$, $f_\beta(\beta)$ is upper-bounded by a constant Δ, the right hand side of (47) is upper-bounded as follows:

$$\begin{aligned} \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right] &\le \Delta\beta_{th}\log\left(1+\frac{K}{N_0W}\right)+\frac{-n\log\beta_{th}}{1-\beta_{th}} \\ &= \frac{\Delta\log\left(1+\frac{K}{N_0W}\right)}{\log K}+\frac{n\log\log K}{1-[\log K]^{-1}} \\ &\overset{(a)}{\sim} O(\log\log K), \end{aligned} \tag{52}$$

where (a) results from the fact that as $M \sim \Theta(K)$, n does not grow with K. Substituting the above equation in (46), we have

$$\begin{aligned}
\bar{R}_{sum}^{(j)} &\leq \frac{W}{M} m_j (1-\alpha)^{m_j} \mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0 W}{M}}\right)\right] + \\
& \quad \frac{W}{M} m_j \left(1-(1-\alpha)^{m_j}\right) O(\log\log K) \\
&\overset{(a)}{\leq} \frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1+\frac{K}{N_0 W}\right) + O\left(\frac{W}{M}\log\log K\right) \\
&= \frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1+\frac{K}{N_0 W}\right)\left[1+o(1)\right], \qquad (53)
\end{aligned}$$

where (a) follows from (51) and the fact that $m_j \in [1, n]$ does not scale with K.

Let us assume that among M clusters, Γ clusters have $m_j = 1$ and for the $M - \Gamma$ of the rest, the number of active links in each cluster is greater than one. Hence by using (42) and (53), an upper bound for the network throughput is obtained as

$$\bar{R}_{sum} \leq \frac{\Gamma W}{M}\log\left(1+\frac{K}{N_0 W}\right) + (M-\Gamma)\frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1+\frac{K}{N_0 W}\right)\left[1+o(1)\right]. \qquad (54)$$

To compare this upper-bounded with the computed network throughput in the case of $M = 1$, we note that as $\varpi \leq 1$ and $\alpha < 1$, we have $\hat{\alpha} < 1$ and consequently,

$$\frac{\Gamma W}{M}\log\left(1+\frac{K}{N_0 W}\right) < \frac{\Gamma W}{M\hat{\alpha}}\log\left(1+\frac{K}{N_0 W}\right).$$

Hence, in order to prove that the maximum achievable throughput obtained in (54) is less than that of $M = 1$ in (37), it is sufficient to prove

$$(M-\Gamma)\frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1+\frac{K}{N_0 W}\right) < (M-\Gamma)\frac{W}{M\hat{\alpha}}\log\left(1+\frac{K}{N_0 W}\right), \qquad (55)$$

or

$$m_j (1-\alpha)^{m_j} < \frac{1}{\hat{\alpha}}.$$

Since $\hat{\alpha} \leq \alpha$, it is sufficient to show that $m_j(1-\alpha)^{m_j} < \frac{1}{\alpha}$. Defining $\Lambda(\alpha) = \alpha m_j (1-\alpha)^{m_j}$, we have

$$\frac{\partial \Lambda(\alpha)}{\partial \alpha} = m_j (1-\alpha)^{m_j - 1}(1-\alpha-\alpha m_j).$$

Thus, the extremum points of $\Lambda(\alpha)$ are located at $\alpha = 1$ and $\alpha = \frac{1}{m_j+1}$. It is observed that

$$\Lambda(1) = 0 < 1,$$

and

$$\Lambda\left(\frac{1}{m_j+1}\right) = \left(\frac{m_j}{m_j+1}\right)^{m_j+1} < 1.$$

Since $\Lambda(\alpha) < 1$, we conclude (55), which implies that the maximum achievable network throughput for $M \sim \Theta(K)$ is less than that of $M = 1$.

Case 3: $1 \leq M \leq K$ and $\alpha = 0$:

According to the shadow-fading model proposed in (1), it is seen that for $\alpha = 0$, with probability one, $\mathcal{L}_{ki} = 0, \; k \neq i$. This implies that no interference exists in each cluster. In the absence of interference, the maximum network throughput is clearly achieved by transmitting at full power for all users in the network. Thus, from (2) and (B-1) and for every value of $1 \leq M \leq K$, the average sum-rate of cluster j for $\alpha = 0$ is simplified as

$$\bar{R}_{sum}^{(j)} = \frac{W}{M} \sum_{i \in \mathbb{C}_j} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}}{\frac{N_0 W}{M}}\right)\right]. \tag{56}$$

where the expectation is computed with respect to h_{ii}. Under a Rayleigh fading channel model, we have

$$\begin{aligned}
\bar{R}_{sum}^{(j)} &= \frac{W}{M} \sum_{i \in \mathbb{C}_j} \int_0^{\infty} e^{-x} \log(1 + \Upsilon x) dx \\
&= \frac{W}{M} n \int_0^{\infty} e^{-x} \log(1 + \Upsilon x) dx \\
&= \frac{W}{M} n e^{\frac{1}{\Upsilon}} \int_{1/\Upsilon}^{\infty} \frac{e^{-x}}{x} dx \\
&= \frac{W}{M} n e^{\frac{1}{\Upsilon}} \mathrm{E}_1\left(\frac{1}{\Upsilon}\right), \qquad (57)
\end{aligned}$$

where $\Upsilon \triangleq \frac{M}{N_0 W}$ and $\mathrm{E}_1(x) = -\mathrm{Ei}(-x) = \int_1^{\infty} \frac{e^{-tx}}{t} dt$, $x > 0$. Hence, the network throughput is obtained as

$$\begin{aligned}
\bar{R}_{sum} &= \sum_{j=1}^{M} \bar{R}_{sum}^{(j)} = \frac{KW}{M} e^{\frac{1}{\Upsilon}} \mathrm{E}_1\left(\frac{1}{\Upsilon}\right) \\
&= \frac{KW}{M} e^{\frac{N_0 W}{M}} \int_1^{\infty} \frac{e^{-t \frac{N_0 W}{M}}}{t} dt. \qquad (58)
\end{aligned}$$

Taking the first-order derivative of (58) with respect to M yields,

$$\frac{\partial \bar{R}_{sum}}{\partial M} = -\frac{KW}{M^2} e^{\frac{N_0 W}{M}} \left(1 + \frac{N_0 W}{M}\right) \mathrm{E}_1 \left(\frac{N_0 W}{M}\right) + \frac{KW}{M^2}.$$

Since for every value of $N_0 W$, $\dfrac{\partial \bar{R}_{sum}}{\partial M}$ is negative, it is concluded that the network throughput is a monotonically decreasing function of M. Consequently, for $\alpha = 0$, the maximum network throughput for every value of $1 \leq M \leq K$ is achieved at $M = 1$. ■

Note that for $M \sim \Theta(K)$, which includes $M = K$, we obtained an upper bound for $\bar{R}_{sum}$. In the next corollary, we derive an explicit expression for the network throughput when $M = K$.

Corollary 2 *Assuming $M = K$, the throughput of the network for every value of $0 \leq \alpha \leq 1$ is obtained by*

$$\bar{R}_{sum} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}), \tag{59}$$

where $\boldsymbol{\gamma}$ is Euler's constant.

Proof: See Appendix IV. ■

Corollary 3 *Note that for $M = 1$, the average number of active links scales as $\Theta(\log^2 K)$, we have total energy saving in the network in comparison with $M = K$ situation, in which all the users transmit with full power.*

So far, we have analyzed the network throughput in terms of M and α, in the asymptotic case of $K \to \infty$. For finite number of users, we evaluate the network throughput versus the number of clusters through simulation results. Fig. 1 illustrates the network throughput versus M for $K = 20$, $K = 40$, $\alpha = 0.1$ and $\varpi = 1$. From the figure, we can see that the network throughput is a monotonically decreasing function of M and the maximum value of $\bar{R}_{sum}$ is achieved at $M = 1$.

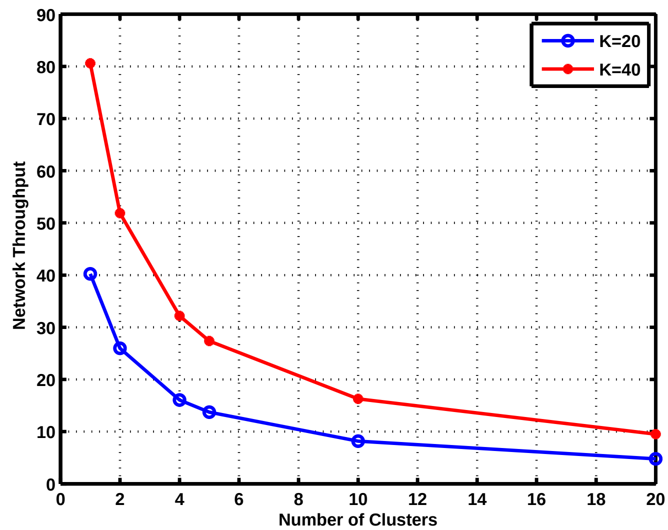

Fig. 1. Network throughput vs. M for $K = 20$, $K = 40$, $\alpha = 0.1$ and $\varpi = 1$.

V. Conclusion and Future Works

In this paper, we considered a distributed single-hop wireless network with K links, where the links are partitioned into M clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. We proved that when the number of links is large, the optimum power allocation strategy for each user is the threshold-based on-off power scheme. We also analyzed asymptotically the network throughput in terms of M and under the shadowing Rayleigh fading model described with parameters (α, ϖ). Under the on-off power scheme, it is demonstrated that for $M \sim o(K)$ and $0 < \alpha \leq 1$, where α is fixed, the network throughput scales as $\frac{W}{\hat{\alpha}} \log \frac{K}{M}$, where $\hat{\alpha} = \alpha\varpi$. For $M \sim \Theta(K)$, we have presented an upper bound for the network throughput. It is proved that the maximum network throughput for every value of $1 \leq M \leq K$, $0 \leq \alpha < 1$ and $\varpi \leq 1$ is achieved at $M = 1$. In other words, partitioning the bandwidth W into M subchannels has no gain in terms of enhancing the throughput. The results are valid for every M-dimensional orthogonal coordinates system such as time, code, etc.

Throughout this work, it is assumed that all the links use a single antenna. A possible future extension of this work would be to analyze the performance of the network with multiple antenna transmitters/receivers [6]. Also, we considered a quasi-static block fading channel model, in which the channel changes independently from block to block. It would be quite interesting to generalize the results by considering correlation between two consecutive blocks of the channel.

APPENDIX I
PROOF OF LEMMA 1

Let us define $\chi_k \triangleq \mathcal{L}_{ki} p_k$, where $\mathcal{L}_{ki}$ is independent of p_k, for $k \neq i$. Under a quasi-static Rayleigh fading channel model, it is concluded that χ_k's are the i.i.d. random variables with

$$\begin{aligned} \mathbb{E}\left[\chi_k\right] &= \mathbb{E}\left[\mathcal{L}_{ki} p_k\right] = \hat{\alpha} q_n, \\ Var\left[\chi_k\right] &= \mathbb{E}\left[\chi_k^2\right] - \mathbb{E}^2\left[\chi_k\right] = 2\alpha\kappa(\sigma_n^2 + q_n^2) - (\hat{\alpha} q_n)^2, \end{aligned}$$

where $\mathbb{E}\left[h_{ki}^2\right] = 2$, $\mathbb{E}\left[p_k\right] = q_n$ and $\hat{\alpha} = \alpha\varpi$. Also, the interference I_i is a random variable with mean μ_n and variance ϑ_n^2, where

$$\begin{aligned} \mu_n &= \mathbb{E}\left[\sum_{\substack{k \in \mathbb{C}_j \\ k \neq i}} \chi_k\right] = \hat{\alpha}(n-1) q_n, \\ \vartheta_n^2 &= Var\left[\sum_{\substack{k \in \mathbb{C}_j \\ k \neq i}} \chi_k\right] = (n-1)(2\alpha\kappa(\sigma_n^2 + q_n^2) - (\hat{\alpha} q_n)^2). \end{aligned}$$

Using the *Chebyshev inequality* [33], we obtain

$$\Pr\{|I_i - \mu_n| < \psi_n\} \geq 1 - \frac{\vartheta_n^2}{\psi_n^2},$$

for all $\psi_n > 0$. Thus,

$$\Pr\{|I_i - \hat{\alpha}(n-1) q_n| < \psi_n\} \geq 1 - \frac{(n-1)(2\alpha\kappa(\sigma_n^2 + q_n^2) - (\hat{\alpha} q_n)^2)}{\psi_n^2}.$$

It is observed that for all

$$\psi_n = \omega\left(\sqrt{(n-1)(2\alpha\kappa(\sigma_n^2+q_n^2)-(\hat{\alpha}q_n)^2)}\right),$$

we have

$$\lim_{n\to\infty} 1 - \frac{(n-1)(2\alpha\kappa(\sigma_n^2+q_n^2)-(\hat{\alpha}q_n)^2)}{\psi_n^2} = 1.$$

Thus,

$$\hat{\alpha}(n-1)q_n - \psi_n < I_i < \hat{\alpha}(n-1)q_n + \psi_n, \qquad w.\ p.\ 1.$$

By choosing $\psi_n = o(\hat{\alpha}(n-1)q_n)$, we can obtain $I_i \sim \hat{\alpha}(n-1)q_n$, w. p. 1.

APPENDIX II
PROOF OF LEMMA 2

Let us denote the average sum-rate of the links in cluster $\mathbb{C}_j$ as $\bar{R}_{sum}^{(j)}$, where

$$\bar{R}_{sum}^{(j)} \triangleq \sum_{i\in\mathbb{C}_j} \mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right]. \tag{B-1}$$

It is observed that $\bar{u}_i$, $i \in \mathbb{C}_j$, is the same as $\bar{R}_{sum}^{(j)}$. Under the on-off power allocation strategy and using $q_n = \Pr\{h_{ii} > \tau_n\}$, we have

$$\begin{aligned}
\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right] &= \mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\middle| h_{ii} > \tau_n\right]\Pr\{h_{ii} > \tau_n\} \\
&+ \mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\middle| h_{ii} \le \tau_n\right]\Pr\{h_{ii} \le \tau_n\} \\
&= q_n\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\middle| h_{ii} > \tau_n\right] + (1-q_n)\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\middle| h_{ii} \le \tau_n\right].
\end{aligned}$$

Since for $h_{ii} \le \tau_n$, $p_i = 0$, it is concluded

$$\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right] = \frac{q_n W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}}{I_i + \frac{N_0 W}{M}}\right)\middle| h_{ii} > \tau_n\right]. \tag{B-2}$$

Since, the number of links in each cluster is large, we can apply Lemma 1 to obtain

$$\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right] \approx \frac{q_n W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}}{\hat{\alpha}(n-1)q_n + \frac{N_0 W}{M}}\right)\middle| h_{ii} > \tau_n\right], \tag{B-3}$$

where the expectation is computed with respect to h_{ii}. For large values of n, we can ignore the noise power $\frac{N_0 W}{M}$. Assuming we can use the approximation $\log(1+z) \approx z - \dfrac{z^2}{2}$ for $|z| \ll 1$, we have[6]

$$\mathbb{E}\left[R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)})\right] \approx \frac{q_n W}{M}\left\{\frac{1}{\hat{\alpha} n q_n}\mathbb{E}\left[h_{ii}\middle|\, h_{ii} > \tau_n\right] - \frac{1}{2}\frac{1}{(\hat{\alpha} n q_n)^2}\mathbb{E}\left[h_{ii}^2\middle|\, h_{ii} > \tau_n\right]\right\}. \quad \text{(B-4)}$$

Under a Rayleigh fading channel model,

$$\mathbb{E}\left[h_{ii}\middle|\, h_{ii} > \tau_n\right] = 1 + \tau_n,$$

$$\mathbb{E}\left[h_{ii}^2\middle|\, h_{ii} > \tau_n\right] = \tau_n^2 + 2\tau_n + 2.$$

Using $q_n = \Pr\left\{h_{ii} > \tau_n\right\} = e^{-\tau_n}$, (B-4) can be simplified as

$$\bar{R}_{sum}^{(j)} \approx \frac{W}{\hat{\alpha} M}\left[1 + \tau_n - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha} n e^{-\tau_n}}\right]. \quad \text{(B-5)}$$

Thus, the optimization problem is

$$\tau_n^* = \arg \max_{\tau_n} \bar{R}_{sum}^{(j)}.$$

Taking the first-order derivative of (B-5) in terms of τ_n yields

$$\frac{\partial \bar{R}_{sum}^{(j)}}{\partial \tau_n} = \frac{W}{\hat{\alpha} M}\left[1 - \frac{\tau_n^2 + 4\tau_n + 4}{2\hat{\alpha} n e^{-\tau_n}}\right]. \quad \text{(B-6)}$$

Since, the second-order derivative of (B-5) is negative, the maximum value of $\bar{R}_{sum}^{(j)}$ is obtained by setting (B-6) equal to zero. So, we have

$$2\hat{\alpha} n e^{-\tau_n} = \tau_n^2 + 4\tau_n + 4,$$

or

$$\tau_n = \log 2\hat{\alpha} n - 2\log\tau_n - \log\left(1 + \frac{4\tau_n + 4}{\tau_n^2}\right). \quad \text{(B-7)}$$

It can be verified that the solution for (B-7) is

$$\tau_n^* = \log\hat{\alpha} n - 2\log\log\hat{\alpha} n + O(1). \quad \text{(B-8)}$$

Also, under the Rayleigh fading channel model, we have $q_n = \Pr\left\{h_{ii} > \tau_n\right\} = e^{-\tau_n}$. Using (B-8), it is concluded

$$q_n = \frac{\log^2 \hat{\alpha} n}{\hat{\alpha} n} \times e^{-O(1)}.$$

Setting $\delta = e^{-O(1)}$ completes the proof of the lemma.

[6]It will be shown that this assumption is valid automatically for the optimum value of q_n.

APPENDIX III
PROOF OF THEOREM 2

Using (B-5), the network throughput is obtained as

$$\begin{aligned}\bar{R}_{sum} &= \sum_{j=1}^{M} \bar{R}_{sum}^{(j)} \\ &= \frac{W}{\hat{\alpha}}\left[1+\tau_n-\frac{\tau_n^2+2\tau_n+2}{2\hat{\alpha}ne^{-\tau_n}}\right]. \qquad \text{(C-1)}\end{aligned}$$

Considering the optimum threshold level obtained in (B-7) and $\hat{\alpha}ne^{-\tau_n}=\hat{\alpha}nq_n=\delta\log^2\hat{\alpha}n$, it can be easily shown that for $M\sim o(K)$

$$1-\frac{\tau_n^2+2\tau_n+2}{2\hat{\alpha}ne^{-\tau_n}} \sim O(1).$$

Hence by using $\tau_n=-\log q_n$, we arrive at the following equation

$$\bar{R}_{sum}\approx \frac{W}{\hat{\alpha}}\left[-\log q_n+O(1)\right]. \qquad \text{(C-2)}$$

Through substituting (30) in (C-2) and using $n=\frac{K}{M}$, we finally obtain

$$\bar{R}_{sum}=\frac{W}{\hat{\alpha}}\left(\log\frac{K}{M}+o\left(\log\frac{K}{M}\right)+O(1)\right).$$

APPENDIX IV
PROOF OF COROLLARY 3

Noting that for $M=K$, only one user exists in each cluster, all the users can communicate with an interference free channel. In this regime, they can transmit with full power over the orthogonal subchannels to achieve large data rates. Hence, since $I_i=0$, for $i=1,...,K$, the throughput of the network is given by

$$\begin{aligned}\bar{R}_{sum} &= \mathbb{E}\left[\sum_{i=1}^{K} R_i(\mathbf{P}^{(j)},\boldsymbol{\mathcal{L}}^{(j)})\right] \\ &= \frac{W}{K}\sum_{i=1}^{K}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\frac{N_0W}{K}}\right)\right],\end{aligned}$$

where the expectation is computed with respect to h_{ii}. Under a Rayleigh fading channel model, we have

$$\bar{R}_{sum} = \frac{W}{K}\sum_{i=1}^{K}\int_0^\infty e^{-x}\log(1+\tilde{\Upsilon}x)dx,$$

where $\tilde{\Upsilon} \triangleq \frac{K}{N_0 W}$. Thus,

$$\begin{aligned}\bar{R}_{sum} &= W\int_0^\infty e^{-x}\log(1+\tilde{\Upsilon}x)dx \\ &= We^{\frac{1}{\tilde{\Upsilon}}}\int_{1/\tilde{\Upsilon}}^\infty \frac{e^{-x}}{x}dx \\ &= We^{\frac{1}{\tilde{\Upsilon}}}\mathrm{E}_1\left(\frac{1}{\tilde{\Upsilon}}\right). \qquad \text{(D-1)}\end{aligned}$$

To simplify (D-1), we use the following series representation for $\mathrm{E}_1(x)$,

$$\mathrm{E}_1(x) = -\boldsymbol{\gamma} - \log x + \sum_{s=1}^{\infty}\frac{(-1)^{s+1}x^s}{s.s!}, \qquad \text{(D-2)}$$

where $\boldsymbol{\gamma}$ is Euler's constant and is defined by the limit [31]

$$\boldsymbol{\gamma} = \lim_{s\to\infty}\left(\sum_{k=1}^{s}\frac{1}{k} - \log s\right) = 0.577215665...$$

Thus, the network throughput is obtained as

$$\bar{R}_{sum} = We^{\frac{1}{\tilde{\Upsilon}}}\left(-\boldsymbol{\gamma} + \log\tilde{\Upsilon} + \sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s.s!}\left(\frac{1}{\tilde{\Upsilon}}\right)^s\right).$$

For sufficiently large values of K, we have $\tilde{\Upsilon} = \frac{K}{N_0 W} \gg 1$, which results in $e^{\frac{1}{\tilde{\Upsilon}}} \approx 1$ and

$$\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s.s!}\left(\frac{1}{\tilde{\Upsilon}}\right)^s \approx 0.$$

Consequently for $M = K$, the network throughput is asymptotically obtained by

$$\bar{R}_{sum} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}).$$

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