On the Delay-Throughput Tradeoff in Multi-User Wireless Networks

by

Jamshid Abouei, Alireza Bayesteh, Masoud Ebrahimi and Amir K. Khandani

Electrical and Computer Engineering Department
University of Waterloo
Waterloo, Ontario, Canada
Email: {jabouei, alireza, masoud, khandani}@cst.uwaterloo.ca

Technical Report UW-ECE #2007-20

August 14, 2007

On the Delay-Throughput Tradeoff in Multi-User Wireless Networks

Jamshid Abouei, Alireza Bayesteh, Masoud Ebrahimi and Amir K. Khandani

Abstract

An asymptotic analysis for the delay-throughput of a single-hop wireless network with K links, operating in bandwidth W, is considered. The links are assumed to be partitioned into M clusters, each operating in a subchannel with bandwidth $\frac{W}{M}$. The analysis relies basically on the distributed *on-off power allocation strategy* proposed in [1] and [2]. Our analysis consists of two parts. The first part deals with the throughput of the network in terms of M and under the shadowing effect with probability α. Assuming the Rayleigh fading channel model, it is proved that the maximum achievable throughput of the network for every value of $1 \leq M \leq K$ and $0 \leq \alpha \leq 1$ is obtained at $M = 1$. In the second part, we present the delay characteristics of the underlying network. It is proved that for $M \sim o(K)$ and $0 < \alpha \leq 1$, where α is fixed, the delay threshold that makes the dropping probability of the link tend to zero, while achieving the maximum throughput, scales as $\omega(\frac{n}{\log^2 n})$, where $n = \frac{K}{M}$. We also present the similar arguments for the minimum delays in each cluster and the whole network. An asymptotic analysis shows that the delay improves without any significant impact on the the throughput.

Index Terms

Sum-rate maximization, delay-throughput tradeoff, dropping probability, shadow-fading.

This work is financially supported by Communications and Information Technology Ontario (CITO), Nortel Networks, and Natural Sciences and Engineering Research Council of Canada (NSERC).

The authors are affiliated with the Coding and Signal Transmission Laboratory, Electrical and Computer Engineering Department, University of Waterloo, Waterloo, ON, N2L 3G1, Canada, Tel: 519-884-8552, Fax: 519-888-4338, Emails: {jabouei, alireza, masoud, khandani}@cst.uwaterloo.ca.

I. Introduction

The main challenge in multi-user wireless networks originates from sharing a common transmission bandwidth by users such that the network throughput is maximized. Effective resource allocation schemes such as spectral sharing and power allocation have long been regarded as efficient tools to mitigate the interference and improve the throughput of the network with limited bandwidth [3]–[10]. In [9], the authors provide a comprehensive survey in the area of resource allocation, in particular in the context of channel assignments for various wireless networks. Etkin and Tse [4] develop power and spectrum allocation strategies in multiple wireless systems. Under strong interference, they show that frequency division multiplexing (FDM) is the optimal scheme in the sense of the throughput maximization. Effective resource allocation schemes should also satisfy quality of service (QoS) requirements such as low transmission delay in buffer-limited networks and fairness for the users. Particularly, for backlogged users with real-time services (e.g., interactive games, live sport videos, etc), too much delay results in dropping some packets. Therefore, the main challenge in wireless networks with real-time services is to utilize efficient resource allocation schemes such that the delay is minimized while achieving a high throughput.

The throughput maximization of cellular and multihop wireless networks has been extensively studied in [11]–[15]. In these works, no delay analysis is performed. However, it is shown that the high throughput is achieved at the cost of a high amount of delay. This problem has motivated the researchers to study the relation between the delay characteristics and the throughput. Particularly, in most recent literature [16]–[23], the tradeoffs between delay and throughput have been investigated as a key measure of the network performance. Sharif and Hassibi [21] analyze the delay characteristics and the throughput in a broadcast channel. They propose an algorithm to reduce the delay without too much degradation in the throughput. The first studies on achieving a high throughput and low delay in ad-hoc wireless networks are framed in [16] and [17]. This line of work is further expanded in [18]–[20] by using different mobility models. El Gamal *et al.* [18] analyze the optimal delay-throughput scaling for some wireless network topologies. In the static random network with n nodes, they prove that the optimal tradeoff between throughput T_n and delay D_n is given by $D_n = \Theta(nT_n)$. They also show that the same result is achieved in random mobile networks, when $T_n = O(1/\sqrt{n \log n})$. Neely and Modiano [20] consider the delay-throughput tradeoff only for mobile ad-hoc networks and under the assumption of the redundant packets transmission through multiple paths.

In this paper, we address the delay-throughput analysis of a single-hop wireless network, in which K links operating in bandwidth W are partitioned into M clusters. Each cluster operates in a subchannel with bandwidth $\frac{W}{M}$. The analysis relies basically on the distributed *on-off power allocation strategy* proposed in [1] and [2], in which the transmission policy for link i is to compare its own channel gain with the prespecified threshold level τ_n. In [1] and [2], the authors study the performance of the network only for $M = 1$ and under a Rayleigh-fading channel model. Also in these works, no delay analysis is considered. It is well-known, however, that the wireless channel can be modeled in a more realistic manner. Here, we consider the shadowing effect that are caused by obstacles. This paper consists of two parts. The main contribution of the first part is to determine the maximum achievable throughput of the network in terms of different values of M and the probability of the shadowing effect denoted by α. Our strategy differs from the model studied in [15] and [24]; primarily we use a distributed on-off power allocation scheme for a single-hop wireless network with M disjoint subchannels, while [15] and [24] present an ad-hoc network model with random connections for $M = 1$ and using relay nodes. Under a Rayleigh fading channel condition, an asymptotic analysis is carried out to show that the maximum throughput of the network for every value of $1 \leq M \leq K$ and $0 \leq \alpha \leq 1$ is achieved at $M = 1$.

To study the delay-throughput tradeoff, we further provide a definition of the transmission delay. We show that the delay depends on the number of clusters, M, and is critical for $M \sim o(K)$. Also, we present the delay characteristics from the link, cluster and the whole network points of view. Our method is different from the delay analysis with the ON/OFF Bernoulli scheme in [25]; primarily we utilize a distributed approach with local information, while [25] considers a central controller to study the channel conditions of all the links in the system. We use a homogeneous network with *quasi-static block fading* without path loss. This differs from the geometric models proposed in [18]–[20] that are based on the distance between the source and the destination.

For $M \sim o(K)$, it is shown that increasing the number of links gives rise to increasing the network throughput, at the cost of increasing the delay. This results in higher packet droppings in real-time applications with limited buffer sizes. We derive the minimum delays in order to make the dropping probabilities of the link, cluster and the whole network tend to zero. Also, we address the question: How can we achieve a better delay performance without sacrificing too much the throughput? It is demonstrated that the tradeoff between the delay and the network throughput is strongly influenced by the threshold level τ_n that depends on the channel model. We show that by relaxing the value of τ_n, the delay is improved without changing the order of the throughput. We further present a new definition of the throughput for any buffer size. Preliminary results of this paper appear in [26] and [27].

The rest of the paper is organized as follows. In Section II, the network model and objectives are described. We analyze the throughput of the network in terms of M and α in Section III. The delay characteristics in terms of the dropping probability are analyzed in Section IV. In Section V, we establish the delay-throughput tradeoff for the network. Finally, in Section VI, an overview of the results and conclusions is presented, and directions for ongoing and future research are mentioned.

Knuth's order notation [28]: For any functions $f(n)$ and $g(n)$:

- $f(n) = O(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| < \infty$.
- $f(n) = o(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = 0$.
- $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = \infty$.

- $f(n) = \Omega(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| > 0$.
- $f(n) = \Theta(g(n))$ means that $\lim_{n\to\infty} |f(n)/g(n)| = c$, where $0 < c < \infty$.
- $f(n) \sim g(n)$ means that $\lim_{n\to\infty} f(n)/g(n) = 1$.

Also throughout the paper, we use $\mathbb{N}_n$ for representing the set $\{1, 2, ..., n\}$ and $\log(.)$ as the natural logarithm function.

II. Network Model and Objectives

A. Network Model

In this work, we consider a single-hop wireless network consisting of K pairs of nodes[1], operating in bandwidth W. The links are assumed to be partitioned into M clusters such that the number of links in each cluster is the same. Also, the links are randomly divided among the clusters. The bandwidth W is divided into M disjoint subchannels, each with bandwidth $\frac{W}{M}$. Letting $\mathcal{C}_j$ denote cluster j, the links in $\mathcal{C}_j$ operate in subchannel j. In this work, we assume that M is a variable parameter in the range of 1 to K. We also assume the number of links in each cluster, $n = \frac{K}{M}$, is a known information for the users. All the nodes in the network are assumed to have a single antenna.

The channel model considered in this paper includes Rayleigh fading along with the shadowing effect. The channel gain between transmitter k and receiver i in $\mathcal{C}_j$ is represented by the random variable $\mathcal{L}_{ki}^{(j)}$. Under a Rayleigh fading channel model, $\mathcal{L}_{ki}^{(j)} = h_{ii}^{(j)}$, for $k = i$. Also for $k \neq i$, the cross-channel gains are defined based on the shadowing model as follows[2]

$$\mathcal{L}_{ki}^{(j)} = \begin{cases} \beta h_{ki}^{(j)}, & \text{with probability} \quad \alpha \\ 0, & \text{with probability} \quad 1 - \alpha. \end{cases} \tag{1}$$

[1]The term “pair” is used to describe the transmitter and the related receiver, and “user” only for the transmitter.

[2]It is worth to mention that the superscript j means that the channel gains belong to cluster j, and it does not mean that the fading channel model is frequency-selective.

where $0 \leq \alpha \leq 1$ is a fixed parameter, and β is a random variable with $\mathbb{E}[\beta] \triangleq \varpi \leq 1$ and $\mathbb{E}[\beta^2] = \kappa$. This definition is a general model of a shadow fading environment. The channel is supposed to be quasi-static block fading, where the channel strength $h_{ki}^{(j)} \triangleq |g_{ki}^{(j)}|^2$ remains constant while transmitting one block and changes independently from block to block. Under a Rayleigh fading channel, $h_{ki}^{(j)}$'s are exponentially distributed with unit mean. We also assume that the channel is flat fading. In other words, all the subchannels are assumed to be constant over the whole bandwidth W. We also assume that each receiver knows only its direct channel gain. This channel-state information (CSI) is fed back to the corresponding transmitter without any error.

In the network model of interest, we assume that all the links utilize the on-off power allocation strategy. Based on this scheme, the average transmit power of user i is assumed to be $p_i \in \{0, 1\}$. The power of additive white Gaussian noise (AWGN) at each receiver is assumed to be $\frac{N_0 W}{M}$. Since the maximum transmit power is one, the quantity $\frac{N_0 W}{M}$ is equivalent to $\frac{1}{SNR}$, where SNR is the signal-to-noise ratio. Assuming Gaussian signal transmission, the interference term in each cluster will be Gaussian with power

$$I_i^{(j)} = \sum_{\substack{k \neq i \\ k=1}}^{n} \mathcal{L}_{ki}^{(j)} p_k, \qquad i, k \in \mathcal{C}_j.$$

Due to the orthogonality of the allocated subchannels, no interference is imposed from links in $\mathcal{C}_k$ on links in $\mathcal{C}_j$, $k \neq j$. Under these assumptions, the achievable data rate of each link is expressed as

$$R_i^{(j)} = \frac{W}{M} \log \left(1 + \frac{h_{ii}^{(j)} p_i}{I_i^{(j)} + \frac{N_0 W}{M}} \right), \quad i \in \mathbb{N}_n. \tag{2}$$

Definition 1 (Network Throughput): In order to analyze the performance of the network, we define the network throughput as the *average sum-rate*. Letting $\bar{R}_{sum}$ denote the average sum-rate of the network, we have

$$\bar{R}_{sum} = \sum_{j=1}^{M} \bar{R}_{sum}^{(j)}, \tag{3}$$

where $\bar{R}_{sum}^{(j)}$ is the average sum-rate of the links in cluster $\mathcal{C}_j$ and is given by

$$\begin{aligned}\bar{R}_{sum}^{(j)} &= \mathbb{E}\left[\sum_{i=1}^{n} R_i^{(j)}\right] \\ &= \sum_{i=1}^{n} \mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ii}^{(j)}p_i}{I_i^{(j)}+\frac{N_0 W}{M}}\right)\right],\end{aligned}$$

where the expectation is computed with respect to $h_{ii}^{(j)}$ and $I_i^{(j)}$.

B. On-Off Power Allocation Strategy

We consider a homogeneous network in the sense that all the links have the same configurations and use the same protocols. Thus, the transmission strategy for all the users are agreed in advance. In this paper, we assume that all the links perform the on-off power allocation strategy proposed in [1] and [2]. Based on this scheme, all users in each cluster perform the following steps during each block:

1- Based on the direct channel gain, the transmission policy is

$$p_i = \begin{cases} 1, & \text{if } \; h_{ii}^{(j)} > \tau_n \\ 0, & \text{Otherwise,} \end{cases}$$

where τ_n is a prespecified threshold level that is a function of n and also depends on the channel model.

2- After adjusting the powers, each active user in $\mathcal{C}_j$ transmits a pilot signal with full power. All the receivers in $\mathcal{C}_j$ measure the interference and compute the rate using (2). Then, each receiver feedbacks the rate to its corresponding transmitter.

3- The active user transmits data with the computed rate and with full power.

According to the above scheme, we define the probability of the link activation in each cluster as $q_n \triangleq Pr\left\{h_{ii}^{(j)} > \tau_n\right\}$ which is a function of n.

C. Delay Concepts

We assume that the time axis is divided into slots with the duration of one block. The slot duration is supposed to be equal for all the links. In this work, we assume each link has the buffer size equal to one packet. We also assume that the packets arrive uniformly with the constant rate $\frac{1}{\lambda}$ packets per block length. This parameter is assumed to be the same for all the links.

Definition 2 (Delay): We define the random variable D_i for link i as the latency between two successive transmissions, expressed as the number of blocks (see Fig. 1).

We consider the packet dropping probability as a performance criterion in the delay sensitive applications. Dropping occurs when the latency between two successive transmissions in each link exceeds a prespecified level. In fact, since the buffer size is one and the packets are arrived with a constant rate $\frac{1}{\lambda}$, it follows that the dropping occurs when $D_i > \lambda$. In order to analyze the dropping probability of the packets, we define the delay threshold levels for the link, cluster and the whole network. In this case, we have the following definition.

Definition 3 (Packet Dropping Probability): Let $\mathscr{B}_i$ represents the event that the dropping occurs in link i. Then, the packet dropping probability in the link is defined as

$$P(\mathscr{B}_i) \triangleq Pr(D_i > \lambda), \quad \forall i \in \mathbb{N}_n. \tag{4}$$

In a similar manner, we define

$$P(\mathscr{B}_{\mathcal{C}}) \triangleq Pr\left(\bigcup_{i=1}^{n}(D_i > \lambda)\right), \tag{5}$$

$$P(\mathscr{B}_{\mathcal{N}}) \triangleq Pr\left(\bigcup_{i=1}^{K}(D_i > \lambda)\right), \tag{6}$$

where $P(\mathscr{B}_{\mathcal{C}})$ and $P(\mathscr{B}_{\mathcal{N}})$ are the packet dropping probabilities in the cluster and the network, respectively.

III. Throughput Maximization

To study the delay-throughput tradeoff of the proposed model, it is essential to analyze the throughput of the network in terms of M and K, where $1 \leq M \leq K$. In this section, we prove that the maximum throughput of the network for every value

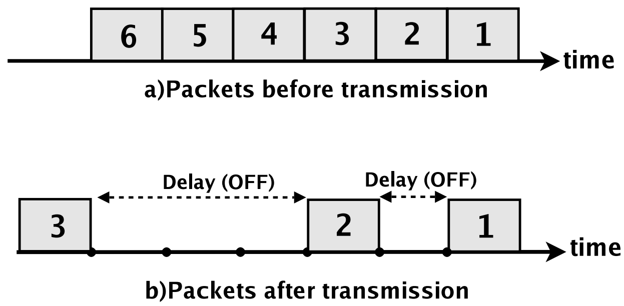


Fig. 1. Queueing model for the on-off power allocation strategy.

of M and $0 \leq \alpha \leq 1$ is obtained at $M = 1$. Following the same approach as in [1] and [2] with $M = 1$ and $\alpha = 1$, we first prove the following lemmas.

Lemma 1: Assuming $0 < \alpha \leq 1$ is fixed, *with probability one* (w. p. 1), we have

$$I_i^{(j)} \sim \hat{\alpha}(n-1)q_n,$$

as $n \to \infty$, where $\hat{\alpha} = \alpha\varpi$.

Proof: See Appendix I. ■

Lemma 2: Let $M \sim o(K)$ and $0 < \alpha \leq 1$ is a fixed parameter. Then for large values of K, the optimum threshold level that maximizes the average sum-rate of each cluster is obtained as

$$\tau_n^* = \log \hat{\alpha}n - 2\log\log \hat{\alpha}n + O(1). \tag{7}$$

Proof: See Appendix II. ■

Lemma 3: Considering the setting of Lemma 2, the probability of the link activation in each cluster is given by

$$q_n = \delta \frac{\log^2 \hat{\alpha}n}{\hat{\alpha}n}, \tag{8}$$

where δ is a constant.

Proof: Under a Rayleigh fading channel condition, we have

$$q_n = Pr\left\{h_{ii}^{(j)} > \tau_n\right\} = e^{-\tau_n}.$$

Using (7), it is concluded

$$q_n = \frac{\log^2 \hat{\alpha} n}{\hat{\alpha} n} \times e^{-O(1)}.$$

Setting $\delta = e^{-O(1)}$, we obtain (8). ■

Corollary 1: In the proposed model, if m_j is the number of active links in cluster $\mathcal{C}_j$, then $\mathbb{E}[m_j] = nq_n \sim \Theta\left(\log^2 n\right)$.

Lemma 4: Assuming $M \sim o(K)$ and $0 < \alpha \leq 1$ is fixed, the maximum achievable throughput of the network is given by

$$\bar{R}_{sum} \approx \frac{W}{\hat{\alpha}}(-\log q_n + O(1)) \tag{9}$$

$$= \frac{W}{\hat{\alpha}}\left(\log\frac{K}{M} + o\left(\log\frac{K}{M}\right) + O(1)\right). \tag{10}$$

Proof: See Appendix III. ■

Lemma 4 states that the throughput of the network for $M \sim o(K)$ depends on the value of $0 < \hat{\alpha} \leq 1$ and scales as $\frac{W}{\hat{\alpha}} \log \frac{K}{M}$. We use this result to analyze the packet dropping probability in the next sections.

Corollary 2: For values of M such that $\log M \sim o(\log K)$, the throughput of the network scales as $\frac{W}{\hat{\alpha}} \log K$.

Theorem 1: Assuming $0 \leq \alpha \leq 1$ is fixed, the maximum achievable throughput of the network with M clusters is obtained at $M = 1$.

Proof: We prove the theorem in the following cases:

Case 1: $M \sim o(K)$ and $0 < \alpha \leq 1$ is fixed:

From (9), the throughput of the network for $M \sim o(K)$ is obtained as

$$\bar{R}_{sum} \approx \frac{W}{\hat{\alpha}}\left(-\log q_n + O(1)\right). \tag{11}$$

Taking the first-order derivative of (11) with respect to M yields,

$$\frac{\partial \bar{R}_{sum}}{\partial M} = -\frac{W}{\hat{\alpha}} \frac{\partial q_n}{\partial M} \frac{1}{q_n}.$$

Noting that $n = \frac{K}{M}$, the probability of the link activation is obtained in terms of M by using (8), i.e.,

$$q_n = \frac{\delta M}{\hat{\alpha} K} \times \left(\log \frac{\hat{\alpha} K}{M}\right)^2. \tag{12}$$

Since,

$$\frac{\partial q_n}{\partial M} = \frac{\delta}{\hat{\alpha}K} \log \frac{\hat{\alpha}K}{M} \times \left(\log \frac{\hat{\alpha}K}{M} - 2 \right) > 0,$$

it is concluded that (11) is a monotically decreasing function of M. Thus, the maximum throughput of the network for $M \sim o(K)$ and $0 < \alpha \leq 1$ is obtained at $M = 1$.

Case 2: $M \sim \Theta(K)$ and $0 < \alpha \leq 1$ is fixed:

Recall m_j denote the number of active links in cluster $\mathcal{C}_j$ and noting that for $M \sim \Theta(K)$, $\lim_{K\to\infty} \frac{M}{K}$ is constant, it is concluded that n and $m_j \in [1, n]$ do not grow with K. To obtain the network throughput, we assume that among M clusters, Γ clusters have $m_j = 1$ and the rest have $m_j > 1$. We first obtain an upper bound of the throughput in each cluster when $m_j = 1$, $1 \leq j \leq M$. Clearly, since only one user in each cluster activates its transmitter, $I_i^{(j)} = 0$. Hence, the maximum achievable throughput of cluster $\mathcal{C}_j$ is obtained as

$$\bar{R}_{sum}^{(j)} = \frac{W}{M} \mathbb{E} \left[\log \left(1 + \frac{M}{N_0 W} h_{max}^{(j)} \right) \right], \tag{13}$$

where $h_{max}^{(j)} = \max_{i=1,\dots,n} h_{ii}^{(j)}$ is a random variable. Since $\log x$ is a concave function of x, an upper bound of (13) is obtained through *Jensen's inequality*, $\mathbb{E}[\log x] \leq \log(\mathbb{E}[x])$, $x > 0$. Thus,

$$\bar{R}_{sum}^{(j)} \leq \frac{W}{M} \log \left(1 + \frac{M}{N_0 W} \mathbb{E}[Y] \right), \tag{14}$$

where $Y \triangleq h_{max}^{(j)}$. Under a Rayleigh fading channel condition and noting that $\{h_{ii}^{(j)}\}$ is a set of i.i.d. random variables over $i \in \mathbb{N}_n$, we have

$$\begin{aligned} F_Y(y) &= Pr\{Y \leq y\}, \quad y > 0 \\ &= \prod_{i=1}^{n} Pr\{h_{ii}^{(j)} \leq y\} \\ &= \left(1 - e^{-y}\right)^n, \end{aligned}$$

where $F_Y(.)$ is the cumulative distribution function (cdf) of Y. Hence,

$$\mathbb{E}[Y] = \int_0^\infty n y e^{-y} \left(1 - e^{-y}\right)^{n-1} dy.$$

Since $(1-e^{-y})^{n-1} \leq 1$, we arrive at the following inequality

$$\mathbb{E}[Y] \leq \int_0^{\infty} n y e^{-y} dy = n. \tag{15}$$

Consequently, the upper bound of (14) can be simplified as

$$\bar{R}_{sum}^{(j)} \leq \frac{W}{M} \log\left(1 + \frac{K}{N_0 W}\right). \tag{16}$$

For $m_j > 1$ and due to the shadowing effect with probability α, the average sum-rate of cluster $\mathcal{C}_j$ can be written as

$$\bar{R}_{sum}^{(j)} = \sum_{i=1}^{m_j} \frac{W}{M} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\sum_{k \neq i}^{m_j} \upsilon_k \varpi h_{ki}^{(j)} + \frac{N_0 W}{M}}\right)\right], \quad k \in \mathcal{C}_j, \tag{17}$$

where υ_k's are Bernoulli random variables with parameter α. Thus,

$$\begin{aligned} \bar{R}_{sum}^{(j)} &= \frac{W}{M} \sum_{i=1}^{m_j} \sum_{l=0}^{m_j} \binom{m_j}{l} \alpha^l (1-\alpha)^{m_j - l} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\Sigma_l + \frac{N_0 W}{M}}\right)\right] \\ &= \frac{W}{M} \sum_{i=1}^{m_j} (1-\alpha)^{m_j} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\frac{N_0 W}{M}}\right)\right] + \\ &\quad \frac{W}{M} \sum_{i=1}^{m_j} \sum_{l=1}^{m_j} \binom{m_j}{l} \alpha^l (1-\alpha)^{m_j - l} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\Sigma_l + \frac{N_0 W}{M}}\right)\right], \end{aligned} \tag{18}$$

where Σ_l is the sum of l i.i.d random variables with $\chi^2(2)$ distribution. For $m_j > 1$, Σ_l is greater than the interference term caused by one interfering link. Thus, an upper bound for the throughput of cluster j is given by

$$\begin{aligned} \bar{R}_{sum}^{(j)} &\leq \frac{W}{M} m_j (1-\alpha)^{m_j} \mathbb{E}\left[\log\left(1 + \frac{Y}{\frac{N_0 W}{M}}\right)\right] \\ &+ \frac{W}{M} \sum_{i=1}^{m_j} \sum_{l=1}^{m_j} \binom{m_j}{l} \alpha^l (1-\alpha)^{m_j - l} \mathbb{E}\left[\log\left(1 + \frac{Y}{Z}\right)\right], \end{aligned} \tag{19}$$

where $Y \triangleq h_{max}^{(j)} = \max_{i=1,\ldots,n} h_{ii}^{(j)}$ and $Z \triangleq \varpi h_{ki}^{(j)}$, $k \neq i$. According to binomial formula, we have

$$\sum_{l=1}^{m_j} \binom{m_j}{l} \alpha^l (1-\alpha)^{m_j - l} = 1 - (1-\alpha)^{m_j}.$$

Thus,

$$\begin{aligned}\bar{R}_{sum}^{(j)} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right] \\ &+ \frac{W}{M}m_j\left(1-(1-\alpha)^{m_j}\right)\mathbb{E}\left[\log\left(1+\frac{Y}{Z}\right)\right]. \end{aligned} \quad (20)$$

Letting $X = \frac{Y}{Z}$, the cdf of X can be evaluated as

$$\begin{aligned}F_X(x) &= Pr\{X \leq x\}, \quad x > 0 \\ &= Pr\{Y \leq Zx\} \\ &= \int_0^\infty Pr\{Y \leq Zx|Z\} f_Z(z)dz \\ &= \int_0^\infty \left(1-e^{-\frac{z}{\varpi}x}\right)^n e^{-\frac{z}{\varpi}}dz \\ &= \varpi\int_0^\infty \left(1-e^{-tx}\right)^n e^{-t}dt \end{aligned} \quad (21)$$

Thus, the probability distribution function (pdf) of X can be written as

$$\begin{aligned}f_X(x) &= \frac{dF_X(x)}{dx} \\ &= \varpi\int_0^\infty nte^{-t(1+x)}\left(1-e^{-tx}\right)^{n-1}dt \\ &\leq \varpi\int_0^\infty nte^{-t(1+x)}dt = \frac{n\varpi}{(1+x)^2}. \end{aligned} \quad (22)$$

Using (13)-(16) and (22), the inequality (20) is simplified as

$$\begin{aligned}\bar{R}_{sum}^{(j)} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j}\log\left(1+\frac{K}{N_0W}\right) \\ &+ \frac{W}{M}m_j(1-(1-\alpha)^{m_j})\int_0^\infty \log\left(1+x\right)f_X(x)dx \\ &\leq \frac{W}{M}m_j(1-\alpha)^{m_j}\log\left(1+\frac{K}{N_0W}\right) \\ &+ \frac{W}{M}n\varpi m_j(1-(1-\alpha)^{m_j})\int_0^\infty \frac{\log\left(1+x\right)}{(1+x)^2}dx. \end{aligned}$$

Since

$$\int_0^\infty \frac{\log(1+x)}{(1+x)^2}dx = 1,$$

we arrive at the following inequality

$$\bar{R}_{sum}^{(j)} \leq \frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1 + \frac{K}{N_0 W}\right) + \frac{W}{M} n\varpi m_j (1 - (1-\alpha)^{m_j}). \quad (23)$$

It is worth mentioning that the second term in (23) does not grow with K. Let assume that among M clusters, Γ clusters have $m_j = 1$ and for the $M - \Gamma$ of the rest, the number of active links in each cluster is greater than one. Hence by using (16) and (23), an upper bound for the network throughput is obtained as

$$\begin{aligned} \bar{R}_{sum} &\leq \Gamma \frac{W}{M} \log\left(1 + \frac{K}{N_0 W}\right) + (M-\Gamma)\frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1 + \frac{K}{N_0 W}\right) \\ &+ (M-\Gamma)\frac{W}{M} n\varpi m_j (1 - (1-\alpha)^{m_j}). \end{aligned} \quad (24)$$

As mentioned earlier, for $M \sim \Theta(K)$, n and $m_j \in [1, n]$ do not grow with K. Consequently, the third term of (24) does not scale with K. Since $0 < \alpha \leq 1$, it is clear that

$$\frac{\Gamma W}{M} \log\left(1 + \frac{K}{N_0 W}\right) \leq \frac{\Gamma W}{M\alpha} \log\left(1 + \frac{K}{N_0 W}\right).$$

Hence, in order to show that the maximum achievable throughput obtained in (24) is less than that of $M = 1$ in (10), it is sufficient to prove

$$(M-\Gamma)\frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1 + \frac{K}{N_0 W}\right) < (M-\Gamma)\frac{W}{M\alpha} \log\left(1 + \frac{K}{N_0 W}\right),$$

or

$$m_j (1-\alpha)^{m_j} < \frac{1}{\alpha}.$$

Letting $f(\alpha) = \alpha m_j (1-\alpha)^{m_j}$, we have

$$\frac{\partial f(\alpha)}{\partial \alpha} = m_j (1-\alpha)^{m_j - 1}(1 - \alpha - \alpha m_j).$$

Thus, the extremum points of $f(\alpha)$ are located at $\alpha = 1$ and $\alpha = \frac{1}{m_j + 1}$. It is seen that

$$f(1) = 0 < 1,$$

and

$$f(\frac{1}{m_j+1}) = \left(\frac{m_j}{m_j+1}\right)^{m_j+1} < 1.$$

Since $f(\alpha) < 1$, we can conclude that the maximum achievable network throughput for $M \sim \Theta(K)$ is less than that of $M = 1$.

<u>*Case 3:*</u> $1 \le M \le K$ and $\alpha = 0$:

According to the shadow fading model proposed in (1), it is seen that for $\alpha = 0$ and with probability one, $\mathcal{L}_{ki}^{(j)} = 0, \ k \neq i$. This implies that no interference exist in each cluster and all the user can transmit with full power. Thus for every value of $1 \le M \le K$, the average sum-rate of cluster j for $\alpha = 0$ is simplified as

$$\bar{R}_{sum}^{(j)} = \frac{W}{M} \sum_{i=1}^{n} \mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\frac{N_0 W}{M}}\right)\right]. \tag{25}$$

where the expectation is computed with respect to $h_{ii}^{(j)}$. Under a Rayleigh fading channel model, we have

$$\bar{R}_{sum}^{(j)} = \frac{W}{M} \sum_{i=1}^{n} \int_0^\infty e^{-x} \log(1 + \Upsilon x) dx,$$

where $\Upsilon \triangleq \frac{M}{N_0 W}$. Thus,

$$\begin{aligned} \bar{R}_{sum}^{(j)} &= \frac{W}{M} n \int_0^\infty e^{-x} \log(1 + \Upsilon x) dx \\ &= \frac{W}{M} n e^{\frac{1}{\Upsilon}} \int_{1/\Upsilon}^\infty \frac{e^{-x}}{x} dx \\ &= \frac{W}{M} n e^{\frac{1}{\Upsilon}} \mathrm{E}_1\left(\frac{1}{\Upsilon}\right), \end{aligned} \tag{26}$$

where $\mathrm{E}_1(x)$ is obtained by the *exponential-integral function* defined as [29]

$$\mathrm{E}_n(x) \triangleq \int_1^\infty \frac{e^{-tx}}{t^n} dt.$$

Hence, the network throughput is obtained as

$$\begin{aligned} \bar{R}_{sum} &= \sum_{j=1}^{M} \bar{R}_{sum}^{(j)} = \frac{KW}{M} e^{\frac{1}{\Upsilon}} \mathrm{E}_1\left(\frac{1}{\Upsilon}\right) \\ &= \frac{KW}{M} e^{\frac{N_0 W}{M}} \int_1^\infty \frac{e^{-t\frac{N_0 W}{M}}}{t} dt. \end{aligned} \tag{27}$$

Taking the first-order derivative of (27) with respect to M yields,

$$\frac{\partial \bar{R}_{sum}}{\partial M} = -\frac{KW}{M^2} e^{\frac{N_0 W}{M}} \left(1 + \frac{N_0 W}{M}\right) \mathrm{E}_1\left(\frac{N_0 W}{M}\right) + \frac{KW}{M^2}.$$

Since for every value of $N_0 W$, $\frac{\partial \bar{R}_{sum}}{\partial M}$ is negative, it is concluded that the network throughput is a monotically decreasing function of M. Consequently for $\alpha = 0$, the network throughput for every value of $1 \leq M \leq K$ is achieved at $M = 1$.

■

Note that for $M \sim \Theta(K)$, which includes $M = K$, we obtained an upper bound for $\bar{R}_{sum}$. In the next corollary, we derive an exact explicit expression for the network throughput when $M = K$.

Corollary 3: Assuming $M = K$, the average sum-rate of the network for every value of $0 \leq \alpha \leq 1$ is obtained by

$$\bar{R}_{sum} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}), \tag{28}$$

where $\boldsymbol{\gamma}$ is *Euler's constant.*

Proof: See Appendix IV. ■

Corollary 4: For $M = K$, the network throughput is of order $\log K$. Through comparing (28) with (10), it is concluded that the throughput of the network with $M = K$ is less than or equal to that of $M = 1$.

So far, we presented an asymptotic analysis for the network throughput in terms of M and α. In the following, we evaluate the throughput of the network versus the number of clusters for finite values of K through simulation results. Fig. 2 illustrates the maximum throughput of the network versus M for $K = 20$, $K = 40$, $\alpha = 0.1$ and $\varpi = 1$. From the figure, we can see that the network throughput is a decreasing function of M. In this case, the maximum value of $\bar{R}_{sum}$ is achieved at $M = 1$.

Corollary 5: In the network with the on-off power allocation strategy, partitioning the bandwidth W into M disjoint subchannels has no gain in terms of enhancing the network throughput.

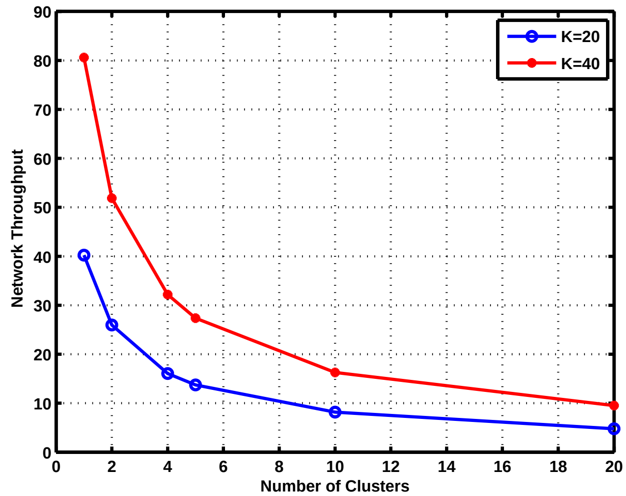


Fig. 2. Network throughput vs. M for $K = 20$, $K = 40$, $\alpha = 0.1$ and $\varpi = 1$.

IV. Delay Analysis

In this section, we analyze the delay characteristics of the underlying network based on the number of clusters, M. First, we obtain the packet dropping probabilities for the link, cluster and the whole network, when $M \sim o(K)$, and under the on-off power allocation strategy. Next, the delay characteristics of the network are obtained for $M \sim \Theta(K)$ with $M \neq K$. Under condition $1 - e^{-1/e} \leq \alpha \leq 1$, where α is fixed, we prove that the *round robin* (RR) policy in each cluster is the optimum scheme from the throughput-delay point of view. We also investigate the throughput-delay of the network for $M = K$. To compare the results, we further present *throughput-delay ratio* as a performance metric for the proposed model.

A. Delay Characteristics for $M \sim o(K)$

Under the on-off power allocation strategy, the latency D_i is a Geometric random variable with the probability mass function $Pr\{D_i = d\} = q_n(1 - q_n)^{d-1}$, where $d = 1, 2, \ldots$. In the following, we derive the delay characteristics of the link, cluster and the whole network based on the arrival rate.

Lemma 5: For the proposed model, the packet dropping probabilities in the link, cluster and the network are obtained as

$$P(\mathscr{B}_i) = (1 - q_n)^{1+\lambda}, \quad i \in \mathbb{N}_n, \tag{29}$$

$$P(\mathscr{B}_{\mathcal{C}}) = 1 - (1 - P(\mathscr{B}_i))^n, \tag{30}$$

$$P(\mathscr{B}_{\mathcal{N}}) = 1 - (1 - P(\mathscr{B}_i))^K, \tag{31}$$

respectively.

Proof: Using (4) and by taking account of the Geometric distribution of D_i, the packet dropping probability in link i is obtained as

$$\begin{aligned} P(\mathscr{B}_i) &= 1 - Pr(D_i \leq \lambda) \\ &= 1 - \sum_{y=0}^{\lambda} q_n (1 - q_n)^y. \end{aligned} \tag{32}$$

It should be noted that (32) is valid only for $q_n < 1$. In the case of $q_n = 1$, i.e., all the links transmit with full power, no delay exists in each link and consequently $P(\mathscr{B}_i) = 0$. Considering the following expression

$$\sum_{k=0}^{L} x^k = \frac{1 - x^{L+1}}{1 - x}, \quad |x| < 1,$$

the dropping probability of link i is obtained as

$$P(\mathscr{B}_i) = (1 - q_n)^{1+\lambda}, \quad i = 1, ..., n. \tag{33}$$

and this completes the proof of the first part of the lemma.

To derive $P(\mathscr{B}_{\mathcal{C}})$, let $\mathscr{E}_{\mathcal{C}}$ denote the occurrence of no dropping event in the cluster. Thus,

$$P(\mathscr{E}_{\mathcal{C}}) = Pr\{\mathscr{B}_1^c \cap \mathscr{B}_2^c \cap .... \cap \mathscr{B}_n^c\}. \tag{34}$$

Noting that the events $\mathscr{B}_i^c$ and $\mathscr{B}_j^c$ are independent of each other for all $i, j \in \mathbb{N}_n$, we have

$$\begin{aligned} P(\mathscr{E}_{\mathcal{C}}) &= \prod_{k=1}^{n} Pr\{\mathscr{B}_k^c\} \\ &= \prod_{k=1}^{n}(1 - P(\mathscr{B}_k)) \\ &= (1 - P(\mathscr{B}_i))^n. \end{aligned}$$

Hence, the packet dropping probability of each cluster is obtained as

$$P(\mathscr{B}_{\mathcal{C}}) = 1 - (1 - P(\mathscr{B}_i))^n. \tag{35}$$

With a similar argument, the dropping probability of the packet in the whole network is computed as

$$P(\mathscr{B}_{\mathcal{N}}) = 1 - (1 - P(\mathscr{B}_i))^K. \tag{36}$$

■

We are now ready to prove the main result of this section. In the next theorem, we derive the parameters λ_ℓ, $\lambda_{\mathcal{C}}$ and $\lambda_{\mathcal{N}}$ as the delay threshold levels for the link, cluster and the whole network, respectively, such that the corresponding dropping probabilities tend to zero, while achieving the maximum throughput of the network.

Theorem 2: Let $M \sim o(K)$ and $0 < \alpha \leq 1$ is fixed. Then, for the optimum q_n given in (8),

i) $\lim_{K\to\infty} P(\mathscr{B}_i) = 0$, if $\lambda_\ell \sim \omega\left(\frac{n}{\log^2 n}\right)$.

ii) $\lim_{K\to\infty} P(\mathscr{B}_{\mathcal{C}}) = 0$, if $\lambda_{\mathcal{C}} \sim \frac{\hat{\alpha}n}{\log \hat{\alpha}n} + \omega\left(\frac{n}{\log^2 n}\right)$.

iii) $\lim_{K\to\infty} P(\mathscr{B}_{\mathcal{N}}) = 0$, if $\lambda_{\mathcal{N}} \sim \frac{\hat{\alpha}n \log K}{\log^2 \hat{\alpha}n} + \omega\left(\frac{n}{\log^2 n}\right)$.

Proof: i) From (29) and using the Taylor series expansion

$$\log(1-z) = -\sum_{k=1}^{\infty} \frac{z^k}{k}, \quad |z| < 1,$$

we have

$$\begin{aligned} P(\mathscr{B}_i) &= e^{(1+\lambda_\ell)\log(1-q_n)} \\ &= e^{-(1+\lambda_\ell)\sum_{k=1}^{\infty} q_n^k/k}, \end{aligned} \tag{37}$$

where $q_n < 1$. Since for $M \sim o(K)$, the number of links in each cluster is large, $q_n \ll 1$. Thus, we can approximate (37) as

$$P(\mathscr{B}_i) \approx e^{-q_n(1+\lambda_\ell)}. \tag{38}$$

Setting $q_n\lambda_\ell = \omega(1)$ makes $e^{-q_n\lambda_\ell} \to 0$. By using the optimum q_n in Lemma 3, if $\lambda_\ell \sim \omega\left(\frac{n}{\log^2 n}\right)$, then $P(\mathscr{B}_i) = o(1)$, i.e., $\lim_{K\to\infty} P(\mathscr{B}_i) = 0$.

ii) From (35) and considering the following binomial series

$$(1-x)^k = 1 - kx + \frac{k(k-1)}{2!}x^2 - ...,$$

the packet dropping probability in each cluster will be

$$P(\mathscr{B}_\mathcal{C}) = nP(\mathscr{B}_i) - \frac{n(n-1)}{2!}P^2(\mathscr{B}_i) + ... \tag{39}$$

If $\lambda_\mathcal{C}$ is the optimum threshold such that $\lim_{K\to\infty} P(\mathscr{B}_\mathcal{C}) = 0$, it guarantees a sufficiently small value for $P(\mathscr{B}_i)$ as well. Hence, (39) can be approximated by

$$P(\mathscr{B}_\mathcal{C}) \approx nP(\mathscr{B}_i).$$

Similar to the proof of part (i), the dropping probability of packets in each cluster can be written as

$$\begin{aligned} P(\mathscr{B}_\mathcal{C}) &\approx n(1-q_n)^{1+\lambda_\mathcal{C}} \\ &\approx e^{-q_n(1+\lambda_\mathcal{C})+\log n}. \end{aligned}$$

Setting $q_n\lambda_\mathcal{C} - \log n = \omega(1)$ makes $e^{-q_n(1+\lambda_\mathcal{C})+\log n} \to 0$. Thus, for the network with the optimum q_n, choosing

$$\lambda_\mathcal{C} \sim \frac{\hat{\alpha}n}{\log \hat{\alpha}n} + \omega\left(\frac{n}{\log^2 n}\right) \tag{40}$$

yields $\lim_{K\to\infty} P(\mathscr{B}_\mathcal{C}) = 0$.

iii) From (36) and with a similar argument, the dropping probability of the whole network can be written as

$$\begin{aligned} P(\mathscr{B}_\mathcal{N}) &\approx KP(\mathscr{B}_i) \\ &\approx K(1-q_n)^{1+\lambda_\mathcal{N}} \\ &\approx e^{-q_n(1+\lambda_\mathcal{N})+\log K}. \end{aligned}$$

Similar to the proof of part (ii), choosing $\lambda_{\mathcal{N}} \sim \frac{\hat{\alpha}n\log K}{\log^2\hat{\alpha}n} + \omega\left(\frac{n}{\log^2 n}\right)$ yields $\lim_{K\to\infty} P(\mathscr{B}_{\mathcal{N}}) = 0$. ■

B. Delay Characteristics for $M \sim \Theta(K)$, $M \neq K$

In the previous subsection, we derived some asymptotic results for the delay based on the on-off power allocation strategy. We showed that for $M \sim o(K)$, the delay is a monotically increasing function of n. However, noting that for every value of $n = \frac{K}{M}$, the number of active links in each cluster is in the range of $1 \leq m_j < n$, it is concluded that for $M \sim \Theta(K)$ with $M \neq K$, the number of active links does not grow with K. As a result, the latency D_i does not increase with K as well. Thus, the distributed on-off power allocation strategy proposed in [1] and [2] is not an optimum policy, as n does not scale as $\omega(1)$. To get a better intuition about the latency, it is essential to investigate the optimum scheduling policy for $M \sim \Theta(K)$, $M \neq K$.

Lemma 6: Let assume $M \sim \Theta(K)$, where $M \neq K$. Then for $1 - e^{-1/e} \leq \alpha \leq 1$, where α is fixed, the maximum throughput of the network is achieved by time sharing policy, i.e., $m_j = 1$.

Proof: Recall from (23), an upper bound of $\bar{R}_{sum}^{(j)}$ for $m_j > 1$ is given by

$$\bar{R}_{sum}^{(j)} \leq \frac{W}{M} m_j (1-\alpha)^{m_j} \log\left(1 + \frac{K}{N_0 W}\right) + \frac{W}{M} n\varpi m_j (1 - (1-\alpha)^{m_j}). \tag{41}$$

It should be noted that the second term of (41) does not grow with K. Defining $\zeta(m_j, \alpha) \triangleq m_j(1-\alpha)^{m_j}$ and for a fixed value of α, we have

$$\frac{\partial\zeta(m_j,\alpha)}{\partial m_j} = m_j(1-\alpha)^{m_j}\log(1-\alpha) + (1-\alpha)^{m_j}. \tag{42}$$

Since the second-order derivative of (42) is negative, the maximum value of $\zeta(m_j, \alpha)$ is obtained by setting (42) equal to zero. So, we have

$$m_{j,opt} = \frac{-1}{\log(1-\alpha)}. \tag{43}$$

On the other hand from (16), an upper bound of the throughput in each cluster for $m_j = 1$ is obtained as

$$\bar{R}_{sum}^{(j)} \leq \frac{W}{M}\log\left(1 + \frac{K}{N_0 W}\right). \tag{44}$$

Using (41)-(44), it is concluded that the maximum achievable throughput of network is obtained for $m_j = 1$, if $\zeta(m_{j,opt}, \alpha) < 1$. Noting that

$$(1-\alpha)^{\frac{-1}{\log(1-\alpha)}} = e^{-1},$$

we arrive at the following inequality

$$\alpha > 1 - e^{-1/e}. \tag{45}$$

Thus for $1 - e^{-1/e} < \alpha \leq 1$, $\zeta(m_j, \alpha) \leq 1$, i.e., the maximum achievable throughput is obtained by time sharing policy. This implicity indicates that only one user in each cluster and in each time slot transmits with full power. ■

Among different types of time-sharing schemes, we consider the *round robin* (RR) (as a delay optimal scheduling approach) and the *best channel* (BS) scheduling schemes as two extreme scheduling policies.

Theorem 3: Suppose $M \sim \Theta(K)$ with $M \neq K$ and $1 - e^{-1/e} < \alpha \leq 1$ is a fixed parameter. Then

$$\lim_{K\to\infty} \frac{\bar{R}^{rr}_{L,sum}}{\bar{R}^{BS}_{U,sum}} = 1.$$

where $\bar{R}^{rr}_{L,sum}$ is the lower bound of the network throughput for the RR scheme and $\bar{R}^{BS}_{U,sum}$ is the upper bound of the network throughput for the BS scheme.

Proof: In the RR scheme, each link in a cluster takes an equal share of service in turn. Since in each time slot, only one link is serviced, the average sum-rate of each cluster is obtained as

$$\begin{aligned}\bar{R}^{(j)}_{sum} &= \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{M}{N_0 W}h\right)\right] \\ &\geq \frac{W}{M}\mathbb{E}\left[\log\left(\frac{M}{N_0 W}h\right)\right] \\ &= \frac{W}{M}\mathbb{E}\left[\log\frac{M}{N_0 W}\right] + \frac{W}{M}\mathbb{E}\left[\log h\right], \end{aligned} \tag{46}$$

where $h \in \{h_{ii}\}_{i=1}^{n}$. Note that

$$\mathbb{E}\left[\log h\right] = \int_0^\infty e^{-x}\log x dx = -\boldsymbol{\gamma},$$

where $\boldsymbol{\gamma}$ is Euler's constant and is finite. Since $M = \frac{K}{n}$, we arrive at the following lower bound for $\bar{R}_{sum}^{(j)}$

$$\bar{R}_{sum}^{(j)} \geq \frac{W}{M}\left\{\log\frac{K}{N_0 W} - \log\frac{n}{N_0 W} - \boldsymbol{\gamma}\right\}. \tag{47}$$

Consequently, the lower bound of $\bar{R}_{sum}$ for the RR scheme can be written as

$$\bar{R}_{sum} \geq W\log\frac{K}{N_0 W} - W\log\frac{n}{N_0 W} - W\boldsymbol{\gamma}. \tag{48}$$

Note that for $M \sim \Theta(K)$, the second term of (48) does not grow with K. Thus, the lower bound of the network throughput for the RR scheme, denoted by $\bar{R}_{L,sum}^{rr}$, scales as $\Theta(\log K)$.

In the best channel scheduling scheme and in each time slot, the link with the best channel condition is allowed to transmit with full power. Thus, the average sum-rate of each cluster is given by

$$\bar{R}_{sum}^{(j)} = \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{M}{N_0 W}h_{max}^{(j)}\right)\right], \tag{49}$$

where $h_{max}^{(j)} = \max_{i=1,\dots,n} h_{ii}^{(j)}$ is a random variable. Using (13)-(16), an upper bound of the network throughput for the best channel scheduling is obtained as

$$\bar{R}_{sum} \leq W\log\frac{K}{N_0 W}. \tag{50}$$

Thus, the upper bound of the network throughput for the best channel scheduling, denoted by $\bar{R}_{U,sum}^{BS}$, is of order $\log K$.

Through comparing (48) with (50), we come up with the following result:

$$\lim_{K\to\infty}\frac{\bar{R}_{L,sum}^{rr}}{\bar{R}_{U,sum}^{BS}} = 1. \tag{51}$$

■

It is worth to mention that for the best channel scheduling scheme, the delay for each link is a random variable and scales as $\omega(1)$. In the next corollary, the performance of two aforementioned scheduling schemes is compared from the delay-throughput point of view.

Corollary 6: Let assume $1 - e^{-1/e} < \alpha \leq 1$ and $M \sim \Theta(K)$, where $M \neq K$. Then, the round robin policy is a delay-optimal scheduling which guarantees a fixed delay of n, while achieving the network throughput of order $\log K$. In this case, the throughput-delay ratio, denoted by ρ, scales as

$$\rho^{rr} \sim \Theta\left(\frac{\log K}{n}\right). \tag{52}$$

Also, since for the best channel scheduling scheme,

$$\rho^{BS} \sim \Theta\left(\frac{\log K}{\omega(1)}\right). \tag{53}$$

it is concluded that $\rho^{rr} > \rho^{BS}$.

Corollary 7: Under the assumptions of Corollary 6, the quantities λ_ℓ, $\lambda_\mathcal{C}$ and $\lambda_\mathcal{N}$ for the round robin policy are the deterministic parameters and are equal to n.

C. Delay Characteristics for $M \sim \Theta(K)$, $M = K$

For $M = K$, each cluster consists of only one user. Also, due to orthogonality of the allocated subchannels, no interference is imposed from the links in one cluster on the links in the other clusters. Thus, it follows that all the links in the network can transmit with full power. In this case, the blocking probabilities of each link, cluster and the whole network tend to zero. Hence, the values of λ_ℓ, $\lambda_\mathcal{C}$ and $\lambda_\mathcal{N}$ are equal to 1. On the other hand from Corollary 3, the maximum achievable throughput of the network scales as $W \log K$. Thus for $M = K$, the maximum throughput-delay ratio scales as $W \log K$, that is more than what is obtained for $M \sim o(K)$ and $M \sim \Theta(K)$ with $M \neq K$. Also, from the fairness point of view, it is seen that $M = K$ is the global optimal fairness case.

V. A Study of Delay-Throughput Tradeoffs

Previously, we investigated the delay charactersitics based on a minimum dropping probability, while achieving the maximum throughput of the network. Also, we showed that the delay depends on the number of clusters, M. For the case of

$M \sim \Theta(K)$, the number of active links and delay does not grow with K. However, the delay is critical for $M \sim o(K)$. In this section, we first study the scaling law of the network throughput with the on-off power allocation strategy for $M \sim o(K)$. Then, we present some results on improving the delay without any significant impact on the network throughput.

Recall from Lemma 3, the optimum value of q_n for $M \sim o(K)$ that achieves the maximum network throughput scales as $\frac{\log^2 \hat{\alpha} n}{\hat{\alpha} n}$. Clearly, increasing the number of the links has the advantage of increasing $\bar{R}_{sum}^{(j)}$. However, due to decreasing the value of q_n, delay increases. The problem is crucial in the static networks with immobile nodes. Particularly, the links in the bad channels experience too much delay. Deploying multiple antennas [30] can improve the delay. The other solution that we investigate in this work is to relax the optimum threshold τ_n^*.

A. *Delay Improvement for* $M \sim o(K)$

Noting that $\tau_n = -\log q_n$, reducing τ_n will increase q_n and the number of the active links in each cluster. Clearly, this reduces the delay in the network, however according to (9), the throughput of the network decreases as well. Therefore, the threshold τ_n is interpreted as a compromise between delay and throughput. In the following, we derive lower and upper bounds on τ_n such that the order of the network throughput for $M \sim o(K)$ is preserved, i.e.,

$$\lim_{n \to \infty} \frac{\bar{R}_{sum}}{\frac{W}{\hat{\alpha}} \log n} = 1. \tag{54}$$

Theorem 4: Assuming $M \sim o(K)$, the throughput of the network is asymptotically of order $\frac{W}{\hat{\alpha}} \log n$, if

$$\Theta\left(\frac{\log^2 n}{n}\right) \leq q_n \leq \frac{\Lambda(n)}{n}, \tag{55}$$

where $\Lambda(n) \sim \Omega(\log^2 n)$ and satisfies $\Lambda(n) = e^{o(\log n)}$.

Proof: For the lower bound, the theorem is easily proved by using the Lemma 3. Substituting $q_n = \frac{\Lambda(n)}{n}$ in (9) yields

$$\bar{R}_{sum} = \frac{W}{\hat{\alpha}} \left(\log n - \log \Lambda(n) + O(1) \right). \tag{56}$$

To achieve $\bar{R}_{sum}$ of order $\frac{W}{\hat{\alpha}} \log n$, the function $\Lambda(n)$ must satisfy

$$\log \Lambda(n) = o(\log n),$$

and this completes the proof of the theorem. ■

Corollary 8: The lower and upper bounds on τ_n that result in $\bar{R}_{sum}$ of order $\Theta\left(\frac{W}{\hat{\alpha}} \log n\right)$ are

$$\log n - \log \Lambda(n) \lesssim \tau_n \lesssim \log n - 2 \log \log n + O(1). \tag{57}$$

An interesting insight provided by the upper bound in (55) is the improvement of the delay threshold levels λ_ℓ, $\lambda_{\mathcal{C}}$ and $\lambda_{\mathcal{N}}$, without changing the order of the network throughput.

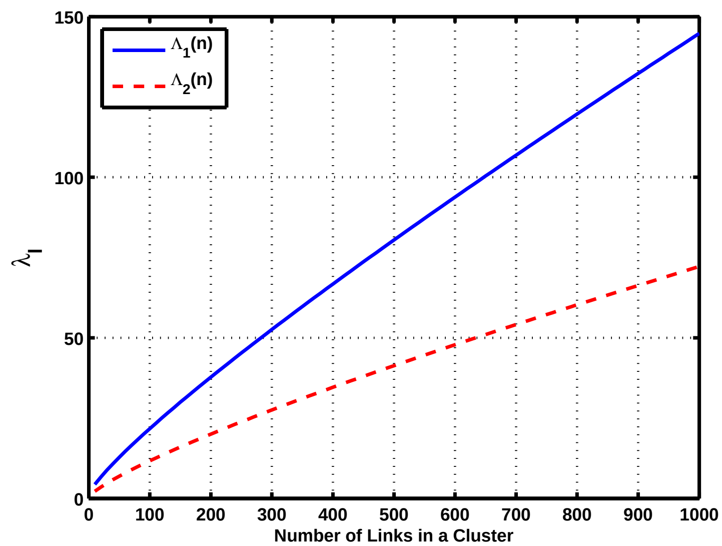
Corollary 9: The optimum values of λ_ℓ, $\lambda_{\mathcal{C}}$ and $\lambda_{\mathcal{N}}$ that make $P(\mathscr{B}_i)$, $P(\mathscr{B}_{\mathcal{C}})$ and $P(\mathscr{B}_{\mathcal{N}})$ tend to zero, while achieving $\bar{R}_{sum}$ of order $\frac{W}{\hat{\alpha}} \log n$ scale as $\lambda_\ell \sim \omega\left(\frac{n}{\Lambda(n)}\right)$, $\lambda_{\mathcal{C}} = \frac{n \log n}{\Lambda(n)} + \omega\left(\frac{n}{\Lambda(n)}\right)$ and $\lambda_{\mathcal{N}} = \frac{n \log K}{\Lambda(n)} + \omega\left(\frac{n}{\Lambda(n)}\right)$, respectively.

Fig. 3-a illustrates λ_ℓ versus the number of links in each cluster for $\Lambda(n) = \log n$ and $\Lambda(n) = e^{\sqrt{\log n}}$, and for $\hat{\alpha} = W = 1$. Also, Fig. 3-b shows the corresponding network throughput given in (56) versus n. From these figures, it is observed that the delay decreases without any significant impact on the throughput of the network.

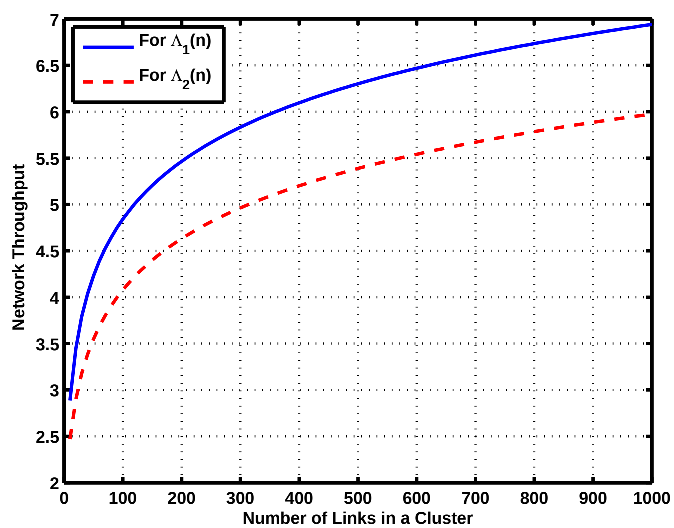
B. *Throughput Maximization for $M \sim o(K)$*

In this section, we present a new definition of the average throughput for the backlogged users, when $M \sim o(K)$. Also, we derive the maximum average throughput in two extreme cases of one and infinite buffer size. Under the on-off power allocation strategy, the average throughput of link i is defined on a per-block basis as

$$T_i \triangleq \lim_{L \to \infty} \frac{1}{L} \sum_{t=1}^{L} R_{i,t}^{(j)} \mathcal{I}_i^{(t)}, \tag{58}$$



(a)



(b)

Fig. 3. a) λ_ℓ, and b) Throughput of the network vs. the number of links in each cluster for $\Lambda_1(n) = \log n$ and $\Lambda_2(n) = e^{\sqrt{\log n}}$ and for $\hat{\alpha} = W = 1$.

where $\mathcal{I}_i^{(t)}$ is an *indicator variable* which is equal 1, if the user i transmits with full power at block t, and 0 otherwise, and $R_{i,t}^{(j)}$ is the instantaneous transmission rate of link i at block t. In the next theorem, we obtain the maximum average throughput for any buffer size.

Theorem 5: For the backlogged user with any buffer size, the optimum solution for the optimization problem

$$q_n^* = \arg \max_{q_n} T_i, \tag{59}$$

is 1. Furthermore, the maximum average throughput asymptotically scales as $\frac{O(W)}{\hat{\alpha}K}$.

Proof: According to the definition of $\mathcal{I}_i^{(t)}$ and for any buffer size, we have

$$\mathcal{I}_i^{(t)} = \begin{cases} 1, & q_n, \\ 0, & 1 - q_n. \end{cases} \tag{60}$$

Since $\mathcal{I}_i^{(t)}$ is a Bernoulli random variable with parameter q_n, we have $\mathbb{E}\{\mathcal{I}_i^{(t)}\} = q_n$. Also, using Lemma 3 and (C-1) in Appendix III, we arrive at the following equation

$$\mathbb{E}[R_{i,t}^{(j)}] = \frac{\bar{R}_{sum}}{K} = \frac{W}{\hat{\alpha}K}\left(1 + \tau_n - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha}ne^{-\tau_n}}\right). \tag{61}$$

Noting that $q_n = e^{-\tau_n}$, the average throughput defined in (58) is obtained in terms of τ_n as

$$\begin{aligned} T_i &= \mathbb{E}\left[R_{i,t}^{(j)}\right]\mathbb{E}[\mathcal{I}_i^{(t)}] \\ &= \frac{W}{\hat{\alpha}K}e^{-\tau_n}\left(1 + \tau_n - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha}ne^{-\tau_n}}\right). \end{aligned} \tag{62}$$

Since

$$\frac{\partial T_i}{\partial \tau_n} = -\frac{W}{\hat{\alpha}K}\left[\tau_n e^{-\tau_n} + \frac{2\tau_n + 2}{2\hat{\alpha}n}\right]. \tag{63}$$

is negative, it is concluded that T_i is a monotically decreasing function of τ_n. Thus, the average throughput achieves the maximum value at $\tau = 0$ or $q_n = 1$. Hence, by using (62), the maximum average throughput for any buffer size is

$$T_{i,max} \sim \frac{O(W)}{\hat{\alpha}K}. \tag{64}$$

■

Interestingly, Theorem 5 indicates the maximum average throughput for $M \sim o(K)$ and with the distributed on-off power allocation strategy is independent of the buffer size. Therefore, we can reduce the hardware complexity while achieving the maximum average throughput.

VI. Conclusion

We have analyzed the delay-throughput of a single-hop wireless network in terms of the number of clusters, M, and under the shadowing effect with probability α. It has been demonstrated that for $M \sim o(K)$ and $0 < \alpha \leq 1$, where α is fixed, the throughput of the network is of order $\frac{W}{\hat{\alpha}} \log \frac{K}{M}$. Also, it has been proved that the maximum network throughput for every value of $0 \leq \alpha \leq 1$ and $1 \leq M \leq K$ is achieved at $M = 1$. In fact, in the proposed model, partitioning the bandwidth W into M subchannels has no gain in terms of enhancing the network throughput.

In addition, we have proved that for $M \sim o(K)$ and $0 < \alpha \leq 1$, where α is fixed,the delay threshold level that results the dropping probability for each link tends to zero, while achieving the maximum throughput scales as $\omega(\frac{n}{\log^2 n})$. Also, the minimum delay in order to make the dropping probabilities for the cluster and for the whole network approach zero scales as $\frac{\hat{\alpha} n}{\log \hat{\alpha} n} + \omega(\frac{n}{\log^2 n})$ and $\lambda_{\mathcal{N}} \sim \hat{\alpha} n \frac{\log K}{\log^2 \hat{\alpha} n} + \omega(\frac{n}{\log^2 n})$, respectively. It was shown that the delay is critical for $M \sim o(K)$. We have also shown that by relaxing the value of threshold τ_n, the delay is significantly improved without changing the order of the network throughput. We have presented a new definition for the throughput and derived the maximum throughput in the cases of one and infinite buffer size. It has been proved that the maximum average throughput of the link with the distributed on-off power allocation strategy is independent of the buffer size.

Throughout this work, it is assumed that all the links use a single antenna. A possible future extension of this work would be to analyze the performance of the network with multiple antenna transmitters/receivers [30]. Also, we considered a quasi static block fading channel model, in which the channel changes independently from block to block. It would be quite interesting to generalize the results by considering correlation between two consecutive blocks of the channel.

Appendix I
Proof of Lemma 1

Let define $\chi_k \triangleq \mathcal{L}_{ki}^{(j)} p_k$, where $\mathcal{L}_{ki}^{(j)}$ is independent of p_k, for $k \neq i$. Under a quasi-static Rayleigh fading channel condition, it is concluded that χ_k's are the i.i.d. random variables with

$$\begin{aligned} \mathbb{E}\left[\chi_k\right] &= \mathbb{E}\left[\mathcal{L}_{ki}^{(j)} p_k\right] = \hat{\alpha} q_n, \\ Var\left[\chi_k\right] &= \mathbb{E}\left[\chi_k^2\right] - \mathbb{E}^2\left[\chi_k\right] = 2\alpha\kappa q_n - (\hat{\alpha} q_n)^2, \end{aligned}$$

where $\mathbb{E}\left[(h_{ki}^{(j)})^2\right] = 2$ and $\mathbb{E}\left[p_k\right] = q_n$. Also, the interference $I_i^{(j)}$ is a random variable with mean μ_n and variance ϑ_n^2, where

$$\begin{aligned} \mu_n &= \mathbb{E}\left[\sum_{\substack{k\neq i \\ k=1}}^{n} \chi_k\right] = \hat{\alpha}(n-1) q_n, \\ \vartheta_n^2 &= (n-1)\left(2\alpha\kappa q_n - (\hat{\alpha} q_n)^2\right). \end{aligned}$$

By using the *Chebyshev inequality*, we obtain

$$Pr\{|I_i^{(j)} - \mu_n| < \psi_n\} \geq 1 - \frac{\vartheta_n^2}{\psi_n^2},$$

for all $\psi_n > 0$. Thus, we have

$$Pr\{|I_i^{(j)} - \hat{\alpha}(n-1)q_n| < \psi_n\} \geq 1 - \frac{(n-1)\left(2\alpha\kappa q_n - (\hat{\alpha} q_n)^2\right)}{\psi_n^2}.$$

It is seen that for

$$\psi_n = \omega\left(\sqrt{(n-1)\left(2\alpha\kappa q_n - (\hat{\alpha} q_n)^2\right)}\right),$$

we have

$$\lim_{n\to\infty} 1 - \frac{(n-1)\left(2\alpha\kappa q_n - (\hat{\alpha} q_n)^2\right)}{\psi_n^2} = 1.$$

Thus,

$$\hat{\alpha}(n-1)q_n - \psi_n < I_i^{(j)} < \hat{\alpha}(n-1)q_n + \psi_n, \qquad w.\ p.\ 1.$$

By choosing $\psi_n = o(\hat{\alpha}(n-1)q_n)$, we can obtain $I_i^{(j)} \sim \hat{\alpha}(n-1)q_n$, w. p. 1.

Appendix II
Proof of Lemma 2

Under the on-off power allocation strategy and using $q_n = Pr\left\{h_{ii}^{(j)} > \tau_n\right\}$, we have

$$\begin{aligned}\mathbb{E}\left[R_i^{(j)}\right] &= \mathbb{E}\left[R_i^{(j)}\middle| h_{ii}^{(j)} > \tau_n\right] Pr\left\{h_{ii}^{(j)} > \tau_n\right\} + \mathbb{E}\left[R_i^{(j)}\middle| h_{ii}^{(j)} \leq \tau_n\right] Pr\left\{h_{ii}^{(j)} \leq \tau_n\right\} \\ &= q_n \mathbb{E}\left[R_i^{(j)}\middle| h_{ii}^{(j)} > \tau_n\right] + (1-q_n)\mathbb{E}\left[R_i^{(j)}\middle| h_{ii}^{(j)} \leq \tau_n\right].\end{aligned}$$

Since for $h_{ii}^{(j)} \leq \tau_n$, $p_i = 0$, it is concluded

$$\mathbb{E}\left[R_i^{(j)}\right] = \frac{q_n W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{I_i^{(j)} + \frac{N_0 W}{M}}\right)\middle| h_{ii}^{(j)} > \tau_n\right]. \tag{B-1}$$

Under condition $M \sim o(K)$ and for large values of K, the number of links in each cluster is sufficiently large. So, we can apply Lemma 1 to obtain

$$\mathbb{E}\left[R_i^{(j)}\right] \approx \frac{q_n W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\hat{\alpha}(n-1)q_n + \frac{N_0 W}{M}}\right)\middle| h_{ii}^{(j)} > \tau_n\right], \tag{B-2}$$

where the expectation is computed with respect to $h_{ii}^{(j)}$. For large values of n, we can ignore the noise power $\frac{N_0 W}{M}$. Hence, by using the approximation $\log(1+z) \approx z - \frac{z^2}{2}$ for $|z| \ll 1$, we have

$$\mathbb{E}\left[R_i^{(j)}\right] \approx \frac{q_n W}{M}\left\{\frac{1}{\hat{\alpha} n q_n}\mathbb{E}\left[h_{ii}^{(j)}\middle| h_{ii}^{(j)} > \tau_n\right] - \frac{1}{2}\frac{1}{(\hat{\alpha} n q_n)^2}\mathbb{E}\left[(h_{ii}^{(j)})^2\middle| h_{ii}^{(j)} > \tau_n\right]\right\}.$$

Under a Rayleigh fading channel model,

$$\mathbb{E}\left[h_{ii}^{(j)}\middle| h_{ii}^{(j)} > \tau_n\right] = 1 + \tau_n,$$

$$\mathbb{E}\left[(h_{ii}^{(j)})^2\middle| h_{ii}^{(j)} > \tau_n\right] = \tau_n^2 + 2\tau_n + 2.$$

Using $q_n = Pr\left\{h_{ii}^{(j)} > \tau_n\right\} = e^{-\tau_n}$, the average sum-rate of the links in $\mathcal{C}_j$ is given by

$$\begin{aligned}\bar{R}_{sum}^{(j)} &= \sum_{i=1}^{n} \mathbb{E}\left[R_i^{(j)}\right] \\ &= \frac{W}{\hat{\alpha}M}\left[1 + \tau_n - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha}ne^{-\tau_n}}\right]. \end{aligned} \tag{B-3}$$

Thus, the optimization problem is

$$\tau_n^* = \arg \max_{\tau_n} \bar{R}_{sum}^{(j)}.$$

Taking the first-order derivative of (B-3) in terms of τ_n yields

$$\frac{\partial \bar{R}_{sum}^{(j)}}{\partial \tau_n} = \frac{W}{\hat{\alpha}M}\left[1 - \frac{\tau_n^2 + 4\tau_n + 4}{2\hat{\alpha}ne^{-\tau_n}}\right]. \tag{B-4}$$

Since, the second-order derivative of (B-3) is negative, the maximum value of $\bar{R}_{sum}^{(j)}$ is obtained by setting (B-4) equal to zero. So, we have

$$2\hat{\alpha}ne^{-\tau_n} = \tau_n^2 + 4\tau_n + 4,$$

or

$$\tau_n = \log 2\hat{\alpha}n - 2\log\tau_n - \log\left(1 + \frac{4\tau_n + 4}{\tau_n^2}\right). \tag{B-5}$$

It can be verified that the solution for (B-5) is

$$\tau_n^* = \log\hat{\alpha}n - 2\log\log\hat{\alpha}n + O(1). \tag{B-6}$$

Appendix III
Proof of Lemma 4

Using (B-3), the network throughput is obtained as

$$\begin{aligned}\bar{R}_{sum} &= \sum_{j=1}^{M} \bar{R}_{sum}^{(j)} \\ &= \frac{W}{\hat{\alpha}}\left[1 + \tau_n - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha}ne^{-\tau_n}}\right]. \end{aligned} \tag{C-1}$$

Considering the optimum threshold level obtained in (B-5) and $\hat{\alpha}ne^{-\tau_n} = \hat{\alpha}nq_n = \delta\log^2\hat{\alpha}n$, it can be easily shown that for $M \sim o(K)$

$$1 - \frac{\tau_n^2 + 2\tau_n + 2}{2\hat{\alpha}ne^{-\tau_n}} \;\sim\; O(1).$$

Hence by using $\tau_n = -\log q_n$, we arrive at the following equation

$$\bar{R}_{sum} \approx \frac{W}{\hat{\alpha}}\left[-\log q_n + O(1)\right]. \tag{C-2}$$

Through substituting (8) in (C-2) and using $n = \frac{K}{M}$, we finally obtain

$$\bar{R}_{sum} = \frac{W}{\hat{\alpha}}\left(\log\frac{K}{M} + o\left(\log\frac{K}{M}\right) + O(1)\right).$$

Appendix IV
Proof of Corollary 3

Noting that for $M = K$, only one user exists in each cluster, all the users can transmit with full power over the orthogonal subchannels. Hence, since $I_i^{(j)} = 0$, for $i = 1, ..., K$, the average sum-rate of the network is given by

$$\begin{aligned}\bar{R}_{sum} &= \mathbb{E}\left[\sum_{i=1}^{K} R_i^{(j)}\right] \\ &= \frac{W}{K}\sum_{i=1}^{K}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(j)}}{\frac{N_0W}{K}}\right)\right],\end{aligned}$$

where the expectation is computed with respect to $h_{ii}^{(j)}$. Under a Rayleigh fading channel model, we have

$$\bar{R}_{sum} = \frac{W}{K}\sum_{i=1}^{K}\int_0^\infty e^{-x}\log(1 + \tilde{\Upsilon}x)dx,$$

where $\tilde{\Upsilon} \triangleq \frac{K}{N_0W}$. Thus,

$$\begin{aligned}\bar{R}_{sum} &= W\int_0^\infty e^{-x}\log(1 + \tilde{\Upsilon}x)dx \\ &= We^{\frac{1}{\tilde{\Upsilon}}}\int_{1/\tilde{\Upsilon}}^\infty \frac{e^{-x}}{x}dx \\ &= We^{\frac{1}{\tilde{\Upsilon}}}\mathrm{E}_1\left(\frac{1}{\tilde{\Upsilon}}\right).\end{aligned} \tag{D-1}$$

To simplify (D-1), we use the following series representation for $\mathrm{E}_1(x)$,

$$\mathrm{E}_1(x) = -\boldsymbol{\gamma} - \log x + \sum_{s=1}^{\infty} \frac{(-1)^{s+1} x^s}{s.s!}, \tag{D-2}$$

where $\boldsymbol{\gamma}$ is Euler's constant and is defined by the limit [29]

$$\boldsymbol{\gamma} = \lim_{s \to \infty} \left(\sum_{k=1}^{s} \frac{1}{k} - \log s \right) = 0.577215665...$$

Thus, the average sum-rate of the network is obtained as

$$\bar{R}_{sum} = W e^{\frac{1}{\tilde{\Upsilon}}} \left(-\boldsymbol{\gamma} + \log \tilde{\Upsilon} + \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s.s!} \left(\frac{1}{\tilde{\Upsilon}} \right)^s \right).$$

For sufficiently large values of K, we have $\tilde{\Upsilon} = \frac{K}{N_0 W} \gg 1$, which results in $e^{\frac{1}{\tilde{\Upsilon}}} \approx 1$ and

$$\sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s.s!} \left(\frac{1}{\tilde{\Upsilon}} \right)^s \approx 0.$$

Consequently for $M = K$, the average sum-rate of the network is asymptotically obtained by

$$\bar{R}_{sum} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}).$$

References

[1] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, "Power allocation and asymptotic achievable sum-rates in single-hop wireless networks," in *Proc. 40th Conference on Information Sciences and Systems (CISS'06)*, Princeton University, Princton, NJ, March 2006.

[2] J. Abouei, M. Ebrahimi, and A. K. Khandani, "A new decentralized power allocation strategy in single-hop wireless networks," in *Proc. IEEE 41st Conference on Information Sciences and Systems (CISS'07)*, Johns Hopkins University, Baltimore, MD, USA, March 2007.

[3] A. Sampath, P. Sarath Kumar, and J. M. Holtzman, "Power control and resource management for a multimedia CDMA wireless system," in *Proc. IEEE PIMRC'95*, Sept. 1995, vol. 1, pp. 21–25.

[4] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE Journal on Selected Areas in Commun.*, vol. 25, no. 3, pp. 517 – 528, April 2007.

[5] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. on Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[6] O. Seong-Jun, D. Zhang, and K. M. Wasserman, "Optimal resource allocation in multiservice CDMA networks," *IEEE Trans. on Wireless Commun.*, vol. 2, no. 4, pp. 811–821, July 2003.

[7] Z. Han, Z. Ji, and K. J. R. Liu, “Fair multiuser channel allocation for OFDMA networks using nash bargaining solutions and coalitions,” *IEEE Trans. on Commun.*, vol. 53, no. 8, pp. 1366–1376, August 2005.

[8] J. Huang, R. A. Berry, and M. L. Honig, “Distributed interference compensation for wireless networks,” *IEEE Journal on Selected Areas in Commun.*, vol. 24, no. 5, pp. 1074 – 1084, May 2006.

[9] I. Katzela and M. Naghshineh, “Channel assignment schemes for cellular mobile telecommunication systems: a comprehensive survey,” *IEEE Personal Communications*, vol. 3, no. 3, pp. 10–31, June 1996.

[10] S. G. Kiani and D. Gesbert, “Maximizing the capacity of large wireless networks: optimal and distributed solutions,” in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Seattle, USA, July 2006, pp. 2501 – 2505.

[11] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Trans. on Information Theory*, vol. 46, no. 2, pp. 388 – 404, March 2000.

[12] M. Grossglauser and D. Tse, “Mobility increases the capacity of ad-hoc wireless networks,” *IEEE/ACM Trans. on Networking*, vol. 10, no. 4, pp. 477–486, August 2002.

[13] S. R. Kulkarni and P. Viswanath, “A deterministic approach to throughput scaling in wireless networks,” *IEEE Trans. on Information Theory*, vol. 50, no. 6, pp. 1041 – 1049, June 2004.

[14] L.-L. Xie and P. R. Kumar, “A network information theory for wireless communication: scaling laws and optimal operation,” *IEEE Trans. on Information Theory*, vol. 50, no. 5, pp. 748 – 767, May 2004.

[15] R. Gowaikar, B. Hochwald, and B. Hassibi, “Communication over a wireless network with random connections,” *IEEE Trans. on Information Theory*, vol. 52, no. 7, pp. 2857–2871, July 2006.

[16] N. Bansal and Z. Liu, “Capacity, delay and mobility in wireless ad-hoc networks,” in *Proc. IEEE INFOCOM*, April 2003, pp. 1553 – 1563.

[17] S. Toumpis and A. J. Goldsmith, “Large wireless networks under fading, mobility, and delay constraints,” in *Proc. IEEE INFOCOM*, March 2004, pp. 609 – 619.

[18] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, “Optimal throughput-delay scaling in wireless networks - part I: The fluid model,” *IEEE Trans. on Information Theory*, vol. 52, no. 6, pp. 2568–2592, June 2006.

[19] G. Sharma L. Xiaojun, R. R. Mazumdar, and N. B. Shroff, “Degenerate delay-capacity tradeoffs in ad-hoc networks with brownian mobility,” *IEEE Trans. on Information Theory*, vol. 52, no. 6, pp. 2777 – 2784, June 2006.

[20] M. J. Neely and E. Modiano, “Capacity and delay tradeoffs for ad hoc mobile networks,” *IEEE Trans. on Information Theory*, vol. 51, no. 6, pp. 1917–1937, June 2005.

[21] M. Sharif and B. Hassibi, “A delay analysis for opportunistic transmission in fading broadcast channels,” in *Proc. IEEE INFOCOM*, March 2005, pp. 2720 – 2730.

[22] A. Bayesteh, M. Ansari, and A. K. Khandani, “Throughput and fairness maximization in wireless downlink systems,” *Submitted to Trans. on Information Theory*, Sept. 2007.

[23] C. Comaniciu and H. V. Poor, “On the capacity of mobile ad hoc networks with delay constraints,” *IEEE Trans. on Wireless Commun.*, vol. 5, no. 8, pp. 2061–2071, August 2006.

[24] C. Bettstetter and C. Hartmann, "Connectivity of wireless multihop networks in a shadow fading environment," *ACM/Kluwer Wireless Networks, Special Issue on Modeling and Analysis of Mobile Networks*, vol. 11, no. 4, July 2005.

[25] M. J. Neely, "Order optimal delay for opportunistic scheduling in multi-user wireless uplinks and downlinks," in *Proc. of 44th Allerton Conference on Communication, Control, and Computing*, Sept. 2006.

[26] J. Abouei, A. Bayesteh, M. Ebrahimi, and A. K. Khandani, "Sum-rate maximization in single-hop wireless networks with the on-off power scheme," in *Proc. IEEE International Symposium on Information Theory (ISIT'07)*, Nice, France, June 2007.

[27] J. Abouei, A. Bayesteh, and A. K. Khandani, "Delay-throughput analysis in decentralized single-hop wireless networks," in *Proc. IEEE International Symposium on Information Theory (ISIT'07)*, Nice, France, June 2007.

[28] D. E. Knuth, "Big omicron and big omega and big theta," in *ACM SIGACT News*, April-June 1967, vol. 8, pp. 18 – 24.

[29] I. S. Gradshteyn, I. M. Ryzhik, and A. Jeffrey, *Table of Integrals, Series, and Products*, Academic Press, 1994.

[30] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. on Information Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.