Optimum Diversity-Multiplexing Tradeoff in the Multiple Relays Network

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Abstract

In this paper, a multiple-relay network in considered, in which K single-antenna relays assist a single-antenna transmitter to communicate with a single-antenna receiver in a half-duplex mode. A new Amplify and Forward (AF) scheme is proposed for this network and is shown to achieve the optimum diversity-multiplexing trade-off curve.

I. SYSTEM MODEL

The system , as in [1], [2], and [3], consists of K relays assisting the transmitter and the receiver in the half-duplex mode, i.e. in each time, the relays can either transmit or receive. The channels between each two node is assumed to be quasi-static flat Rayleigh-fading, i.e. the channel gains remain constant during a block of transmission and changes independently from one block to another. However, we assume that there is no direct link between the transmitter and the receiver. This assumption is reasonable when the transmitter and the receiver are far from each other or when the receiver is supposed to have connection with just the relay nodes to avoid the complexity of the network. As in [2] and [4], each node is assumed to know the state of its backward channel and, moreover, the receiver is supposed to know the equivalent channel gain from the transmitter to the receiver. No feedback to the transmitting node is permitted.

All nodes have the same power constraint. Also, we assume that a capacity achieving gaussian random codebook can be generated at each node of the network. Hence, the code design problem is not considered in this paper.

II. PROPOSED K-SLOT SWITCHING N-SUB-BLOCK MARKOVIAN SCHEME (SM)

In the proposed scheme, the entire block of transmission is divided into N sub-blocks. Each sub-block consists of K slots. Each slot has T' symbols. Hence, the entire block consists of $T = NKT'$ symbols. In order to transmit a message w, the transmitter selects the corresponding codeword of a gaussian random codebook consisting of $2^{NKT'r}$ codewords of length $\frac{NK-1}{NK}T$ and transmits the codeword during the first $NK - 1$ slots. In each sub-block, each relay receives the signal in one of the slots and transmits the received signal in the next slot. So, each relay is off in $\frac{K-2}{2}$ of time. More precisely, in the k' slot of the n'the sub-block ($1 \leq n \leq N, 1 \leq k \leq K, nk \neq NK$), the k'th relay receives the signals the transmitter is sending, and amplifies and forwards it to the receiver in the next slot. The receiver starts receiving the signal from the second slot. After receiving the last slot (NK'th slot) signal, the receiver decodes the transmitted message by using the signal of $NK - 1$ slot received from K relays. It will be shown in the next section that the equivalent point-to-point channel from the transmitter to the receiver would act as a lower-triangular MIMO channel.

III. DIVERSITY-MULTIPLEXING TRADEOFF

In this section, we show that the proposed method achieves the optimum achievable diversity-multiplexing curve. First, according to the cut-set bound theorem [5], the point-to-point capacity of the uplink channel (the channel from the transmitter to the relays) is an upper-bound for the capacity of this system. Accordingly, the diversity-multiplexing curve of a $1 \times K$ SIMO system which is a straight line from multiplexing gain 1 to the diversity gain K is an upper-bound for the diversity-multiplexing curve of our system. In this section, we prove that the tradeoff curve of the proposed method achieves the upper-bound and thus, it is optimum. First, we prove the statement for the case that there is no link between the relays. Next, we prove the statement for the general case.

A. *No Interfering Relays*

Assume, the link gain between the k'th relay and the transmitter and the k'th relay and the receiver are h_k and g_k, respectively. Furthermore, assume that there is no link between the relays. Accordingly, at the k'th relay we have

$$\mathbf{r}_k = h_k\mathbf{x} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{r}_k$ is the received signal vector of the k'th relay, $\mathbf{x}$ is the transmitter signal vector and $\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{I}_{T'})$ is the noise vector of the channel. At the receiver side, we have

$$\mathbf{y} = \sum_{k=1}^{K} g_k\mathbf{t}_k + \mathbf{z}, \tag{2}$$

where $\mathbf{t}_k$ is the transmitted signal vector of the k'th relay, $\mathbf{y}$ is the received signal vector at the receiver side and $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I}_{T'})$ is the noise vector of the downlink channel. The output power constraint $\mathbb{E}\left\{\|\mathbf{x}\|^2\right\}, \mathbb{E}\left\{\|\mathbf{t}_k\|^2\right\} \leq T'P$ holds at the transmitter and relays side. To obtain the DM tradeoff curve of the proposed scheme, we are looking for the end-to-end probability of outage from the rate $r\log(P)$, as P goes to infinity.
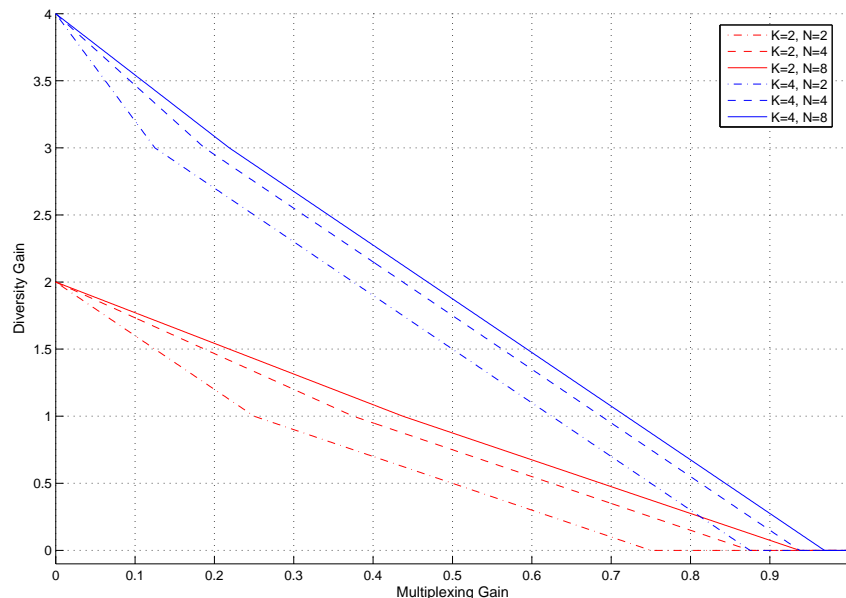

Fig. 1.   DM Tradeoff for the proposed Switching Markovian Scheme and various values of (K,N), No interfering relays case

Theorem 1 *Assume a half-duplex parallel relay scenario with K no interfering relays. The proposed SM scheme achieves the diversity gain*

$$d_{SM,NI}(r) = \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}, \tag{3}$$

which achieves the optimum achievable DM tradeoff curve $d_{opt}(r) = K(1-r)$ as $N \to \infty$.

Proof: Let us define $\mathbf{x}_{n,k}, \mathbf{n}_{n,k}, \mathbf{r}_{n,k}, \mathbf{t}_{n,k}, \mathbf{z}_{n,k}, \mathbf{y}_{n,k}$ as the signal/noise transmitted/received by the transmitter/relay/receiver to the k'th relay/receiver in the k'th slot of the n'th sub-block. Also, let us define $(k) \equiv k - 2 \mod K + 1$ and $(n) \equiv n - \lfloor \frac{(k)}{K} \rfloor$. Thus, we have

$$\begin{aligned}
\mathbf{y}_{n,k} &= g_k \mathbf{t}_{n,k} + \mathbf{z}_{n,k} \\
&= g_k \alpha_{(k)} \left(h_{(k)} \mathbf{x}_{(n),(k)} + \mathbf{n}_{(n),(k)} \right) + \mathbf{z}_{n,k}, \tag{4}
\end{aligned}$$

where $\alpha_k = \frac{P}{|h_k|^2 P + 1}$ is the amplification coefficient performed in the k'th relay. Defining the event $\mathcal{E}_k$ as the event of outage from the rate $r \log(P)$ in the k'th sub-channel consisting of the transmitter, the k'th relay, and the receiver, we have

$$\begin{aligned}
\mathbb{P}\{\mathcal{E}_k\} &= \mathbb{P}\left\{ \log\left[1 + P|g_k|^2|\alpha_k|^2|h_k|^2 \left(1 + |g_k|^2|\alpha_k|^2\right)^{-1}\right] \leq r \log(P) \right\} \\
&\doteq \min\left\{ \operatorname{sign}(r), \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \left(1 + |g_k|^2|\alpha_k|^2\right)^{-1} \leq P^{r-1} \right\} \right\} \\
&\overset{(a)}{\doteq} \min\left\{ \operatorname{sign}(r), \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \min\left\{ \frac{1}{2}, \frac{1}{2|g_k|^2|\alpha_k|^2} \right\} \leq P^{r-1} \right\} \right\} \\
&\overset{(b)}{\doteq} \min\left\{ \operatorname{sign}(r), \mathbb{P}\left\{ |h_k|^2 \leq 2P^{r-1} \right\} + \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \leq 2P^{r-1} \right\} \right\} \\
&\overset{(c)}{\doteq} \min\left\{ \operatorname{sign}(r), P^{-(1-r)} + \mathbb{P}\left\{ |g_k|^2 \min\left\{ \frac{1}{2}, \frac{|h_k|^2 P}{2} \right\} \leq 2P^{r-1} \right\} \right\} \\
&\overset{(d)}{\doteq} \min\left\{ \operatorname{sign}(r), P^{-(1-r)} + \mathbb{P}\left\{ |g_k|^2 \leq 4P^{r-1} \right\} + \mathbb{P}\left\{ |g_k|^2|h_k|^2 \leq 4P^{r-2} \right\} \right\} \\
&\overset{(e)}{\doteq} \min\left\{ \operatorname{sign}(r), P^{-(1-r)} \right\}, \tag{5}
\end{aligned}$$

where $\operatorname{sign}(r)$ is the sign function, i.e. $\operatorname{sign}(r) = 1, r \geq 0, \operatorname{sign}(r) = 0, r < 0$. Here, (a) follows from the fact that $\frac{1}{1+|g_k|^2|\alpha_k|^2} \doteq \min\left\{ \frac{1}{2}, \frac{1}{2|g_k|^2|\alpha_k|^2} \right\}$, (b) and (d) follow from the union bound inequality, (c) follows from the fact that $|\alpha_k|^2|h_k|^2 \doteq \min\left\{ \frac{1}{2}, \frac{|h_k|^2 P}{2} \right\}$ and the pdf distribution of the rayleigh-fading parameter near zero, and (e) follows from the fact that the product of two independent rayleigh-fading parameters behave as a rayleigh-fading parameter near zero.

(5) shows that each sub-channel's tradeoff curve performs as a single-antenna point-to-point channel.

Defining $R_k(P)$ as the random variable showing the rate of the k'th sub-channel consisting of the transmitter, the k'th relay, and the receiver in terms of P, the outage event of the entire channel from the $r \log(P)$, the event $\mathcal{E}$, is equal to

$$\mathbb{P}\{\mathcal{E}\} = \mathbb{P}\left\{N \sum_{k=1}^{K-1} R_k(P) + (N-1)R_K(P) \leq NKr \log(P)\right\} \tag{6}$$

Assuming $R_k(P) = r_k \log(P)$, we have

$$\mathbb{P}\{\mathcal{E}\} \doteq \mathbb{P}\left\{N \sum_{k=1}^{K-1} r_k + (N-1)r_K \leq NKr\right\} \tag{7}$$

$\mathbb{P}\{R_k(P) \leq r_k \log(P)\}$ is known by (5). Defining the region $\mathcal{R}$ as

$$\mathcal{R} = \left\{(r_1, r_2, \cdots, r_K) \,|\, 0 \leq r_k \leq 1, N \sum_{k=1}^{K-1} r_k + (N-1)r_K \leq NKr\right\} \tag{8}$$

it is easy to check that all the vectors $(r_1, r_2, \cdots, r_K)$ that result in the outage event almost surely lie in $\mathcal{R}$. In fact, according to (5), for all k we know $r_k \geq 0$. Also, for $r_k > 1$, $\mathbb{P}\{R_k(P) \geq r_k \log(P)\} \leq e^{-P^{r-1}}$ which is exponential in terms of P. Hence, $r_k > 1$ can be disregarded for the outage region. As a result, $\mathbb{P}\{\mathcal{E}\} \doteq \mathbb{P}\{\mathbf{r} \in \mathcal{R}\}$.

On the other hand, by (5) and the fact that r_k's are independent, we have

$$\mathbb{P}\left\{r_1 \leq r_1^0, r_2 \leq r_2^0, \cdots, r_K \leq r_K^0\right\} \doteq P^{-\left(K - \sum_{k=1}^{K} r_k^0\right)} \tag{9}$$

Now, we show that $\mathbb{P}\{\mathcal{E}\} \doteq P^{-\min_{\mathbf{r} \in \mathcal{R}} K - \mathbf{1} \cdot \mathbf{r}}$. First of all, by taking derivative of (9) with respect to $r_1, r_2, \cdots, r_K$, it is easy to see that the probability density function of $\mathbf{r}$ behaves the same as the probability function in (9), i.e. $f_r(\mathbf{r}) \doteq P^{-(K - \mathbf{1} \cdot \mathbf{r})}$. Hence, the outage probability is equal to

$$\begin{aligned} \mathbb{P}\{\mathcal{E}\} &\doteq \int_{\mathbf{r} \in \mathcal{R}} f_r(\mathbf{r}) d\mathbf{r} \\ &\stackrel{.}{\leq} vol(\mathcal{R}) P^{-\min_{\mathbf{r} \in \mathcal{R}} K - \mathbf{1} \cdot \mathbf{r}} \\ &\stackrel{(a)}{\doteq} P^{-\min_{\mathbf{r} \in \mathcal{R}} K - \mathbf{1} \cdot \mathbf{r}} \end{aligned} \tag{10}$$

Here, (a) follows from the fact that $\mathcal{R}$ is a fixed bounded region whose volume is independent of P. On the other hand, by continuity of $P^{-(K - \mathbf{1} \cdot \mathbf{r})}$ over $\mathbf{r}$, we have $\mathbb{P}\{\mathcal{E}\} \stackrel{.}{\geq} P^{-\min_{\mathbf{r} \in \mathcal{R}} K - \mathbf{1} \cdot \mathbf{r}}$ which combining with (10), results into $\mathbb{P}\{\mathcal{E}\} \doteq P^{-\min_{\mathbf{r} \in \mathcal{R}} K - \mathbf{1} \cdot \mathbf{r}}$. Defining $l(\mathbf{r}) = K - \mathbf{1} \cdot \mathbf{r}$, we

have to solve the following linear programming optimization problem $\min_{\mathbf{r} \in \mathcal{R}} l(\mathbf{r})$. Notice that the region $\mathcal{R}$ is defined by a set of linear inequality constraints. To solve the problem, we have

$$
\begin{aligned}
l(\mathbf{r}) &\overset{(a)}{\geq} \max\left\{0, K - \frac{NKr + r_K}{N}, K - \frac{NKr - \sum_{k=1}^{K-1} r_k}{N-1}\right\} \\
&\overset{(b)}{\geq} \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}.
\end{aligned}
\tag{11}
$$

Here, (a) follows from the inequality constraint in (8) governing $\mathcal{R}$, and (b) follows from the fact that $r_K \leq 1$ and $\forall k < K : r_k \geq 0$. Now, we partition the range $0 \leq r \leq 1$ into three intervals. First, in the case that $r > 1 - \frac{1}{NK}$, the feasible point $\mathbf{r} = \mathbf{1}$ achieves the lower bound 0. Second, in the case that $r < \frac{1}{K} - \frac{1}{NK}$, the feasible point $\mathbf{r} = \left(0, 0, \cdots, 0, \frac{NKr}{N-1}\right)$, achieves the lower bound $K(1-r) - \frac{Kr}{N-1}$. Finally, in the case that $\frac{1}{K} - \frac{1}{NK} \leq r \leq 1 - \frac{1}{NK}$, The lower bound $K(1-r) - \frac{1}{N}$ is achievable by the feasible point $\mathbf{r}, \forall k < K : r_k = \frac{NKr - N + 1}{N(K-1)}, r_K = 1$. Hence, we have $\min_{\mathbf{r} \in \mathcal{R}} l(\mathbf{r}) = \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}$. This completes the proof. ∎

Remark - It is worth noting that as long as the graph $G(V, E)$ whose vertices are the relay nodes and edges are the non interfering relay node pairs includes a hamiltonian cycle [1], the result of this subsection remains valid.

B. General Case

In the general case, an interference term due to the neighboring relay adds at the receiver antenna of each relay.

$$
\mathbf{r}_k = h_k \mathbf{x} + i_{(k)} \mathbf{t}_{(k)} + \mathbf{n}_k,
\tag{12}
$$

where $i_{(k)}$ is the interference link gain between the k'th and (k)'th relays. Hence, the amplification coefficient is bounded as $\alpha_k \leq \frac{P}{P\left(|h_k|^2 + |i_{(k)}|^2\right) + 1}$. Here, we observe that in the case that $\alpha_k > 1$, the noise n_k at the receiving side of the k'th relay can be boosted at the receiving side of the next relay. Hence, we bound the amplification coefficient as $\alpha_k = \min\left\{1, \frac{P}{P\left(|h_k|^2 + |i_{(k)}|^2\right) + 1}\right\}$. In this way, it is guaranteed that the noise of relays are not boosted up through the system. This is at the expense working with the output power less than P. On the other hand, we know that almost surely [2] $|h_k|^2, |i_{(k)}|^2 \dot{\leq} 1$. Hence, almost surely we have $\alpha_k \doteq 1$. Another change we make

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[1]By hamiltonian cycle, we mean a simple cycle $v_1 v_2 \cdots v_K v_1$ that goes exactly one time through each vertex of the graph.

[2]By almost surely, we mean its probability is greater than $1 - P^{-\delta}$, for all values of δ.

in this part is that we assume that the entire time of transmission consists of $NK + 1$ slots, and the transmitter sends the data during the first NK slots while the relays send in the last NK slots (from the second slot up to the $NK + 1$'th slot). Hence, we have $T = (NK + 1)T'$. This assumption makes our analysis easier and the lower bound on the diversity curve tighter. Now, we prove the main theorem of this section.

Theorem 2 *Consider a half-duplex multiple relays scenario with K interfering relays whose gains are independent rayleigh fading variables. The proposed SM scheme achieves the diversity gain*

$$d_{SM,I}(r) \geq \max \left\{ 0, K \left(1 - r \right) - \frac{r}{N} \right\}, \tag{13}$$

which achieves the optimum achievable DM tradeoff curve $d_{opt}(r) = K(1 - r)$ as $N \to \infty$.

Proof: First, we show that the entire channel matrix acts as a lower triangular matrix. At the receiver side, we have

$$
\begin{aligned}
\mathbf{y}_{n,k} &= g_k \mathbf{t}_{n,k} + \mathbf{z}_{n,k} \\
&= g_k \alpha_{(k)} \left(\sum_{0 < n_1, k_1, n_1(K+1) + k_1 < n(K+1) + k} p_{n-n_1, k, k_1} \left(h_{k_1} \mathbf{x}_{n_1, k_1} + \mathbf{n}_{n_1, k_1} \right) \right) + \mathbf{z}_{n,k} \quad (14)
\end{aligned}
$$

Here, p_{n,k,k_1} has the following recursive formula $p_{0,k,k} = 1, p_{n,k,k_1} = i_{(k)}\alpha_{(k)}p_{(n),(k),k_1}$. Defining the square $NK \times NK$ matrices as $\mathbf{G} = \mathbf{I}_N \otimes diag\{g_1, g_2, \cdots, g_K\}$, $\mathbf{H} = \mathbf{I}_N \otimes diag\{h_1, h_2, \cdots, h_K\}$, $\mathbf{\Omega} = \mathbf{I}_N \otimes diag\{\alpha_1, \alpha_2, \cdots, \alpha_K\}$, and

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ p_{0,2,1} & 1 & 0 & 0 & \cdots \\ p_{0,3,1} & p_{0,3,2} & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ p_{N-1,K,1} & p_{N-1,K,2} & \cdots & p_{0,K,K-1} & 1 \end{pmatrix}, \tag{15}$$

where $\otimes$ is the Kronecker product [6] of matrices and $\mathbf{I}_N$ is the $N \times N$ identity matrix, and the $NK \times 1$ vectors $\mathbf{x}(s) = [x_{1,1}(s), x_{1,2}(s), \cdots, x_{N,K}(s)]^T$, $\mathbf{n}(s) = [n_{1,1}(s), n_{1,2}(s), \cdots, n_{N,K}(s)]^T$, $\mathbf{z}(s) = [z_{1,2}(s), z_{1,3}(s), \cdots, z_{N+1,1}(s)]^T$, and $\mathbf{y}(s) = [y_{1,2}(s), y_{1,3}(s), \cdots, y_{N+1,1}(s)]^T$, we have

$$\mathbf{y}(s) = \mathbf{G}\mathbf{\Omega}\mathbf{F}(\mathbf{H}\mathbf{x}(s) + \mathbf{n}(s)) + \mathbf{z}(s). \tag{16}$$

Here, we observe that the matrix of the entire channel acts as a lower triangular matrix of a $NK \times NK$ MIMO channel whose noise is colored. The probability of outage of such a channel for the multiplexing gain r is defined as

$$\mathbb{P}\left\{\mathcal{E}\right\} = \mathbb{P}\left\{\log\left|\mathbf{I}_{KN} + P\mathbf{H}_T\mathbf{H}_T^H\mathbf{P}_n^{-1}\right| \leq (NK+1)r\log(P)\right\}, \tag{17}$$

where $\mathbf{P}_N = \mathbf{I}_{NK} + \mathbf{G}\mathbf{\Omega}\mathbf{F}\mathbf{F}^H\mathbf{\Omega}^H\mathbf{G}^H$, and $\mathbf{H}_T = \mathbf{G}\mathbf{\Omega}\mathbf{F}\mathbf{H}$. Assume $|h(k)|^2 = P^{-\mu(k)}$, $|g(k)|^2 = P^{-\nu(k)}$, $|i(k)|^2 = P^{-\omega(k)}$, and $\mathcal{R}$ as the region in $\mathbb{R}^{3K}$ that defines the outage event $\mathcal{E}$ in terms of the vector $[\mu, \nu, \omega]$, where $\mu = [\mu(1)\mu(2)\cdots\mu(K)]^T$, $\nu = [\nu(1)\nu(2)\cdots\nu(K)]^T$, $\omega = [\omega(1)\omega(2)\cdots\omega(K)]^T$. The probability distribution function (and also the inverse of cumulative distribution function) decays exponentially as $P^{-P^{-\delta}}$ for positive values of δ. Hence, the outage region $\mathcal{R}$ is almost surely equal to $\mathcal{R}_+ = \mathcal{R}\bigcap\mathbb{R}_+^{3K}$. Now, we have

$$\mathbb{P}\left\{\mathcal{E}\right\} \overset{(a)}{\leq} \mathbb{P}\left\{|\mathbf{H}_T|^2|\mathbf{P}_n|^{-1} \leq P^{-NK(1-r)+r}\right\}$$

$$\overset{(b)}{\leq} \mathbb{P}\left\{-N\sum_{k=1}^{K}\mu(k) + \nu(k) - \min\left\{0, \mu(k), \omega((k))\right\} + \frac{NK\log(3) - \log|\mathbf{P}_N|}{\log(P)} \leq -NK(1-r) + r\right\}$$

$$\overset{(c)}{\leq} \mathbb{P}\left\{NK\frac{\log(3) - \log(N^2K^2+1)}{\log(P)} + NK(1-r) - r \leq N\sum_{k=1}^{K}\mu(k) + \nu(k),\right.$$

$$\left.\mu(k), \nu(k), \omega(k) \geq 0\right\}. \tag{18}$$

Here, (a) follows from the fact that for a positive semidefinite matrix $\mathbf{A}$ we have $|\mathbf{I} + \mathbf{A}| \geq |\mathbf{A}|$, (b) follows from the fact that

$$\alpha(k) = \min\left\{1, \frac{P}{P^{1-\mu(k)} + P^{1-\omega((k))} + 1}\right\} \geq \frac{1}{3}\min\left\{1, P, P^{\mu(k)}, P^{\omega((k))}\right\}$$

and assuming P is large enough such that $P \geq 1$, and (c) follows from the fact that $\alpha(k) \leq 1$ and accordingly, $p_{n,k,k_1} \leq 1$, and knowing that the sum of the entries of each row in $\mathbf{F}\mathbf{F}^H$ is less than N^2K^2, we have[3] $\mathbf{F}\mathbf{F}^H \preccurlyeq N^2K^2\mathbf{I}_{NK}$, and $\mathbb{P}\left\{\mathcal{R}\right\} \doteq \mathbb{P}\left\{\mathcal{R}_+\right\}$, and conditioned on $\mathcal{R}_+$, we have $\min\left\{0, \mu(k), \omega((k))\right\} = 0$ and $\nu(k) \geq 0$ and consecutively $\mathbf{P}_N \preccurlyeq (N^2K^2+1)\mathbf{I}_{KN}$.

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[3]This can be verified by the fact that every symmetric real matrix $\mathbf{A}$ which has the property that for every i, $a_{i,i} \geq \sum_{i \neq j}|a_{i,j}|$ is positive semidefinite.

On the other hand, we know for vectors $\mu^0, \nu^0, \omega^0 \geq \mathbf{0}$, we have $\mathbb{P}\{\mu \geq \mu^0, \nu \geq \nu^0, \omega \geq \omega^0\} \doteq$ $P^{-\mathbf{1}\cdot(\mu^0+\nu^0+\omega^0)}$. Similarly to the proof of Theorem 1, by taking derivative with respect to μ, ν we have $f_{\mu,\nu}(\mu, \nu) \doteq P^{-\mathbf{1}\cdot(\mu+\nu)}$. Defining the lower bound l_0 as $l_0 = \frac{\log(3)-\log(N^2K^2+1)}{\log(P)}+(1-r)-\frac{r}{NK}$, the new region $\hat{\mathcal{R}}$ as $\hat{\mathcal{R}} = \{\mu, \nu \geq \mathbf{0}, \frac{1}{K}\mathbf{1}\cdot(\mu+\nu) \geq l_0\}$, the cube $\mathcal{I}$ as $\mathcal{I} = [0, Kl_0]^{2K}$, and for $1 \leq i \leq 2K$, $\mathcal{I}_i^c = [0,\infty)^{i-1} \times [Kl_0, \infty) \times [0,\infty)^{2K-i}$, we observe

$$
\begin{aligned}
\mathbb{P}\{\mathcal{E}\} &\overset{(a)}{\leq} \mathbb{P}\{\hat{\mathcal{R}}\} \\
&\leq \int_{\hat{\mathcal{R}} \cap \mathcal{I}} f_{\mu,\nu}(\mu,\nu)\,d\mu d\nu + \sum_{i=1}^{2K} \mathbb{P}\left\{[\mu,\nu] \in \hat{\mathcal{R}} \cap \mathcal{I}_i^c\right\} \\
&\overset{\cdot}{\leq} vol(\hat{\mathcal{R}} \cap \mathcal{I}) P^{-\min_{[\mu^0,\nu^0]\in\hat{\mathcal{R}}\cap\mathcal{I}} \mathbf{1}\cdot(\mu^0+\nu^0)} + 2KP^{-Kl_0} \\
&\overset{(b)}{\doteq} P^{-Kl_0} \\
&\doteq P^{-\left[K(1-r)-\frac{r}{N}\right]}.
\end{aligned}
\tag{19}
$$

Here, (a) follows from (18) and (b) follows from the fact that $\hat{\mathcal{R}} \bigcap \mathcal{I}$ is a bounded region whose volume is independent of P. (19) completes the proof. ∎
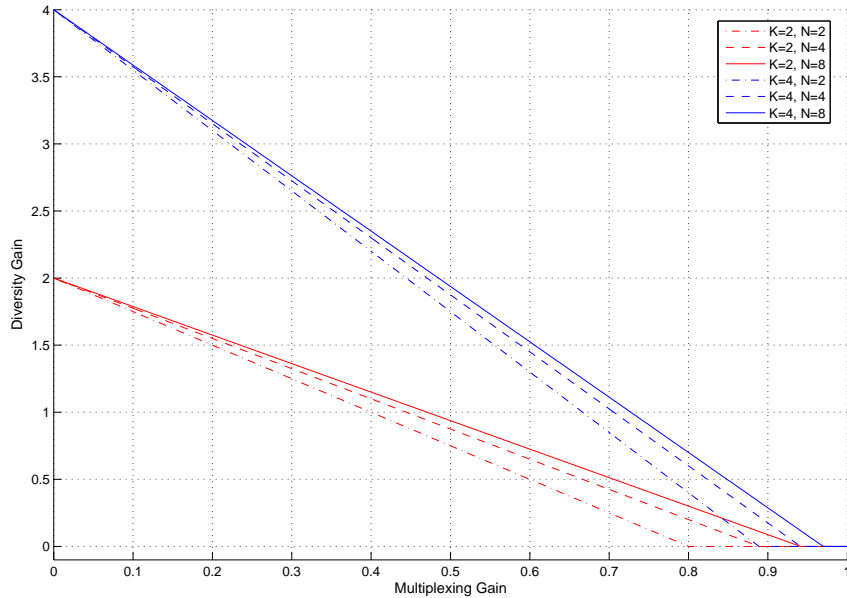

Fig. 2. DM Tradeoff for the proposed Switching Markovian Scheme and various values of (K,N), Interfering relays case

Remark - The statement in the above theorem holds for the general case in which any arbitrary set of relay pairs are non-interfering. Hence, the proposed scheme achieves the upper-bound of the tradeoff curve in the asymptotic case of $N \to \infty$ for any graph topology on the interfering relay pairs.

Figure (2) shows the D-M tradeoff curve of the scheme for the case of interfering relays and varying number of K and N.

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