



## **Scheduling and Codeword Length Optimization in Time Varying Wireless Networks**

Mehdi Ansari Sadrabadi, Alireza Bayesteh, and Amir K. Khandani

Coding & Signal Transmission Laboratory  
Department of Electrical & Computer Engineering

University of Waterloo

Waterloo, Ontario, Canada, N2L 3G1

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# Scheduling and Codeword Length Optimization in Time Varying Wireless Networks

Mehdi Ansari Sadrabadi, Alireza Bayesteh and Amir K. Khandani

Coding & Signal Transmission Laboratory([www.cst.uwaterloo.ca](http://www.cst.uwaterloo.ca))

Dept. of Elec. and Comp. Eng., University of Waterloo

Waterloo, ON, Canada, N2L 3G1

Tel: 519-884-8552, Fax: 519-888-4338

e-mail: {mehdi, alireza, khandani}@cst.uwaterloo.ca

## Abstract

In this paper, a downlink scenario in which a single-antenna base station communicates with  $K$  single antenna users, over a time-correlated fading channel, is considered. It is shown that using the conventional scheduling that transmits to the user with the maximum signal to noise ratio, a gap of  $\Theta(\sqrt{\log \log \log K})$  exists between the achievable throughput and the maximum possible throughput of the system. We show that by using a simple scheduling that considers both the signal to noise ratio and the channel time variation, this gap approaches  $O(1)$ . Finally, the delay of the system under the proposed strategies are compared.

## I. INTRODUCTION

In wireless networks, diversity is a means to combat the time varying nature of the wireless communication link. Conventional diversity techniques over point-to-point links, such as spatial diversity and frequency diversity, offer performance improvements. In multiuser wireless systems, there exists another form of diversity, called *multiuser diversity* [1]. In a broadcast channel where users have independent fading and feedback their

Signal to Noise Ratio (SNR) to the Base Station (BS), system throughput is maximized by transmitting to the user with the strongest SNR.

Multiuser diversity was introduced first by Knopp and Humblet [2]. It is shown that the optimal transmission strategy in the uplink of multiuser system using power control is to only let the user with largest SNR transmit. The similar result is shown to be valid for the downlink [3]. Bender *et al.* [4] examined practical aspects of downlink multiuser diversity in the context of IS-95 CDMA standard.

In wireless mobile networks, the rate of channel variations is characterized by maximum Doppler frequency which is proportional to the velocity. Utilizing multiuser diversity in such an environment needs to be revised since the throughput depends not only on the received SNR, but also on how fast the channel varies over time.

In this paper, we consider a broadcast channel in which a BS transmits data to a large number of users in a time correlated flat fading environment. It is assumed that the Channel State Information (CSI) is perfectly known to the receivers, while BS only knows the statistical characteristics of the fading process for all the users (which is assumed to be constant during a long period). Moreover, each user feeds back its channel gain to the BS at the beginning of each frame. Based on this information, BS selects one user for transmission in each frame, in order to maximize the throughput. For the case of Additive White Gaussian Noise (AWGN) or block fading, it is well known that increasing the codeword length results in improving the achievable throughput. However, in a time varying channel, it is not possible to obtain arbitrary small error probability by increasing the codeword length. In fact, increasing the codeword length also results in increasing the fading fluctuation over the frame, and consequently, the throughput will decrease. Therefore, it is of interest to find the optimum codeword length which maximizes the throughput.

We analyze different user selection strategies; i) the BS transmits data to the user with the strongest SNR using fixed length codewords (conventional multiuser scheduling), ii) the BS transmits data to the user with the strongest SNR using variable length codewords, and iii) the BS transmits data to the user that achieves the maximum throughput

using variable length codewords. We show that in all cases the achievable throughput scales as  $\log \log K$ . Moreover, in cases (i) and (ii), the gap between the achievable throughput and the maximum throughput scales as  $\sqrt{\log \log \log K}$ , while in case (iii), this gap behaves like  $O(1)$ .

The rest of the paper is organized as follows. In Section II, the model of time correlated fading channel is described. In Section III, different user selection strategies are discussed and the corresponding throughput of the system is derived for each strategy, for  $K \rightarrow \infty$ . Section IV is devoted to the delay analysis of the system for each strategy. Finally in Section V, we conclude the paper.

Throughout this paper,  $\mathbb{E}\{\cdot\}$  represents the expectation, “log” is used for the natural logarithm, and the rates are expressed in *nats*. For any functions  $f(N)$  and  $g(N)$ ,  $f(N) = O(g(N))$  is equivalent to  $\lim_{N \rightarrow \infty} \left| \frac{f(N)}{g(N)} \right| < \infty$ ,  $f(N) = o(g(N))$  is equivalent to  $\lim_{N \rightarrow \infty} \left| \frac{f(N)}{g(N)} \right| = 0$ ,  $f(N) = \omega(g(N))$  is equivalent to  $\lim_{N \rightarrow \infty} \frac{f(N)}{g(N)} = \infty$ , and  $f(N) = \Theta(g(N))$  is equivalent to  $\lim_{N \rightarrow \infty} \frac{f(N)}{g(N)} = c$ , where  $0 < c < \infty$ .

## II. SYSTEM MODEL

A single-antenna broadcast channel with  $K$  users is considered. The channel of any given user is modelled as a time-correlated fading process. It is assumed that the channel gain is constant over each channel use (symbol) and varies from symbol to symbol following a Markovian random process. Assume the fading gain of user  $k$  is  $\mathbf{h}_k = [h_{1,k}, \dots, h_{N_k,k}]^T$  where  $h_{i,k}$ ,  $1 \leq i \leq N_k$  are complex Gaussian random variables with zero mean and unit variance and  $N_k$  is the codeword length of user  $k$ . Dropping the time index for simplicity, the received signal for the  $k$ th user is given by

$$\mathbf{r}_k = \mathbf{S}_k \mathbf{h}_k + \mathbf{n}_k, \quad (1)$$

where  $\mathbf{S}_k = \text{diag}(s_{1,k}, s_{2,k}, \dots, s_{N_k,k})$  is the transmitted codeword with the power constraint<sup>1</sup>  $\mathbb{E}\{|s_{i,k}|^2\} \leq P$ ,  $\mathbf{n}_k$  is AWGN with zero mean and covariance matrix  $\mathbf{I}$ . Assume  $h_{0,k}$  is the fading gain at the time instant before  $\mathbf{S}_k$  is transmitted. The sequence  $u_{i,k} =$

<sup>1</sup>Obviously, for maximizing throughput, the energy constraint translates to  $\mathbb{E}\{|s_{i,k}|^2\} = P$ .

$|h_{i,k}|$ ,  $0 \leq i \leq N_k$ , is assumed to be a stationary ergodic chain with the following probability density function [5]:

$$p(u_{0,k}) = \begin{cases} 2ue^{-u^2} & u \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

$$p(u_{1,k}, u_{2,k}, \dots, u_{N_k,k} | u_{0,k}) = \prod_{i=1}^{N_k} p_k(u_{i,k} | u_{i-1,k}), \quad (3)$$

where,

$$p_k(u|v) = \begin{cases} \frac{2u}{(1-\alpha_k^2)} \exp\left\{-\frac{u^2 + \alpha_k^2 v^2}{(1-\alpha_k^2)}\right\} \mathcal{I}_0\left(\frac{2\alpha_k uv}{(1-\alpha_k^2)}\right) & u \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

in which  $0 < \alpha_k < 1$  describes the channel correlation coefficient for user  $k$ . It is assumed that  $\alpha_k$ ,  $1 \leq k \leq K$ , are i.i.d random variables which remain fixed during the whole transmission, and  $\mathcal{I}_0(\cdot)$  denotes the modified Bessel function of order zero.

It is assumed that the CSI is perfectly known at each receiver, while the statistical characteristics of the fading process and  $u_{0,k}$ ,  $1 \leq k \leq K$  are known to the transmitter.

### III. THROUGHPUT ANALYSIS

In this section, we derive the achievable throughput of the system in the asymptotic case of  $K \rightarrow \infty$ . We define the user  $k$ 's throughput per channel use, denoted by  $T_k$ , as

$$T_k \triangleq R_k(1 - p_e(k)), \quad (4)$$

where  $R_k$  is the transmitted rate and  $p_e(k)$  is the decoding error probability for this user. Using the concept of random coding exponent [6],  $p_e(k)$  can be upper-bounded as

$$p_e(k) \leq \inf_{0 \leq \rho \leq 1} e^{-N(E_k(\rho) - \rho R_k)}. \quad (5)$$

For simplicity of analysis, we use this upper-bound in evaluating the throughput.

Assuming  $s_{i,k}$ ,  $1 \leq i \leq N_k$ , are Gaussian and i.i.d., it is shown that the random coding error exponent for user  $k$ ,  $E_k(\rho)$ , is given by [7],

$$E_k(\rho) = -\frac{1}{N_k} \log \mathbb{E}_{\mathbf{u}_k} \left\{ \prod_{i=1}^{N_k} \left( \frac{1}{1 + \frac{P}{1+\rho} u_{i,k}^2} \right)^\rho \right\}. \quad (6)$$

where  $\mathbf{u}_k = [u_{1,k}, \dots, u_{N_k,k}]$ .

**Theorem 1** *For the channel model described in the previous section, and assuming  $u_{0,k}$  is known, we have*

$$E_k(\rho) = \frac{1}{N_k} \sum_{i=1}^{N_k} \rho \log \left( 1 + \frac{P u_{0,k}^2 \alpha_k^{2i}}{(1+\rho)} \right) + O \left( \frac{1}{\sqrt{u_{0,k}}} \right). \quad (7)$$

**Proof:** See Appendix A.

It follows that the value of  $\rho$  which minimizes the right hand side of (5) is 1. Since  $K \rightarrow \infty$ ,  $\Pr(u_{0,k} = \log K) \rightarrow 1$ . It is easy to show that for large values of  $u_{0,k}$ ,  $\rho = 1$  maximizes the throughput in (7). Using (4), (5), and (7), we have

$$T_k = R_k \left[ 1 - e^{-N_k \left( \log \left( \frac{P u_{0,k}^2}{2} \right) - (N_k + 1) \log(\alpha_k) + R_k \right)} \right]. \quad (8)$$

It is easy to show that  $T_k$  is a convex function of variables  $R_k$  and  $N_k$ , and the values of  $R_k$  and  $N_k$  which maximize the throughput ( $R_k^{\text{opt}}$  and  $N_k^{\text{opt}}$ ) satisfy the following equations:

$$R_k^{\text{opt}} = \log \left( \frac{P u_{0,k}^2}{2} \right) + (2N_{k,\text{opt}} + 1) \log(\alpha_k), \quad (9)$$

$$N_k^{\text{opt}} = \sqrt{\frac{\log(1 + N_{k,\text{opt}} R_{k,\text{opt}})}{\log(\alpha_k^{-1})}}. \quad (10)$$

It follows that

$$T_k = \log \left( \frac{P u_{0,k}^2}{2} \right) - 2 \sqrt{\log(\alpha_k^{-1}) \log \log \left( \frac{P u_{0,k}^2}{2} \right)} \times \left( 1 + O \left( \frac{\log \log \log(u_{0,k})}{\log \log(u_{0,k})} \right) \right). \quad (11)$$

From the above equation, it is concluded that the throughput not only depends on the initial fading gain,  $u_{0,k}$ , but also on the fading correlation coefficient. Moreover, throughput is an increasing function of the channel correlation coefficient.

In the following, we introduce three scheduling strategies in order to maximize the throughput; i) Traditional scheduling in which the user with the largest channel gain

(SNR-based scheduling) is selected and the codeword length is assumed to be fixed. ii) SNR-based scheduling with optimized codeword length regarding the channel condition of the selected user, and iii) Scheduling which exploits both the channel gain and channel correlation coefficient of the users. The asymptotic throughput of the system is derived under each strategy for  $K \rightarrow \infty$ .

*A. Strategy I: SNR-based scheduling with fixed codeword length*

The BS transmits to the user with the maximum initial fading gain and  $N_1 = N_2 = \dots = N_K = N$  while adapting the data rate in order to maximize the throughput of the selected user. The following theorem gives the throughput of the system under this scheduling.

**Theorem 2** *The asymptotic throughput of the system under strategy I scales as*

$$T_1 \sim \log\left(\frac{P \log K}{2}\right) - 2\sqrt{\mathbb{E}\{\log(\alpha^{-1})\}}\sqrt{\log \log\left(\frac{P \log K}{2}\right)}, \quad (12)$$

as  $K \rightarrow \infty$ .

**Proof:** For simplicity of notation, we define  $v_k \triangleq u_{0,k}^2$ . Let  $v = \max_{1 \leq l \leq K} v_l$  and  $\alpha$  be the corresponding correlation coefficient of the selected user. Using (8), we find the rate of the selected user as follows:

$$R = \log\left(\frac{Pv}{2}\right) + (N+1)\log(\alpha) - \frac{\log(1+NR)}{N}. \quad (13)$$

Substituting (13) in (8), we have

$$T_{v,\alpha} = \left(\log\left(\frac{Pv}{2}\right) + (N+1)\log(\alpha) - \frac{\log(1+NR)}{N}\right) \times \left(1 - \frac{1}{1+NR}\right), \quad (14)$$

where  $T_{v,\alpha}$  is the system throughput, conditioned on  $v$  and  $\alpha$ . It is easy to see that  $p_v(x) = e^{-x}u(x)$ . Having the fact that  $\Pr\{v \sim \log(K) + O(\log \log K)\} \rightarrow 1$  as  $K$  tends

to infinity [8], the throughput of the system is computed as,

$$\begin{aligned}
T_1 &= \mathbb{E}\{T_{v,\alpha}\} \\
&= \log\left(\frac{P \log K}{2}\right) + N \mathbb{E}\{\log(\alpha)\} \\
&\quad - \frac{\log \log\left(\frac{P \log K}{2}\right)}{N} + O\left(\frac{\log N}{N}\right) + O\left(\frac{\log \log K}{\log K}\right).
\end{aligned} \tag{15}$$

The codeword length  $N$  is computed such that the system throughput achieved in (15) is maximized. It can be easily shown that the maximizing value of  $N$  scales as

$$N_{\text{opt}} \sim \sqrt{\frac{\log \log\left(\frac{P \log K}{2}\right)}{\mathbb{E}\{\log(\alpha^{-1})\}}}. \tag{16}$$

The proof is completed by substituting (16) in (15).

### B. Strategy II: SNR-based scheduling with adaptive codeword length

In this scheme, the BS transmits to the user with the maximum initial fading gain. The rate and codeword length are adapted to maximize the throughput of the selected user.

**Theorem 3** *Assuming  $K \rightarrow \infty$ , the asymptotic throughput of the system under strategy II scales as follows:*

$$T_2 \sim \log\left(\frac{P \log K}{2}\right) - 2\mathbb{E}\{\sqrt{\log(\alpha^{-1})}\} \sqrt{\log \log\left(\frac{P \log K}{2}\right)}. \tag{17}$$

**Proof:** Given  $v = \max_k u_{0,k}^2$  and  $\alpha$  (the channel correlation coefficient of the selected user) similar to (11), the throughput is computed as

$$\begin{aligned}
T_{v,\alpha} &\sim \log\left(\frac{Pv}{2}\right) - 2\sqrt{\log(\alpha^{-1}) \log \log\left(\frac{Pv}{2}\right)} \times \\
&\quad \left(1 + O\left(\frac{\log \log \log(v)}{\log \log(v)}\right)\right).
\end{aligned} \tag{18}$$



Noting that  $v \sim \log K + O(\log \log K)$ , and  $v$  and  $\alpha$  are independent, we have

$$\begin{aligned} T_2 &= \mathbb{E}\{T_{v,\alpha}\} \\ &\sim \log\left(\frac{P \log K}{2}\right) - 2\mathbb{E}\{\sqrt{\log(\alpha^{-1})}\} \sqrt{\log \log\left(\frac{P \log K}{2}\right)} \times \\ &\quad O\left(\frac{\log \log \log \log K}{\sqrt{\log \log \log K}}\right). \end{aligned} \quad (19)$$

■

*Remark 1-* Since  $\mathbb{E}\{\sqrt{x}\} \leq \sqrt{\mathbb{E}\{x\}}$ , for  $x > 0$ , it is concluded that the achievable rate of Strategy II is higher than that of Strategy I. More precisely,

$$T_2 - T_1 \sim 2\left(\sqrt{\mathbb{E}\{\log(\alpha^{-1})\}} - \mathbb{E}\{\sqrt{\log(\alpha^{-1})}\}\right) \times \sqrt{\log \log \log K}. \quad (20)$$

For the case of uniform distribution for  $\alpha$ , we have

$$T_2 - T_1 \sim 0.228 \sqrt{\log \log \log K}. \quad (21)$$

*Remark 2-* Although  $\lim_{K \rightarrow \infty} \frac{T_1}{T_{\max}} = \lim_{K \rightarrow \infty} \frac{T_2}{T_{\max}} = 1$ , where  $T_{\max} \sim \log(P \log K)$  is the maximum achievable throughput in a quasi-static fading channel [8], there exists a gap of  $\Theta(\sqrt{\log \log \log K})$  between the achievable throughput of Strategies I and II, and the maximum throughput. As we show later, this gap is due to the fact that the channel correlation coefficients of the users is not considered in the scheduling. In fact, this gap approaches  $O(1)$  by exploiting the channel correlation, which is discussed in Strategy III.

### C. Strategy III: Scheduling based on both SNR and channel correlation coefficient with adaptive codeword length

To maximize the throughput of the system, the user which maximizes the expression in (11) should be serviced. Here, for simplicity of analysis, we propose a sub-optimum scheduling that considers the effect of both SNR and channel correlation in the user selection. In this strategy, each user is required to feed back its initial fading gain only if

it is greater than a pre-determined threshold  $\sqrt{\Upsilon}$ , where  $\Upsilon$  is a function of the number of users. Among these users, the BS selects the one with the maximum channel correlation coefficient. The data rate and codeword length are adapted to maximize the throughput of the selected user. The following theorem gives the system throughput under this strategy.

**Theorem 4** *Let  $\alpha_k$ ,  $k = 1, \dots, K$ , be i.i.d random variables with uniform distribution. Using strategy III, with threshold  $\Upsilon$  ( $\Upsilon \gg 1$ ), the throughput of system scales as*

$$T_3 \gtrsim \log\left(\frac{P\Upsilon}{2}\right) - 2\sqrt{\mu(\Upsilon, K)}\sqrt{\log\log\left(\frac{P\Upsilon}{2}\right)} \times O\left(\frac{\log\log\log(\Upsilon)}{\sqrt{\log\log(\Upsilon)}}\right), \quad (22)$$

where

$$\mu(\Upsilon, K) \triangleq \sum_{n=1}^K \frac{1}{n} (1 - e^{-\Upsilon})^{K-n} - (1 - e^{-\Upsilon})^K \sum_{n=1}^K \frac{1}{n}.$$

Moreover, for values of  $\Upsilon$  satisfying

$$\begin{aligned} \Upsilon &\sim \log K - o(\log K), \\ \Upsilon &\sim \log K - \log\log\log\log K - \omega(1), \end{aligned} \quad (23)$$

we have

$$\lim_{K \rightarrow \infty} T_{\max} - T_3 = O(1). \quad (24)$$

**Proof-** Define  $\mathcal{A} \triangleq \{k | v_k \geq \Upsilon\}$  and  $\alpha_{\max} \triangleq \max_{k \in \mathcal{A}} \alpha_k$ . Let  $v$  be the squared initial fading gain of the user corresponding to  $\alpha_{\max}$ . Using (11) and noting that  $T_k$  in (11) is

an increasing function of  $u_{0,k}$ , we have

$$\begin{aligned}
T &= \mathbb{E} \left\{ \log\left(\frac{Pv}{2}\right) - 2\sqrt{\log(\alpha_{\max}^{-1})} \right. \\
&\quad \left. \sqrt{\log \log\left(\frac{Pv}{2}\right)} \left[ 1 + O\left(\frac{\log \log \log(v)}{\log \log(v)}\right) \right] \right\} \\
&\geq \log\left(\frac{P\Upsilon}{2}\right) - 2\mathbb{E}\{\sqrt{\log(\alpha_{\max}^{-1})}\} \\
&\quad \sqrt{\log \log\left(\frac{P\Upsilon}{2}\right)} \left[ 1 + O\left(\frac{\log \log \log(\Upsilon)}{\log \log(\Upsilon)}\right) \right] \\
&\geq \log\left(\frac{P\Upsilon}{2}\right) - 2\sqrt{\mathbb{E}\{\log(\alpha_{\max}^{-1})\}} \\
&\quad \sqrt{\log \log\left(\frac{P\Upsilon}{2}\right)} \left[ 1 + O\left(\frac{\log \log \log(\Upsilon)}{\log \log(\Upsilon)}\right) \right]. \tag{25}
\end{aligned}$$

$\mathbb{E}\{\log(\alpha_{\max}^{-1})\}$  can be evaluated as

$$\mathbb{E}\{\log(\alpha_{\max}^{-1})\} = \sum_{n=1}^K \mathbb{E}\{\log(\alpha_{\max}^{-1}) \mid |\mathcal{A}| = n\} \Pr\{|\mathcal{A}| = n\} \tag{26}$$

Since  $\alpha_k$ ,  $k = 1, \dots, K$ , are i.i.d random variables with uniform distribution over  $[0, 1]$ , we can write

$$\begin{aligned}
F_{\alpha_{\max}}(\alpha \mid |\mathcal{A}| = n) &= \alpha^n \\
\Rightarrow \mathbb{E}\{\log(\alpha_{\max}^{-1}) \mid |\mathcal{A}| = n\} &= \int_0^1 \log(\alpha^{-1}) n \alpha^{n-1} d\alpha \\
&= \frac{1}{n}, \tag{27}
\end{aligned}$$

where  $F_X(\cdot)$  denotes the cumulative density function of the random variable  $X$ . Indeed,  $|\mathcal{A}|$  is a binomial random variable with parameters  $K$  and  $e^{-\Upsilon}$ . Note that  $\Pr(v_k \geq \Upsilon) = e^{-\Upsilon}$ . Hence,

$$\Pr\{|\mathcal{A}| = n\} = \binom{K}{n} e^{-n\Upsilon} (1 - e^{-\Upsilon})^{K-n} \tag{28}$$

Substituting (27) and (28) in (26), we have

$$\mathbb{E}\{\log(\alpha_{\max}^{-1})\} = \sum_{n=1}^K \binom{K}{n} \frac{1}{n} e^{-n\Upsilon} (1 - e^{-\Upsilon})^{K-n}. \tag{29}$$

Define  $\mu(\Upsilon, K) \triangleq \mathbb{E}\{\log(\alpha_{\max}^{-1})\}$ . After some manipulations, we obtain

$$\mu(\Upsilon, K) = (1 - e^{-\Upsilon})\mu(\Upsilon, K - 1) + \frac{1}{K} - \frac{(1 - e^{-\Upsilon})^K}{K} \quad (30)$$

Solving the recursive function in (30) and noting  $\mu(\Upsilon, 1) = e^{-\Upsilon}$ , we have

$$\mu(\Upsilon, K) = \sum_{n=1}^K \frac{1}{n} (1 - e^{-\Upsilon})^{K-n} - (1 - e^{-\Upsilon})^K \sum_{n=1}^K \frac{1}{n}. \quad (31)$$

By Substituting (31) in (25), the first part of the theorem easily follows. From (22), it is easy to see that for values of  $\Upsilon$  satisfying

$$\begin{aligned} \Upsilon &\sim \log K - o(\log K), \\ \mu(\Upsilon, K) \log \log \Upsilon &\sim o(1), \end{aligned} \quad (32)$$

we have  $\lim_{K \rightarrow \infty} T_{\max} - T_3 = O(1)$ , where  $T_{\max} \sim \log(P \log K)$  is the maximum achievable throughput of the system. For large values of  $K$ , we can approximate (31) as

$$\begin{aligned} \mu(\Upsilon, K) &= (1 - e^{-\Upsilon})^K \left( \int_1^K \frac{(1 - e^{-\Upsilon})^{-x} dx}{x} - \int_1^K \frac{dx}{x} \right) \\ &= \frac{1}{K \log(1 - e^{-\Upsilon})} \left( 1 + O\left(\frac{1}{K \log(1 - e^{-\Upsilon})}\right) \right) + \\ &\quad (1 - e^{-\Upsilon})^K (\Upsilon - \log K) \\ &\simeq \frac{1}{K e^{-\Upsilon}} \left( 1 + O\left(\frac{1}{K e^{-\Upsilon}}\right) \right) + e^{-K e^{-\Upsilon}} (\Upsilon - \log K). \end{aligned} \quad (33)$$

From the above equation, it is realized that having  $\Upsilon \sim \log K - \log \log \log \log K - \omega(1)$ , results in having  $\mu(\Upsilon, K) \log \log \Upsilon \sim o(1)$ , which completes the proof of the second part of the theorem. ■

*Remark 1-* The uniform distribution of the correlation coefficients is not a necessary condition for Theorem 4. In fact, Theorem 4 is valid if  $\mathbb{E}\{\log(\alpha_{\max}^{-1}) | \mathcal{A}| = n\} = o(\frac{1}{n})$ . Hence, there exists a larger class of distributions that satisfy the requirements for this theorem.

#### IV. DELAY ANALYSIS

In this section, we analyze the delay of the system under the mentioned scheduling strategies. The delay is defined as the minimum value of  $D$  that satisfies the following:

$$\Pr\{\mathcal{B}_D\} \rightarrow 1, \quad (34)$$

where  $\mathcal{B}_D$  is the event that all users receive at least one packet during  $D$  blocks of transmission.

Since in the Strategies I and II, the user with the best channel condition is serviced, by the same argument as in [9] the delay of the system can be shown to scale at least as  $K \log K$ . More precisely,

$$D_1 = D_2 \sim K \log K + \omega(1), \quad (35)$$

where  $D_1$  and  $D_2$  stand for the delay of the system under the strategies I, and II, respectively. The following theorem gives the scaling law for the delay of the system using strategy III:

**Theorem 5** *Under the condition of Theorem 4, we have*

$$D_3 \gtrsim e^{\Upsilon + K e^{-\Upsilon} + \log \Upsilon}, \quad (36)$$

where  $D_3$  denotes the delay of the system under the strategy III.

**Proof-** Without loss of generality assume  $\alpha_1 \leq \alpha_2 \cdots \leq \alpha_K$ . Note that for large number of users  $\Upsilon \gg 1$ . First, we obtain the probability of selecting the  $k^{\text{th}}$  user in each frame, denoted by  $p_k$ .

$$\begin{aligned} p_k &= \frac{\Pr(v_k \geq \Upsilon) \prod_{i=k+1}^K \Pr(v_i \leq \Upsilon)}{1 - p_0} \\ &= \frac{e^{-\Upsilon} (1 - e^{-\Upsilon})^{K-k}}{1 - (1 - e^{-\Upsilon})^K}, \end{aligned} \quad (37)$$

where  $p_0 = (1 - e^{-\Upsilon})^K$  is the probability of existing no users with channel norm above the threshold, which is negligible <sup>2</sup>. Hence,

$$p_k \simeq e^{-\Upsilon} (1 - e^{-\Upsilon})^{K-k}. \quad (38)$$

<sup>2</sup>It is assumed that  $K e^{-\Upsilon} \gg 1$ .

Defining  $\mathcal{F}_{D,i}$  as the event that user  $i$  is not serviced during  $D$  blocks of transmission and  $\eta_D \triangleq \Pr\{\mathcal{B}_D\}$ , we have

$$\begin{aligned}\eta_D &= 1 - \sum_{k=1}^K (-1)^{k+1} \sum_{1 \leq i_1 < i_2 < \dots < i_k \leq K} \Pr\left\{\bigcap_{m=1}^k \mathcal{F}_{D,i_m}\right\} \\ &= 1 - \sum_{k=1}^K (-1)^{k+1} \sum_{1 \leq i_1 < i_2 < \dots < i_k \leq K} \left(1 - \sum_{m=1}^k p_{i_m}\right)^D.\end{aligned}\quad (39)$$

A lower-bound and upper-bound for  $\eta_D$  can be given as,

$$\eta_D \geq 1 - \sum_{k=1}^K (1 - p_k)^D, \quad (40)$$

and

$$\begin{aligned}\eta_D &\leq 1 - \sum_{k=1}^K (1 - p_k)^D + \sum_{1 \leq i < j \leq K} (1 - p_i - p_j)^D \\ &\leq 1 - \sum_{k=1}^K (1 - p_k)^D + \frac{1}{2} \left( \sum_{k=1}^K (1 - p_k)^D \right)^2.\end{aligned}\quad (41)$$

From the above equations, it is realized that in order to get  $\Pr\{\mathcal{B}_D\} \rightarrow 1$ , we must have  $\sum_{k=1}^K (1 - p_k)^D \rightarrow 0$ .  $\Phi \triangleq \sum_{k=1}^K (1 - p_k)^D$  can be computed as follows:

$$\begin{aligned}\Phi &= \sum_{i=1}^K (1 - e^{-\Upsilon} (1 - e^{-\Upsilon})^{K-i})^D \\ &\simeq \int_1^K \exp\{-De^{-\Upsilon} (1 - e^{-\Upsilon})^{K-x}\} dx \\ &\sim e^{\Upsilon} \frac{e^{-De^{-(\Upsilon+Ke^{-\Upsilon})}}}{De^{-(\Upsilon+Ke^{-\Upsilon})}}.\end{aligned}\quad (42)$$

Noting (42), in order to have  $\Phi \rightarrow 0$ , the following condition must be satisfied:

$$D \sim e^{\Upsilon+Ke^{-\Upsilon}} (\Upsilon + \log \Upsilon + \omega(1)), \quad (43)$$

which incurs

$$D_3 \gtrsim e^{\Upsilon+Ke^{-\Upsilon}+\log \Upsilon}. \quad (44)$$

■

Assuming  $\Upsilon \sim \log K - \varphi(K)$ , where  $\varphi(K) = o(\log K)$ , it can be easily shown that  $\frac{D_3}{D_1} = e^{e^{\varphi(K)} - \varphi(K)}$ . It follows from Theorem 4 that in order to achieve  $\lim_{K \rightarrow \infty} T_{\max} - T_3 = 0$ ,  $\varphi(K)$  must be greater than  $\log \log \log \log K$ . Hence,

$$\frac{D_3}{D_1} > \frac{\log \log K}{\log \log \log K}. \quad (45)$$

In other words, although using strategy III one can approach the maximum throughput of the system, it produces much more delay in the system than the strategies I and II.

## V. CONCLUSION

A multiuser downlink communication over a time-correlated fading channel has been considered. We have proposed three scheduling schemes in order to maximize the throughput of the system. Assuming a large number of users in the system, we show that using SNR-based scheduling, a gap of  $\Theta(\sqrt{\log \log \log K})$  exists between the achievable throughput and the maximum throughput of the system. We propose a simple scheduling considering both the SNR and channel correlation of the users. We show that the throughput of the proposed scheme reaches the maximum throughput of the system as the number of users tends to infinity. Moreover, the delay of the system under the proposed strategies are compared. It is realized that the third strategy, despite achieving the maximum throughput of the system, produces much more delay than the conventional scheduling.

## VI. APPENDIX A

Noting (6), we define  $E_0(\rho) = -\frac{1}{N} \log I_N$  where

$$I_N = \int_{u_N} \dots \int_{u_1} \prod_{i=1}^N \left( \frac{1}{1 + \frac{a}{1+\rho} u_i^2} \right)^{\rho/2} p(\mathbf{u}|u_0) du_i. \quad (46)$$

Using 4, we have

$$I_N = \int_{u_N} \dots \int_{u_1} \prod_{i=1}^N \frac{2u_i}{1 - \alpha^2} \exp\left\{-\frac{u_i^2 + \alpha^2 u_{i-1}^2}{1 - \alpha^2}\right\} \mathcal{I}_0\left(\frac{2\alpha u_i u_{i-1}}{1 - \alpha^2}\right) \left(\frac{1}{1 + \frac{a}{1+\rho} u_i^2}\right)^{\rho} du_i \quad (47)$$

Substituting  $v_i = \frac{u_i}{u_0\sqrt{(1-\alpha^2)/2}}$ ,  $0 \leq i \leq N$ , we have

$$\begin{aligned} I_N &= \int_{v_N} \dots \int_{v_1} \prod_{i=1}^N u_0^2 v_i e^{-\frac{v_i^2 + \alpha^2 v_{i-1}^2}{2/u_0^2}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \int_{v_N} \dots \int_{v_1} \prod_{i=1}^N u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2}} e^{-\alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i, \end{aligned} \quad (48)$$

where,

$$f(v_i) = \left( \frac{1}{1 + \frac{\alpha u_0^2 (1-\alpha^2)}{2(1+\rho)} v_i^2} \right)^\rho. \quad (49)$$

Since  $K \rightarrow \infty$ , then  $\Pr(u_0 = \log K) \rightarrow 1$ . For large values of  $u_0$ , we evaluate the following integral.

$$\begin{aligned} I &= \int_0^\infty v u_0^2 \mathcal{I}_0(u_0^2 v \mu) e^{-u_0^2 v \mu} f(v) e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\ &= \int_{\mu - \frac{1}{\sqrt{u_0}}}^{\mu + \frac{1}{\sqrt{u_0}}} v h_0^2 \mathcal{I}_0(u_0^2 v \mu) e^{-u_0^2 v \mu} f(v) e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \epsilon \end{aligned} \quad (50)$$

where  $\epsilon$  can be bounded as follows:

$$\begin{aligned} \epsilon &= \int_0^{\mu - \frac{1}{\sqrt{u_0}}} v h_0^2 \mathcal{I}_0(u_0^2 v \mu) e^{-u_0^2 v \mu} f(v) e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty v h_0^2 \mathcal{I}_0(u_0^2 v \mu) e^{-u_0^2 v \mu} f(v) e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\ &\leq \frac{c}{\sqrt{\mu}} \int_0^{\mu - \frac{1}{\sqrt{u_0}}} \frac{\sqrt{v}}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv + \frac{c}{\sqrt{\mu}} \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty \frac{\sqrt{v}}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\ &\leq \frac{2c}{\sqrt{\mu}} \int_{\mu + \frac{1}{\sqrt{u_0}}}^\infty \frac{\sqrt{v}}{\sqrt{2\pi/u_0^2}} e^{-\frac{(v-\mu)^2}{2/u_0^2}} dv \\ &\leq \frac{2ce^{-u_0}}{\sqrt{\mu}} (\sqrt{\mu} + \frac{u_0 \sqrt{u_0}}{\sqrt{\mu}}) = O(u_0 \sqrt{u_0} e^{-u_0}) \end{aligned} \quad (51)$$

In deriving (51), we use the facts that  $f(v) \leq 1$ ,  $v \geq 0$  and  $\mathcal{I}_0(z) e^{-z} \sqrt{2\pi z} \leq c$  where  $c$  is a constant [10].

Defining  $g(v) = \sqrt{2\pi} v u_0 \mathcal{I}_0(u_0^2 v \mu) e^{-u_0^2 v \mu} f(v)$  and using (50), we have

$$\begin{aligned} I &= \int_{\mu - \frac{1}{\sqrt{u_0}}}^{\mu + \frac{1}{\sqrt{u_0}}} g(v) \frac{e^{-\frac{(v-\mu)^2}{2/u_0^2}}}{\sqrt{2\pi/u_0^2}} dv + \epsilon \\ &= g(\mu) + O\left(\frac{g'(\mu)}{\sqrt{u_0}}\right) + \epsilon \end{aligned} \quad (52)$$



Since  $\mathcal{I}_0(z)e^{-z}\sqrt{2\pi z} = 1 + O(\frac{1}{z})$  [10], then

$$g(\mu) = f(\mu)(1 + O(\frac{1}{u_0^2})), \quad (53)$$

and

$$O(g'(\mu)) = O(g(\mu)). \quad (54)$$

Using (52), (53) and (54), we have

$$I = f(\mu)(1 + O(\frac{1}{\sqrt{u_0}})) + \epsilon \quad (55)$$

Applying (55) in (48), we have

$$\begin{aligned} I_N &= \int_{v_{N-1}} \dots \int_{v_1} f(\alpha v_{N-1})(1 + O(\frac{1}{\sqrt{u_0}})) \times \\ &\quad \prod_{i=1}^{N-1} u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2} - \alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \int_{v_{N-2}} \dots \int_{v_1} f(\alpha^2 v_{N-2}) f(\alpha v_{N-2})(1 + O(\frac{1}{\sqrt{u_0}}))^2 \times \\ &\quad \prod_{i=1}^{N-2} u_0^2 v_i e^{-\frac{(v_i - \alpha v_{i-1})^2}{2/u_0^2} - \alpha u_0^2 v_i v_{i-1}} \mathcal{I}_0(\alpha u_0^2 v_i v_{i-1}) f(v_i) dv_i \\ &= \prod_{i=1}^N f(\alpha^i v_0)(1 + O(\frac{1}{\sqrt{u_0}})) \end{aligned} \quad (56)$$

Substituting  $v_0 = \frac{1}{\sqrt{(1-\alpha^2)/2}}$ , we have

$$I_N = \prod_{i=1}^N f\left(\frac{\sqrt{2}\alpha^i}{\sqrt{(1-\alpha^2)}}\right)(1 + O(\frac{1}{\sqrt{u_0}})). \quad (57)$$

Using (57) and noting the definition  $E_0(\rho) = -\frac{1}{N} \log I_N$ , we conclude Theorem (1).

## REFERENCES

- [1] P. Viswanath, D.N.C. Tse, R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1277–1294, June 2002.
- [2] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," *IEEE ICC'95*, vol. 1, pp. 331 – 335, June 1995.

- [3] D. Tse, "Optimal power allocation over parallel gaussian channels," in *Proc. Int. Symp. on Inform. Theory*, p. 27, June 1997.
- [4] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushayana, and A. Viterbi, "A bandwidth efficient high-speed wireless data service for nomadic users," *IEEE Communications Magazine*, pp. 70–77, July 2000.
- [5] A. N. Tsvetkov, "Convolutional codes for channels with fading," vol. 27, pp. 155–165, Oct. 1991.
- [6] R. G. Gallager, *Information theory and Reliable communication*. J. Wiley, New York, 1968.
- [7] G. Kaplan and S. Shamai, "Achievable performance over the correlated rician channel," *IEEE Trans. Commun.*, vol. 42, pp. 2967 – 2978, Nov. 1994.
- [8] M. Sharif and B. Hassibi, "On the capacity of mimo broadcast channels with partial side information," *IEEE Trans. Inform. Theory*, vol. 51, pp. 506 – 522, Feb. 2005.
- [9] M. sharif and B. Hassibi, "Delay analysis of throughput optimal scheduling in broadcast fading channels." to appear in Proc. IEEE INFOCOM, 2005.
- [10] Y. L. Luke, *The special functions and their approximations*. New York, Academic Press, 1969.