Throughput Scaling in Decentralized Single-Hop Wireless Networks with Fading Channels

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Abstract

A network of n communication links operating over a shared wireless channel is considered. Fading is assumed to be the dominant factor affecting the strength of the channels between nodes. The objective is to maximize the throughput of the network. A scheme that can be simply implemented in a distributed and single-hop fashion is proposed and analyzed. It is shown that under Rayleigh and log-normal fading conditions, the proposed scheme achieves sum-rates that scale as $\log n$ and $e^{\sqrt{2}S\sqrt{\log n}}$, respectively, where S is a channel parameter and a constant. This is the same as what is obtained by a centralized and multihop method in the work of Gowaikar et al. Nevertheless, an analysis in the shadow fading model points out that when the network is not well-connected, single-hop methods are suboptimum and invoking multihop communication is inevitable to achieve the maximum throughput. An interesting aspect of the proposed method is the tradeoff it introduces between the scaling factor and the rate per link; it is shown that the proposed method allows rate-per-links of order $\Theta(1)$ while keeping the

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order of the sum-rate unchanged. This is in contrary to the most previous works that enforce a vanishingly small rate for each link.

Index Terms

Wireless network, decentralized power allocation, fading channel, sum-rate, scaling law.

I. Introduction

In a wireless network, a number of source nodes transmit data to their designated destination nodes through a shared wireless channel. Followed by the pioneering work of Gupta and Kumar [1], considerable attention has been paid to investigate how the throughput of wireless networks scales with n, the number of nodes. This has been done assuming different network topologies, traffic patterns, protocol schemes, and channel models [1]–[12].

From the information theoretic point of view, a wireless network is a generalized version of the *interference channel* [13], whose capacity region has not been fully characterized, yet. Consequently, there are only a few papers that take a pure information theoretic approach to the throughput of wireless networks [2]–[4]; instead, most researchers base their throughput analyses on certain simplifying assumptions including Gaussian signal transmission, linear receiver structures (which excludes interference cancelation), and point-to-point coding (which excludes for example multi-access and broadcast schemes). In this case, interference at each receiver is treated as *additive white Gaussian noise* (AWGN) and the *signal to interference-plus-noise ratio* ($SINR$) along with the Shannon capacity formula determine the achievable rate of each link. We will follow this paradigm throughout this paper.

In [1], it is assumed that n nodes are distributed over a constant area. Also, each node is the source of exactly one data stream to be sent to a destination randomly or arbitrarily chosen from the other $n - 1$ nodes. With this traffic pattern and under the fairness constraint that all data streams be transmitted at the same rate, it is shown that a rate of $1/\sqrt{n}$ and $1/\sqrt{n\log(n)}$ per node is achievable in arbitrary and random networks[1], respectively. An extension of this result for the case that nodes are mobile or under delay constraints was provided in [5]. The result of [1] was later improved by [6], [7] by showing that even random networks can achieve the rate of $1/\sqrt{n}$ per node. In all of these works, rate per nodes decrease to zero as the number of nodes grows. However, it is practically appealing to have rate per nodes that remain constant when n grows. In [8], it is shown that a nondecreasing rate per node is achievable when nodes are mobile. In this work, we address this issue and show that even for fixed networks, it is possible to have rate per links like $\Theta(1)$ while keeping the order of the maximum sum-rate unchanged.

Most of the works analyzing the throughput of large wireless networks consider a channel model in which the signal power decays according to a distance-based attenuation law [1]–[4], [6]–[10]. However, in a wireless environment the presence of obstacles and scatterers adds some randomness to the received signal. In addition, the power attenuation laws may not be valid when the receiver is not in the far field of the transmitter as in the dense networks. This random behaviour of the channel, known as fading, can drastically change the scaling laws of a network. This can be verified by reviewing the results of the few papers that have considered fading in their study of the throughput scaling law [5], [11], [12]. In [5], it is shown that for the same setup as in [1], the presence of fading decreases the order of the lower bound on rate per node by a factor of $\log n$.

In this paper, we follow the model of [11], where fading is assumed to be the dominant factor affecting the strength of the channels between nodes. In [11], two nodes are considered as a potential transmitter-receiver pair only if the fading channel between them is *good* enough. The transmission protocol, which is basically a scheduling, is based on a result from random graph theory about noncolliding paths between randomly chosen pairs of vertices. Despite most of the previous works, only a fraction of data streams are scheduled for transmission; however, the scheduled data streams are transmitted at the

[1]In an arbitrary network, the nodes locations can be chosen optimally, but, in a random network the nodes are located randomly.

same rate. This data rate, the number of active source-destination pairs, and the channel *goodness* criterion are chosen such that the total throughput of the network is maximized. It is shown that the throughput of the network strongly depends on the fading channel distribution. We perform throughput analysis for Rayleigh, log-normal, and shadow fading channels.

Generally, a realistic model of wireless network channels should take into account both randomness and distance-based effects. Recently, a generalization of the work in [11] has appeared in [12], where the authors consider a general fading model on top of a power decay law.

A common attribute of the works in [1]–[12] is that the information flow is routed through some intermediate nodes, named relay or router, to reach the final destination. Such a strategy, which is called multihop communication, necessitates a central unit with full knowledge of all channel conditions that decides on the routing paths and the schedule of transmissions. Furthermore, it induces delay in the network that can not be tolerated in some applications. Also, implicit in multihop communication is the excess power consumption by the relay nodes. This latter issue can be a critical disadvantage in applications where the total power is constrained. In this work, we present a strategy that allows data to be transmitted directly from sources to their corresponding receivers without utilizing any other nodes as routers. This model includes single-hop ad hoc networks, cellular networks, and code division multiple access (CDMA) systems as its special cases. This single-hop communication scheme can be implemented in a decentralized fashion. Each source node needs to know only its direct channel to the corresponding destination. The generality of this method allows it to be applied to any fading channel models. It turns out that in the popular models of Rayleigh and log-normal fading, the proposed method achieves the same sum-rate as what is obtained in the centralized and multihop method of [11].

Contrary to the aforementioned works on large wireless networks, there are a lot of works in the literature investigating the throughput optimization in networks with arbitrarily small sizes. Based on the network structure, throughput optimization can be executed in different ways, e.g. by power control [14], bandwidth allocation [15], [16], transmission scheduling [17], routing [18], [19], base station selection [20], etc. Among these various challenging problems, power control has a prominent role in the past and ongoing research in this area. The problem of minimizing the transmit power subject to satisfying some quality of service requirements can be formulated as a linear program [17] and can be solved in a decentralized fashion [14], [21]. However, the problem of sum-rate maximization subject to some power constraints, that has been frequently appeared in the literature [22]–[25], turns out to be a nonlinear nonconvex problem. One approach to solve this problem is to utilize numerical optimization methods (see e.g. [24]). Another approach is to adopt an approximation of the objective function such that the problem can be converted to a convex program [22]–[24]. As will be explained in the next section, this work is based on power allocation and some related observations reported in the literature.

The rest of the paper is organized as follows. In Section II, the system description, objective, and problem formulation, are presented . We derive a lower bound on the average sum-rate in Section III. The analyses of the proposed method for some specific fading models is studied in Section IV. We discuss the tradeoff between sum-rate and rate per link in Section V. Finally, we conclude the paper in Section VI.

Notation: Bold face lower case (upper case) letters denote vectors (matrices); $\mathbf{0}_n$ and $\mathbf{1}_n$ stand for the all-zero and all-one column vectors of length n, respectively; $\mathbb{N}_n$ represents the set of natural numbers less than or equal to n; $\boldsymbol{x}_{-i}$ is a vector obtained by eliminating the ith element of $\boldsymbol{x}$; $\boldsymbol{x} \leq \boldsymbol{y}$ or $\boldsymbol{x} < \boldsymbol{y}$ denote element-wise inequality; $\log$ is the natural logarithm function; $\approx$ means approximate equality; for any functions $f(n)$ and $h(n)$, $h(n) = O(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| < \infty$, $h(n) = \Omega(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| > 0$, $h(n) = o(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = 0$, $h(n) = \omega(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = \infty$, $h(n) = \Theta(f(n))$ is equivalent to $\lim_{n\to\infty} |h(n)/f(n)| = c$, where $0 < c < \infty$, and

$h(n) \sim f(n)$ is equivalent to $\lim_{n\to\infty} h(n)/f(n) = 1$.

II. System Description and Preliminaries

We consider a wireless communication network with n pairs of transmitters and receivers. Each transmitter aims to send data to its corresponding receiver. We denote the vector of transmit powers by $\mathbf{p} = (p_1, \cdots, p_n)$, where p_i is the transmit power of link i. The following constraint is imposed on the transmit power

$$\mathbf{0}_n \leq \mathbf{p} \leq \mathbf{a}, \tag{1}$$

where $\mathbf{a} = (a_1, \cdots, a_n)$ represents the vector of maximum allowed transmit powers.

The channels are represented by coefficients $G_{ji} = |\alpha_{ji}|^2$, where α_{ji} is the channel gain between receiver i and transmitter j. This means the received power from transmitter j at the receiver i equals $G_{ji}p_j$. The channel gains, in general, depend on small scale and large scale fadings, path attenuation, processing gain of the CDMA system, etc. In this paper, we assume the channel coefficients are i.i.d. random variables drawn from a pdf $f(x)$ with mean μ and variance σ^2.

We consider an additive white Gaussian noise (AWGN) with variance η_i at the receiver i. The receivers are conventional, linear receivers, i.e., without multiuser detection. Since the transmissions occur simultaneously within the same environment, the signal from each transmitter acts as interference for other links. Assuming Gaussian signal transmission from all links, the distribution of the interference will be Gaussian as well. Thus, we can define the *SINR* of the receiver i as

$$\gamma_i(\mathbf{p}) = \frac{G_{ii}p_i}{\eta_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} G_{ji}p_j}. \tag{2}$$

Throughout the paper, we occasionally use γ_i instead of $\gamma_i(\mathbf{p})$. The *SINR* determines different QoS measures such as the maximum possible data rate, or the error probability of link i.

In this paper, we are interested in rates at which the transmitters can send data to their corresponding receivers without any error. According to the Shannon capacity formula [13], the maximum rate of link i is equal to

$$r_i(\mathbf{p}) = \log\left(1 + \gamma_i(\mathbf{p})\right) \quad \mathit{nats/channel\ use}. \tag{3}$$

The network rate vector is defined as $\mathbf{r} = (r_1, \cdots, r_n)$. In a network, we desire to have all rates as large as possible. However, due to the interplay between the rates of different links (see (2) and (3)), it is not possible to maximize all the rates simultaneously. Instead, one may consider maximizing a utility function of the network which is increasing in all rates. A common utility function is the sum-rate of the network.

The problem of sum-rate maximization is formulated as follows:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{n} r_i(\mathbf{p}), \\ \text{s.t.} \quad & \mathbf{0}_n \leq \mathbf{p} \leq \mathbf{a}, \end{aligned} \tag{4}$$

which is a non-convex optimization problem. Thus, the algorithms developed for convex problems may converge to local optimum points for this problem. The following proposition identifies one of the characteristics of problem (4).

Proposition 1: In the optimum solution $\mathbf{p}^*$ of (4), the power of at least one link takes its maximum allowed value.

Proof: For the sake of contradiction, assume $\mathbf{p}^* < \mathbf{a}$. Consider the index set $\mathcal{I} = \{i \in \mathbb{N}_n : p_i^* > 0\}$ and define

$$\alpha^* = \min_{i \in \mathcal{I}} \left\{ \frac{a_i}{p_i^*} \right\}. \tag{5}$$

Choose a new power vector $\hat{\mathbf{p}} = \alpha^* \mathbf{p}^*$, which obviously satisfies the constraints $\mathbf{0}_n \leq \hat{\mathbf{p}} \leq \mathbf{a}$. Since $\alpha^* > 1$, it follows that

$$\sum_{i=1}^{n} r_i(\hat{\mathbf{p}}) > \sum_{i=1}^{n} r_i(\mathbf{p}), \tag{6}$$

which is in contradiction to the optimality of $\mathbf{p}^*$. ∎

There are some special cases, where even stronger statements can be expressed about the solution of (4). These special cases are listed below:

Special Case 1 (Uplink CDMA): The result in [25] indicates that in the uplink CDMA, where $G_{ji} = G_j$ for all $j \in \mathbb{N}_n$, the power of all links take the value of zero or the maximum allowed value except for at most one link.

Special case 2 ($n = 2$): When there are only two links sharing a wireless channel, we have the following interesting result.

Proposition 2: The optimum solution of (4) for $n = 2$ is obtained when one of the transmitters transmits with maximum power and the other one is silent, or both transmitters transmit with maximum power[2].

Proof: Assume for simplicity that channel coefficients and noise powers are scaled such that the maximum allowed power of both links and also the direct channel coefficients G_{ii} are equal to one. According to Proposition 1, in the optimum solution of (4) the power of at least one link should be equal to one; without loss of generality assume $p_2 = 1$. It suffices to show that the maximum of the function

$$f(p_1) = \log\left(1 + \frac{p_1}{\eta_1 + G_{21}}\right) + \log\left(1 + \frac{1}{\eta_2 + G_{12}p_1}\right) \tag{7}$$

is obtained either at $p_1 = 0$ or $p_1 = 1$. By computing the derivative of $f(p_1)$ and simplifying it we obtain

$$f'(p_1) = \frac{Ap_1^2 + Bp_1 + C}{d(p_1)}, \tag{8}$$

where $A = G_{12}^2$, $B = 2\eta_2 G_{12}$, $C = \eta_2(\eta_2+1) - G_{12}(\eta_1 + G_{21})$, and $d(p_1)$ is a polynomial in p_1 with all coefficients non-negative. Thus, the sign of $f'(p_1)$ is determined by its numerator. Note that $A, B \geq 0$. If $C \geq 0$, the numerator (and thus $f'(p_1)$) is always non-negative for $p_1 \geq 0$. Thus, $f(p_1)$ is increasing in p_1 and achieves its maximum at $p_1 = 1$. If $C < 0$, the numerator has exactly one positive (p_1') and one negative (p_1'') roots. Thus, $f(p_1)$ has a minimum at p_1' and attains its maximum at 0 or 1. ■

[2]After the authors published this result in [26], it was independently reported in [27].

Obviously, if in the optimum solution only one link is active, it should be the link with the largest direct channel coefficient.

Special case 3 (low SINR regime): If we know that the *SINR* of all links is small, we can use the approximation $\log(1+x) \approx x$ to write (4) as follows

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{n} \gamma_i(\mathbf{p}), \\ \text{s.t.} \quad & \mathbf{0}_n \leq \mathbf{p} \leq \mathbf{a}. \end{aligned} \tag{9}$$

Although this problem is again non-convex, the following result can be concluded that allows for obtaining the optimum solution by enumerating the vertices of the hypercube of the power domain.

Proposition 3: In the optimum solution $\mathbf{p}^*$ of (9), all transmit powers are either zero or the maximum allowed value, i.e., $p_i^* \in \{0, a_i\}$ for all $i \in \mathbb{N}_n$.

Proof: Define the objective function

$$T(\mathbf{p}) = \sum_{i=1}^{n} \gamma_i(\mathbf{p}). \tag{10}$$

If for some $i \in \mathbb{N}_n$, $\boldsymbol{p}_{-i}^* = \mathbf{0}_{n-1}$, clearly $p_i^* = a_i$ maximizes the sum-rate and the proof is complete. If $\boldsymbol{p}_{-i}^* \neq \mathbf{0}_{n-1}$, by substituting the values of $\gamma_i(\boldsymbol{p})$ from (2) in the objective function (10) and computing the second order partial derivative with respect to p_i we obtain

$$\frac{\partial^2 T(\boldsymbol{p})}{\partial p_i^2} = 2 \sum_{j \neq i} G_{ij}^2 \frac{\gamma_j(\boldsymbol{p})}{d_j^2(\boldsymbol{p})}, \tag{11}$$

which is positive for all $\boldsymbol{p}_{-i} \neq \mathbf{0}_{n-1}$. Thus, $T(\boldsymbol{p})$ is convex with respect to p_i. As a result, the maximizing value of p_i lies on one end of the interval $[0, a_i]$. ∎

The low *SINR* scenario is of special interest in this paper. In fact, when there are many links sharing a wireless channel, most likely they should operate in the low *SINR* regime. In this case, Proposition 3 suggests an *on-off* strategy to maximize the sum-rate. This strategy is the basis of our proposed method in the next section.

III. A LOWER BOUND ON SUM-RATE

It is interesting to know how the throughput of a wireless network scales with the number of nodes, when this number is large. In this section, we present a simple heuristic power allocation scheme, which yields to a lower bound on the average sum-rate of the wireless network described before. Motivated by the special cases listed at the end of Section II, particularly Proposition 3, this scheme is based on the *on-off power allocation strategy*.

Definition 1: A power allocation is called an *on-off strategy* if the power of link i is selected from the set $\{0, a_i\}$.

We provide a power allocation strategy and obtain the sum-rate which is *asymptotically almost surely* (*a.a.s.*) achievable by this scheme. In the following, we assume all links have power constraints equal to P, i.e., $\mathbf{a} = P\mathbf{1}_n$. Furthermore, the noise powers at all receivers are limited and the same, i.e., $\eta_i = \eta < \infty$.

Consider a threshold t and assume that link i is activated and transmits with full power if $G_{ii} > t$; otherwise, it remains silent. Note that the performance of this on-off strategy depends on the value of the threshold t; if t is very large, the quality of the selected links will be very good, but the number of such links is small and as a result the achieved sum-rate will be small; on the other hand, if t is very small, many links are chosen, but it causes a large interference and again the sum-rate will be small. Thus, it is crucial to choose a proper value for t.

Let k denote the number of active links. Without loss of generality, we assume that the active links are indexed by $1, 2, \cdots, k$. The corresponding sum-rate is equal to

$$R = \sum_{i=1}^{k} \log\left(1 + \frac{G_{ii}}{\rho + \sum_{\substack{j=1\\ j\neq i}}^{k} G_{ji}}\right), \tag{12}$$

where $\rho = \eta/P$. Considering the fact that $G_{ii} > t$ for the selected links and by defining the interference term $I_i = \sum_{\substack{j=1\\ j\neq i}}^{k} G_{ji}$ for all $i = 1, 2, \cdots, k$, we obtain the following lower bound on the sum-rate

$$R \geq \sum_{i=1}^{k} \log\left(1 + \frac{t}{\rho + I_i}\right). \tag{13}$$

This lower bound can be further simplified by using the Jensen's inequality, i.e.,

$$R \geq k \log\left(1 + \frac{t}{\rho + \frac{1}{k}\sum_{i=1}^{k} I_i}\right), \tag{14}$$

Let's define $I = \frac{1}{k}\sum_{i=1}^{k} I_i$, which is the empirical average of the interference terms. From the definition of I_i, we conclude that

$$\mathrm{E}[I] = (k-1)\mu \tag{15}$$

$$\mathrm{Var}[I] = \frac{k-1}{k}\sigma^2. \tag{16}$$

Since the mean of I_i grows with k, the classical form of the strong law of large numbers can not be applied to conclude that I tends to its mean as $k \to \infty$. However, the following Lemma indicates that I is in some sense concentrated around its mean.

Lemma 1: Assume I_i ($i = 1, 2, \cdots, k$) are i.i.d. random variables with mean $(k-1)\mu$ and variance $(k-1)\sigma^2$. Then, for $I = \frac{1}{k}\sum_{i=1}^{k} I_i$, we have

$$|I - (k-1)\mu| < \psi_k, \quad a.a.s., \tag{17}$$

for any $\psi_k \to \infty$ as $k \to \infty$.

Proof: By using the mean and variance of I from (15) in the Chebyshev inequality, we obtain

$$\mathrm{P}\left\{|I - (k-1)\mu| < \psi_k\right\} \geq 1 - \frac{k-1}{k}\frac{\sigma^2}{\psi_k^2} > 1 - \frac{\sigma^2}{\psi_k^2}. \tag{18}$$

If $\psi_k \to \infty$ as $k \to \infty$, the right hand side of the above inequality tends to 1. This proves the lemma. ■

According to Lemma 1, the lower bound in (14) can be written as

$$R \geq k \log\left(1 + \frac{t}{\mu k + \psi_k}\right), \quad a.a.s., \tag{19}$$

for any $\psi_k \to \infty$ as $k \to \infty$. Note that the constant $\rho - \mu$ is absorbed in the function ψ_k. This lower bound is only a function of t and k. Parameter t is a constant to be chosen optimaly later, however, k is a random variable. Assume the probability of a link being active is q. Due to our link activation strategy, which selects links independently and with probability q, the number of active links, k, is a binomial random variable with parameters n and q. The following lemma indicates that k is asymptotically almost surely concentrated around its mean.

Lemma 2: If k is a binomial random variable with parameters n and q, then we have

$$|k - nq| < \xi_n \sqrt{nq}, \quad a.a.s. \tag{20}$$

for any $\xi_n \to \infty$ as $n \to \infty$.

Proof: Since k is binomial with parameters n and q, its mean and variance are equal to nq and $nq(1-q)$, respectively. Using these values in the Chebyshev inequality, we obtain

$$\begin{aligned} 1 - \frac{1}{\xi_n^2} &\leq \mathrm{P}\left\{|k - nq| < \xi_n \sqrt{nq(1-q)}\right\} \\ &\leq \mathrm{P}\left\{|k - nq| < \xi_n \sqrt{nq}\right\}, \end{aligned} \tag{21}$$

where the second inequality is due to the fact that $1 - q < 1$. If $\xi_n \to \infty$ as $n \to \infty$, the left hand side of the above inequality tends to 1. This proves the lemma. ∎

As mentioned before, Lemmas 1 and 2 are valid for any ψ_k and ξ_n that grow unboundedly with k and n, respectively. However, to benefit from these results, in the following, we are only interested in such cases that the growth rate of these two functions is slower than some certain order.

Now, we are ready to prove the main result of this section, which is an achievability result on the sum-rate of wireless networks under consideration.

Theorem 1: Consider a wireless network with n links and i.i.d. random channel coefficients with pdf $f(x)$, cdf $F(x)$, and mean μ. Choose any $t > 0$ and define

$q = 1 - F(t)$. Then, a sum-rate of

$$R(t) = (nq - \xi_n\sqrt{nq}) \log\left(1 + \frac{t}{\mu(nq - \xi_n\sqrt{nq}) + \varphi_n}\right) \tag{22}$$

is a.a.s. achievable for any $\xi_n = o(\sqrt{nq})$ that approach infinity as $n \to \infty$ and any $\varphi_n = \psi(nq - \xi_n\sqrt{nq})$, where $\psi(n)$ makes the function

$$n \log\left(1 + \frac{c}{\mu n + \psi(n)}\right) \tag{23}$$

increasing in k for any constant c, and $\psi(n) \to \infty$ as $n \to \infty$.

Proof: By applying the on-off power allocation strategy, we obtained a lower bound on the achievable sum-rate in (19). According to condition (23), this lower bound is increasing in k. Thus, if we replace the number of active links, k, by its possible lower bound, the achievability result remains valid. However, from Lemma 2 we know that

$$k > nq - \xi_n\sqrt{nq}, \quad a.a.s. \tag{24}$$

By the assumption $\xi_n = o(\sqrt{nq})$, the lower bound in (24) is non-negative and can replace k in (19) to give (22) with $\varphi_n = \psi(nq - \xi_n\sqrt{nq})$. ■

We can conclude the following corollaries from the above discussion and from the proof of Theorem 1.

Corollary 1: In the on-off strategy that achieves the sum-rate given in Theorem 1, the number of active links scales as

$$k \sim nq \quad a.a.s. \tag{25}$$

Proof: The proof is starightforward by using the assumption $\xi_n = o(\sqrt{nq})$ in Lemma 2. ■

An important parameter in a wireless network is the rate at which each scheduled link can transmit its own data stream. We refer to this parameter as *rate per link* and denote it by λ. The following corollary quantifies this parameter for the proposed method.

Corollary 2: In the on-off strategy that achieves the sum-rate given in Theorem 1, the rate per link scales as

$$\lambda \sim \log\left(1 + \frac{t}{\mu(nq - \xi_n\sqrt{nq}) + \varphi_n}\right) \quad a.a.s. \tag{26}$$

Proof: The result is simply obtained by dividing the sum-rate $R(t)$ from (22) by the number of users k from (25). ∎

As specified in (22), the achievable sum-rate in Theorem 1 is a function of the parameter t. Thus, t can be chosen such that the achievable sum-rate in Theorem 1 is maximized. Let's define

$$t^* = \arg\max_t R(t), \tag{27}$$

and

$$R^* = \max_t R(t) \tag{28}$$

to be the optimum threshold and the maximum achievable sum-rate, respectively.

Corollary 3: In the on-off strategy, the average achieved sum-rate satisfies

$$\mathrm{E}[R] \geq R^*, \tag{29}$$

where R^* is as defined in (28).

Note that Corollary 3 expresses a deterministic statement, in contrary to previous statements which hold asymptotically almost surely.

Proof: By definition of conditional expectation, we have

$$\begin{aligned} \mathrm{E}[R] &= \mathrm{E}[R|R \geq R^*] \Pr[R \geq R^*] \\ &\quad + \mathrm{E}[R|R < R^*] \Pr[R < R^*] \\ &\geq R^* \times \Pr[R \geq R^*] + 0 \times \Pr[R < R^*] \\ &= R^*, \end{aligned} \tag{30}$$

where the last equality holds asymptotically by choosing $t = t^*$ in Theorem 1. ∎

It is worth mentioning that in the suggested on-off strategy, no coordination is required between the links. All one transmitter needs to know is wether its direct channel coefficient is above the threshold t^*. Based on this information, it decides wether to transmit with full power or remain silent. In general, the values of t^* and R^* are functions of n, but how they scale with n strongly depends on the channel distribution function

$f(x)$. Specifically, one needs to know the relation between q and t as well as the value of μ to obtain t^* and R^*. In the next Section we provide some examples and show how the achievable sum-rates depend on the fading model.

IV. Case Study

In this section, we provide some examples of fading channels and show how to apply the method and result of the previous section to each case. We start with the popular channel model of Rayleigh fading and study it in detail. We also adopt two more models from [11], i.e., shadow fading and log-normal fading models. The results mention that the scaling laws of the sum-rate drastically depend on the fading channel model under consideration. It also turns out that the sum-rate scaling law achieved by our decentralized scheme is the same as what is obtained in [11] for the case of Rayleigh fading and log-normal fading models. However, it is smaller in the case of shadow fading model. This observation points out the need for multihop in cases where the interference term is strong.

A. Rayleigh Fading

In a Rayleigh fading channel, the coefficients G_{ji} are exponentially distributed with $f(x) = e^{-x}$ and $\mu = 1$. Thus, using the corresponding cumulative distributed function, the relation between q and t is described as $q = e^{-t}$. By substituting this value in (22), we obtain the function to be maximized as

$$R(t) = \left(ne^{-t} - \xi_n\sqrt{ne^{-t}}\right)\log\left(1 + \frac{t}{ne^{-t} - \xi_n\sqrt{ne^{-t}} + \hat{\psi}_n}\right). \tag{31}$$

Lemma 3: In a wireless network with Rayleigh fading channels and n links, the optimum threshold for the on-off strategy is equal to

$$t^* = \log n - 2\log\log n + \log 2 + \frac{4\log\log n}{\log n} + O\left(\frac{\xi_n}{\log n}\right), \tag{32}$$

where $\xi_n = o(\log n)$.

Proof: see Appendix A. ■

Corollary 4: In a wireless network with Rayleigh fading channels and n links, we have

$$R^* = \log n - 2 \log\log n + O(1), \quad a.a.s. \tag{33}$$

Proof: Substitute the value of t^* from (32) in (31). ■

By applying the result of Lemma 3 to Corollaries 1, 2, and 3, we obtain the following corollary.

Corollary 5: In a wireless network with n links and under Rayleigh fading channel model, on-off strategy yields

$$k \sim \frac{1}{2}\log^2 n - 2\log\log n \times \log n + O(\xi_n \log n), a.a.s., \tag{34}$$

$$\lambda \sim \frac{2}{\log n}, \qquad a.a.s., \tag{35}$$

$$\mathrm{E}[R] \geq \log n - 2\log\log n + O(1). \tag{36}$$

In the above, we analyzed the scaling laws of the wireless network, when the the threshold t and the number of users k are optimaly chosen. However, a natural question is how the scaling laws change when the network serves more or less users (links) than the optimum obtained above. The following Lemma addresses this issue.

Lemma 4: In a wireless network with Rayleigh fading channels, if the k best links (links with largest self channel coefficients) are permitted transmission, then, the achievable sum-rate R is related to the number of active links k according to

$$R \sim \begin{cases} o(\log n) & \text{if} \quad k = o(\log n) \\ \alpha \log\left(1 + \frac{1}{\alpha}\right)\log n & \text{if} \quad k \sim \alpha \log n \text{ for some } \alpha > 0 \\ \log n & \text{if} \quad k = \omega(\log n) \text{ and } k = o(n^{\alpha}), \quad \forall\, 0 < \alpha < 1 \\ (1-\alpha)\log n & \text{if} \quad k \sim n^{\alpha+\epsilon_n} \text{ for some } 0 < \alpha < 1 \text{ and some } \epsilon_n \to 0 \\ o(\log n) & \text{if} \quad k \sim n^{1-\epsilon_n} \text{ for some } \epsilon_n \to 0^+ \end{cases}, \tag{37}$$

asymptotically almost surely, where the limits are taken as $n \to \infty$.

Proof: The proof relies on the following two equations, that we have already seen, but are repeated here for simplicity

$$\begin{aligned} k &\sim ne^{-t} && (38)\\ R &= ne^{-t}\log\left(1+\frac{t}{ne^{-t}}\right). && (39)\end{aligned}$$

The first equation describes the relation between t and k, and the second one gives the sum-rate R in terms of t. We proceed by the proof in a case by case basis.

1) If $k = o(\log n)$, we can write $k = \frac{\log n}{\zeta_n}$, where $\zeta_n \to \infty$ as $n \to \infty$. Hence, we obtain

$$t = \log n - \log\log n + \log(\zeta_n), \tag{40}$$

Consequently, from (39) we obtain

$$\begin{aligned} R &\sim \frac{\log n}{\zeta_n}\log\left(\frac{\log n}{\frac{\log n}{\zeta_n}}\right)\\ &= \frac{\log \zeta_n}{\zeta_n}\log n\\ &= o(\log n) && (41)\end{aligned}$$

2) If $k \sim \alpha \log n$ for some constant $\alpha > 0$, we have

$$t \sim \log n - \log\log n - \log\alpha. \tag{42}$$

As a result, we obtain

$$\begin{aligned} R &\sim \alpha\log n\log\left(1+\frac{\log n}{\alpha\log n}\right)\\ &= \alpha\log\left(1+\frac{1}{\alpha}\right)\log n && (43)\end{aligned}$$

3) If $k = \omega(\log n)$ and $k = o(n^\alpha)$ for all $0 < \alpha < 1$, we should have

$$t \sim \log n. \tag{44}$$

Hence, we obtain

$$\begin{aligned} R &\sim \omega(\log n)\log\left(1+\frac{\log n}{\omega(\log n)}\right)\\ &\sim \log n, && (45)\end{aligned}$$

where the last equality is based on the approximation $\log(1+x) \approx x$ for small values of x.

4) If $k \sim n^{\alpha+\epsilon_n}$ for some $0 < \alpha < 1$ and some $\epsilon_n \to 0$, we should have

$$\begin{aligned} t &\sim \log n - \log n^{\alpha+\epsilon_n} \\ &= (1-\alpha-\epsilon_n)\log n \\ &\sim (1-\alpha)\log n \end{aligned} \tag{46}$$

Substituting these values of k and t in (39) gives

$$R \sim (1-\alpha)\log n. \tag{47}$$

5) If $k \sim n^{1-\epsilon_n}$ for some $\epsilon_n \to 0^+$, we have

$$\begin{aligned} t &\sim \log n - \log n^{1-\epsilon_n} \\ &= \epsilon_n \log n \\ &= o(\log n) \end{aligned} \tag{48}$$

Substituting these values of k and t in (39) gives

$$R = o(\log n). \tag{49}$$

■

This lemma points out that there is a vast range for values of k that can result in sum-rates of order $\log n$. This range starts from $\Theta(\log n)$ and includes all values of k as long as $k = o(n)$. This effect is shown in Fig. 1, where R is depicted versus k for three different values of n. It is seen that while decreasing k from its optimum value rapidly decreases the sum-rate, the rate of variations in larger values of k is fairly slow. The range of logarithmic variations of sum-rate is also of interest to us due to the rate per link issue to be discussed in Section V.

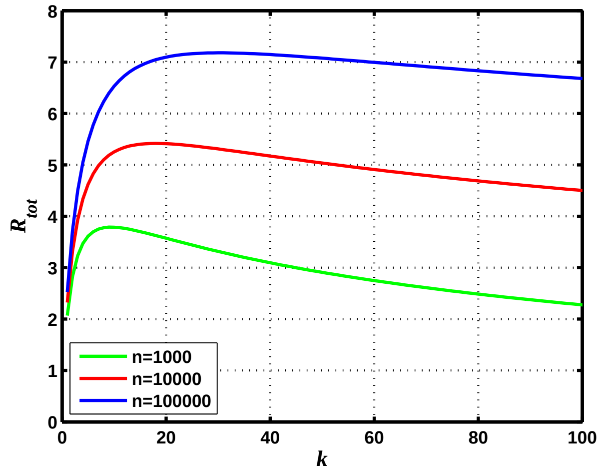

Fig. 1. Sum-rate vs. number of active links for $n = 1000,\ 10000,\ 100000$.

B. *Log-Normal Fading*

Consider a network with channel strengths drawn i.i.d. from a log-normal distribution with pdf

$$f(x) = \frac{1}{\sqrt{2\pi}Sx} e^{-\frac{(\log x - M)^2}{2S^2}},\ x \geq 0 \tag{50}$$

with S and M being the parameters of the distribution [11]. The following proposition establishes the relation between q and t in the log-normal fading model for large values of t.

Proposition 4: Assume X is a log-normal random variable with the pdf given in (50) and let $q = \Pr[X > t]$. Then, for large values of t we have

$$q \approx \frac{S}{\sqrt{2\pi}(\log t - M)} e^{-\frac{(\log t - M)^2}{2S^2}}. \tag{51}$$

Proof: By the definition of q, we have

$$q = \int_t^\infty \frac{1}{\sqrt{2\pi}Sx} e^{-\frac{(\log x - M)^2}{2S^2}} dx \quad (52)$$

$$= \int_{\log t}^\infty \frac{1}{\sqrt{2\pi}S} e^{-\frac{(x-M)^2}{2S^2}} dx \quad (53)$$

$$= \frac{1}{2}\,\text{erfc}\left(\frac{\log t - M}{\sqrt{2}S}\right), \quad (54)$$

where $\text{erfc}(x) = \dfrac{2}{\sqrt{\pi}} \int_x^\infty e^{-z^2} dz$ is the complementary error function. By using the approximation $\text{erfc}(x) \approx \frac{1}{\sqrt{\pi}x} e^{-x^2}$ for large values of x, the result is obtained. ■

By substituting the value of q from Proposition 4 in (22), the sum-rate, as a function of t, is obtained as

$$R(t) = \frac{S}{\sqrt{2\pi}} \frac{n}{u} e^{-\frac{u^2}{2S^2}} \log\left(1 + \frac{Bue^u e^{\frac{u^2}{2S^2}}}{n}\right), \quad (55)$$

where, for the brevity of notation, we have defined $u = \log t - M$. Also, B is a constant depending on the distribution parameters.

Lemma 5: In a wireless network with log-normal fading channels and n links, the optimum threshold for the on-off strategy satisfies

$$t^* \sim e^{M - S^2} e^{\sqrt{2}S\sqrt{\log n}}. \quad (56)$$

Proof: See Appendix B. ■

Corollary 6: In a wireless network with log-normal fading channels and n links, we have

$$R^* \sim e^{-\frac{3S^2}{2}} e^{\sqrt{2}S\sqrt{\log n}} \quad a.a.s. \quad (57)$$

Proof: Substitute the value of t^* from (56) in (55). ■

By applying the result of Lemma 5 to Corollaries 1, 2, and 3, we obtain the following corollary.

Corollary 7: In a wireless network with n links and under log-normal fading channel model, on-off strategy yields

$$\begin{aligned} k &\sim \frac{e^{-\frac{3S^2}{2}}}{\sqrt{8}S}\sqrt{\log n}e^{\sqrt{2}S\sqrt{\log n}}, \quad a.a.s. && (58)\\ \lambda &\sim \frac{\sqrt{8}S}{\sqrt{\log n}}, \quad a.a.s. && (59)\\ \mathrm{E}[R] &\geq e^{-\frac{3S^2}{2}}e^{\sqrt{2}S\sqrt{\log n}}. && (60)\end{aligned}$$

The sum-rate scaling order of $e^{\sqrt{2}S\sqrt{\log n}}$ is the same as what is obtained in [11].

C. Shadow Fading

Consider a network whose channels coefficients are drawn i.i.d. from the pdf [11]

$$f(x) = (1-p)\cdot\delta(x) + p\cdot\delta(x-1), \tag{61}$$

where $\delta(\cdot)$ is the Dirac's delta function. For this distribution $\mu = p$. This pdf simply models a shadow fading environment in which, for any links, with probability $1-p$ there exists an obstacle that completely suppresses the signal and with probability p such an obstacle does not exist and the transmitted signal is received without any fading effect.

For a constant p, since there is only two possibilities for the channel coefficients, the threshold optimization is trivial; one should choose $t^* = 1$ to maximize the sum-rate. This gives $q = p$ and consequently,

$$R = np\log\left(1+\frac{1}{np^2}\right). \tag{62}$$

An interesting scenario is when it is possible to choose p as a function of n such that the sum-rate is maximized. Intuitively, when p is very small, the effect of interference is low, but the number of unblocked links is low as well resulting in a small sum-rate. On the other hand, if p is very large, there are many unblocked links but the number of links interfering with each link is also high and the achieved sum-rate will be small again. Thus, there should be some optimum value for p in between.

Lemma 6: In a wireless network with shadow fading channels and n links, the optimum value of p for the on-off strategy is equal to

$$p^* = \frac{c}{\sqrt{n}}, \tag{63}$$

where $c \approx 0.5050$ is a constant.

Proof: See Appendix C. ■

Corollary 8: In a wireless network with shadow fading channels and n links, under optimum channel conditions, i.e., $p = p^*$, we have

$$R^* \sim c \log\left(1 + \frac{1}{c^2}\right)\sqrt{n}, \quad a.a.s. \tag{64}$$

Proof: Substitute the value of p^* from (63) in (62). ■

As observed, the sum-rate scales as $\sqrt{n}$, which is smaller than the linear scaling with n that was obtained in [11]. This shows the necessity of a multihop scheme to achieve higher orders of sum-rate.

By applying the result of Lemma 6 to Corollaries 1, 2, and 3, we obtain the following corollary.

Corollary 9: In a wireless network with n links and under shadow fading channel model with p^* given in (63), the on-off strategy yields

$$k \sim c\sqrt{n}, \tag{65}$$

$$\lambda \sim \log\left(1 + \frac{1}{c^2}\right), \tag{66}$$

$$\mathrm{E}[R] \geq c \log\left(1 + \frac{1}{c^2}\right)\sqrt{n}. \tag{67}$$

It should be noted that under optimum channel conditions, the rate per link does not go to zero.

V. TRADEOFF BETWEEN SUM-RATE AND RATE PER LINK

We have seen that in the proposed scheme, when the number of users is chosen properly such that the sum-rate is maximized, rate per link scales as $\frac{2}{\log n}$ and $\frac{\sqrt{8}S}{\sqrt{\log n}}$ for Rayleigh fading and log-normal fading models, respectively. In both cases, rate per link approaches zero as $n \to \infty$, which is not desirable in practice where each link requires a reasonable amount of rate in the order of 1. Fortunately, the proposed strategy allows for having rates per users equal to $\Theta(1)$, while still keeping the scaling of the sum-rate as $\Theta(R_n^*)$. For example, accrding to Lemma 4 for the Rayleigh fading model, when $k = \Theta(\log n)$, sum-rate is $\Theta(\log n)$ as well. In this case, rate per link scales as $\Theta(1)$. The same scenario also holds for the log-normal fading model. In the following, we address this issue for Rayleigh and log-normal fading channels.

Assume in a network with n links for some threshold t and the corresponding number of active users k, a sum-rate R_n is achievable. We define the scaling factor as

$$\alpha = \lim_{n\to\infty} \frac{R_n}{R_n^*}, \tag{68}$$

where R_n and R_n^* are as defined before, but we have added a subscript n to stress the functionality of n. We are interested in values of α for which $\alpha = \Theta(1)$. In Rayleigh and log-normal fading channels, R_n^* is obtained in a situation where $t^* \gg \mu k^*$, which results in R_n^* and $\frac{t^*}{\mu}$ having the same order. On the other hand, in both cases, if we decrease the order of the number of users such that $k = \beta\frac{t^*}{\mu}$, for some constant $\beta > 0$, the corresponding t will still have the same order as t^*. Consequently, we achieve $R_n = \beta \log\left(1 + \frac{1}{\beta}\right)\frac{t^*}{\mu}$, which means

$$\alpha = \beta \log\left(1 + \frac{1}{\beta}\right), \tag{69}$$

and

$$\lambda = \log\left(1 + \frac{1}{\beta}\right). \tag{70}$$

As a result of (69) and (70), at the range of $R_n = \Theta\left(R_n^*\right)$, the scaling factor and the rate per link are related as

$$\alpha = \frac{\lambda}{e^{\lambda} - 1}. \tag{71}$$

This equation reveals a tradeoff between rate per link λ and the scaling factor α; one can increase rate per link by decreasing the scaling factor and vice versa. Figure 2 illustrates the tradeoff between the scaling factor and the rate per link.

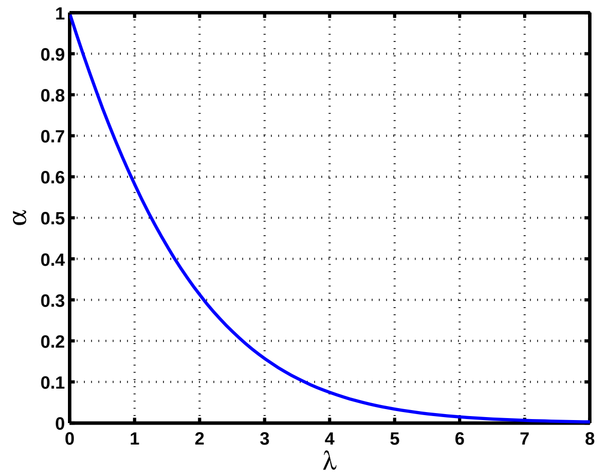

Fig. 2. Tradeoff between the scaling factor and the rate per link.

VI. Summary

In this paper, power allocation was considered to improve the sum-rate of wireless networks with fading channels. A decentralized single-hop scheme based on on-off strategy was proposed and analyzed for a general fading model. The analysis resulted in a sum-rate which is asymptotically almost surely achievable. It was observed that in the popular models of Rayleigh and log-normal fading the achievable sum-rate of this simple method is the same as what was obtained previously in other works by complex centralized multihop methods. We also showed that the proposed method is capable of providing each active link an arbitrary large constant rate, while keeping the sum-rate at the same order as its maximum. In this case,we could identify an interesting tradeoff between the rate per link and the scaling factor of the sum-rate.

Appendix A
Optimum Threshold for Rayleigh Fading Model

The optimum value of the threshold, t^*, is the value that maximizes the achievable sum-rate in (31). As it is seen, $R(t)$ is a complicated function of t. However, since ξ_n

can grow as slow as desired, we can set $\xi_n = 0$ to obtain a more tractable form for $R(t)$ from which a *zero order approximation* of the solution is obtained. In the next stage, we will improve the solution using this zero order approximation.

a) Zero order approximation: By setting $\xi_n = 0$, the objective function in (31) is transformed to

$$\hat{R}(t) = ne^{-t} \log\left(1 + \frac{t}{ne^{-t}}\right). \tag{A.72}$$

Using the approximation $\log(1+x) \approx x - \frac{x^2}{2}$, the above function can be approximated as

$$\hat{R}(t) \approx t - \frac{t^2}{2ne^{-t}}. \tag{A.73}$$

The maximum of this function can be found using the first derivative test as follows. By taking derivative of both sides of (A.73), we obtain

$$\hat{R}'(t) \approx 1 - \frac{t}{ne^{-t}} - \frac{t^2}{2ne^{-t}}, \tag{A.74}$$

which is an increasing function in t. Consequently, the root of the equation $\hat{R}'(t) = 0$ gives the value $t^*_{(0)}$. This equation is equivalent to

$$2ne^{-t} = 2t + t^2. \tag{A.75}$$

Noting that the solution to this equation is increasing with n (i.e. $t^*_{(0)}$ is large), by taking logarithm of both sides of (A.75) we arrive at the following equation

$$t = \log n - 2\log t + \log 2 - \frac{2}{t}, \tag{A.76}$$

whose solution can be verified to be

$$t^*_{(0)} = \log n - 2\log\log n + \log 2 + O\left(\frac{\log\log n}{\log n}\right). \tag{A.77}$$

b) First order approximation: Using $t^*_{(0)}$ in (A.77), the term containing ξ_n in (31) is approximated as[3]

$$\xi_n \sqrt{ne^{-t}} = \xi_n \log n. \tag{A.78}$$

Since $\hat{\psi}_n$ can be chosen of order $o(\log n)$, it is negligible in comparison with $\xi_n \log n$. Thus, the function to be maximized takes the form

$$R^{(1)}(t) = \left(ne^{-t} - \xi_n \log n\right) \log\left(1 + \frac{t}{ne^{-t} - \xi_n \log n}\right). \tag{A.79}$$

Assuming $\xi_n = o(\log n)$, and taking the same approach as for obtaining $t^*_{(0)}$ in the zero order approximation, we can obtain

$$t^*_{(1)} = \log n - 2\log\log n + \log 2 + \frac{4\log\log n}{\log n} + O\left(\frac{\xi_n}{\log n}\right). \tag{A.80}$$

As it is observed, as long as $\xi_n = o(\log n)$, the parameter ξ_n does not contribute in the dominant terms of t^*. Thus, we have

$$t^* = \log n - 2\log\log n + \log 2 + \frac{4\log\log n}{\log n} + O\left(\frac{\xi_n}{\log n}\right). \tag{A.81}$$

Appendix B
Optimum Threshold for Log-Normal Fading Model

The function to be maximized is given in (55). We first consider it as a function of u. Once we have obtained u^*, the value that maximizes this function, we will be able to find t^* from

$$t^* = e^{u^* + M}. \tag{B.82}$$

Using the approximation $\log(1+x) \approx x - \frac{1}{2}x^2$, the objective function is written as

$$g(u) = \frac{SB}{\sqrt{2\pi}} \left(e^u - \frac{Bue^{2u}e^{\frac{u^2}{2S^2}}}{n}\right). \tag{B.83}$$

[3]With a little abuse of notation we have replaced $\frac{\xi_n}{\sqrt{2}}$ by ξ_n. This is acceptable because we are only interested in the order of the term that ξ_n introduces to the solution.

By taking the derivative of $g(u)$ and setting it equal to zero, we obtain

$$Be^{u}e^{\frac{u^2}{2S^2}}\left(1+2u+\frac{u^2}{S^2}\right)=n. \tag{B.84}$$

Since the right hand side of this equatin is n, which is assumed to be large, the solution u^* should be large as well. Thus, we can simplify the equation as

$$\frac{Bu^2e^{u}e^{\frac{u^2}{2S^2}}}{S^2}=n. \tag{B.85}$$

By taking logarithm of both sides of this equation and rearranging the terms, we obtain

$$\frac{u^2}{2S^2}=\log n-u-2\log u-\log\frac{B}{S^2}. \tag{B.86}$$

The solution of this equation can be verified to be

$$u^*=\sqrt{2}S\sqrt{\log n}-S^2+O\left(\frac{\log\log n}{\sqrt{\log n}}\right). \tag{B.87}$$

Then, from (B.82), t^* is obtained as given in (56).

APPENDIX C

OPTIMUM p FOR SHADOW FADING MODEL

By defining $x=np$, the function to be maximized is

$$g(x)=x\log\left(1+\frac{n}{x^2}\right). \tag{C.88}$$

The first derivative of $g(x)$ equals

$$g'(x)=\log\left(1+\frac{n}{x^2}\right)-\frac{2n}{x^2+n}. \tag{C.89}$$

The equation $g'(x)=0$ holds when $\dfrac{x^2}{n}=c^2$ or equivalently

$$p=\frac{c}{\sqrt{n}}, \tag{C.90}$$

where $c\approx 0.5050$ is a constant. It is easy to verify that for the aforementioned value of x, $g''(x)>0$. Thus, p given in (C.90) gives the maximum value of the function under consideration.

REFERENCES

[1] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Trans. Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[2] M. Gastpar and M. Vetterli, “On the capacity of large gaussian relay networks,” *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 765–779, March 2005.

[3] L.-L. Xie and P. R. Kumar, “A network information theory for wireless communication: scaling laws and optimal operation,” *IEEE Trans. Information Theory*, vol. 50, no. 5, pp. 748–767, May 2004.

[4] O. Lévêque and E. Telatar, “Information theoretic upper bounds on the capacity of large extended ad hoc wireless networks,” *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 858–865, March 2005.

[5] S. Toumpis and A. J. Goldsmith, “Large wireless networks under fading, mobility, and delay constraints,” in *Proc. IEEE Infocom*, vol. 1, Hong Kong, 2004, pp. 609–619.

[6] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, “Closing the gap in the capacity of random wireless networks,” in *Proc. IEEE International Symposium on Information Theory*, Chicago, IL, 27 Jun.-2 Jul. 2004, p. 438.

[7] ——, “On the throughput capacity of random wireless networks,” *Submitted to IEEE Trans. Information Theory*, 2004.

[8] M. Grossglauser and D. N. C. Tse, “Mobility increases the capacity of ad hoc wireless networks,” *IEEE/ACM Trans. Networking*, vol. 10, no. 4, pp. 477–486, August 2002.

[9] O. Dousse, M. Franceschetti, and P. Thiran, “On the throughput scaling of wireless relay networks,” *IEEE Trans. Information Theory*, vol. 52, no. 6, pp. 2756–2761, June 2006.

[10] S. R. Kulkarni and P. Viswanath, “A deterministic approach to throughput scaling in wireless networks,” *IEEE Trans. Information Theory*, vol. 50, no. 6, pp. 1041–1049, June 2004.

[11] R. Gowaikar, B. Hochwald, and B. Hassibi, “Communication over a wireless network with random connections,” *IEEE Trans. Information Theory*, vol. 52, no. 7, pp. 2857–2871, July 2006.

[12] R. Gowaikar and B. Hassibi, “On the achievable throughput in two-scale wireless networks,” in *Proc. IEEE ISIT*, 2006.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, Inc., 1991.

[14] R. D. Yates, “A framework for uplink power control in cellular radio systems,” *IEEE J. Selec. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[15] K. Kumaran and H. Viswanathan, “Joint power and bandwidth allocation in downlink transmission,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1008 – 1016, May 2005.

[16] R. Etkin, A. Parekh, and D. Tse, “Spectrum sharing for unlicensed bands,” in *Proc. IEEE DySPAN*, Baltimore, MD, Nov. 8-11 2005, pp. 338–345.

[17] T. ElBatt and A. Ephremides, “Joint scheduling and power control for wireless ad hoc networks,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 74–85, Jan. 2004.

[18] S. Toumpis and A. J. Goldsmith, “Capacity regions for wireless ad hoc networks,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 736–748, July 2003.

[19] L. Xiao, M. Johansson, and S. P. Boyd, "Simultaneous routing and resource allocation via dual decomposition," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1136–1144, July 2004.

[20] S. V. Hanly, "An algorithm for combined cell-cite selection and power control to maximize cellular spread spectrum capacity," *IEEE J. Selec. Areas Commun.*, vol. 13, no. 7, pp. 1332–1340, Sep. 1995.

[21] S. V. Hanly and D. N. Tse, "Power control and capacity of spread spectrum wireless networks," *Automatica*, vol. 35, no. 12, pp. 1987–2012, 1999.

[22] D. Julian, M. Chiang, D. O'Neill, and S. Boyd, "Qos and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks," in *Proc. IEEE Infocom*, vol. 2, New York, NY, Jun. 23-27 2002, pp. 477–486.

[23] H. Boche and S. Stanczak, "Optimal qos tradeoff and power control in cdma systems," in *Proc. IEEE Infocom*, vol. 2, March 2004, pp. 1078–1088.

[24] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 884–895, June 1999.

[25] S.-J. Oh and A. C. K. Soong, "QoS-constrained information-theoretic sum capacity of reverse link CDMA systems," *IEEE Trans. Wirless Commun.*, vol. 5, no. 1, pp. 3–7, Jan. 2006.

[26] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, "Power allocation and asymptotic achievable sum-rates in single-hop wireless networks," in *Proc. Conference on Information Sciences and Systems*, Princeton, NJ, March 2006.

[27] A. Gjendemsj, D. Gesbert, G. E. ien, and S. G. Kiani, ""optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. IEEE Workshop on Resource Allocation in Wireless NETworks (RAWNET '06)*, Boston, MA, April 2006.