An Efficient User Removal Algorithm for Interference Channels with Constraints on Power

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Abstract

In this paper, the problem of maximizing the number of active users satisfying a required quality of service (QoS) in n-user interference channels is investigated. This problem is known as an NP-complete problem. We introduce an efficient suboptimal algorithm, relying on the results for the boundary of the rate region, we derived in [1]. The algorithm is developed for different sorts of constraints on the transmit powers, including constraint on the power of the individual transmitters and constraint on the total power of the transmitters. Simulation results show that the performance of the proposed algorithm is very close to the optimal solution, and outperforms alternative algorithms.

I. Introduction

High spectral efficiency wireless technology is relied greatly on channel sharing schemes. While such a scheme increases the capacity and the coverage area of com-

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munication systems, it suffers from the interference of the concurrent links over each other, known as the co-channel interference. Consequently, the signal-to-interference-plus-noise-ratio (SINR) of the links are upper-bounded, even if there is no constraint on the transmit powers.

There have been some effort to evaluate the maximum achievable SINR in the interference channels. In [2], the maximum achievable SINR in a satellite network with no power constraint is presented in terms of the Perron-Frobenius (PF) eigenvalue of a non-negative matrix. This result was deployed in many other applications by [3]–[6], afterwards.

Recently, the authors have extended this result to the case that the power of the transmitters are subject to some constraints, including the constraints on power of the individual transmitters, and the constraint on the total transmit power [1]. Furthermore, this result is generalized to the time-varying channels.

In practical scenarios, it is desired that the active users satisfy a required QoS. On the other hand, due to the deteriorative effect of the co-channel interference, it is not possible for all users to satisfy such a requirement. Therefore, some of the users should be removed to the advantage of the others. Finding a feasible subset of users (i.e., a subset of users which satisfy the required QoS) with maximum cardinality is claimed to be an NP-complete problem [7]. In the literature, some heuristic algorithms are presented for this problem. In [4], a stepwise removal algorithm (SRA) has been proposed for the case that the transmit power is unbounded. In this algorithm, in each iteration, for each user, the maximum of the aggregation of the normalized channel gains from that user to the others and the aggregation of the normalized channel gains from the other users to that user is computed. Then, the user with the largest of the computed parameter is removed from the set of active users. The removal algorithm continues in an iterative manner until the maximum achievable signal-to-interference-ratio (SIR) meets the required threshold. Later, in [5], a distributed balancing algorithm (DBA) for noiseless systems was proposed. The DBA algorithm is utilized to develop a new algorithm known as limited information SRA (LI-SRA).

In this algorithm, the user to be removed is the one with the smallest SIR, while all the users are allocated a fixed power. Then, in [5], an algorithm known as limited information SRA (LI-SRA) which utilizes a partial information to remove the users is presented. In [8], another algorithm, named as stepwise-maximum-interference-removal-algorithm (SMIRA) is proposed, in which the maximum of the aggregate interference power from each user to the other ones and from the other users to that user is computed and the user with maximum computed value is removed. This procedure continues iteratively until the maximum achievable SIR meets the target SIR. It is shown that this algorithm outperforms SRA. For congested systems with constraint on the power of the individual transmitters, an algorithm known as gradually-removal-distributed-constrained-power-control (GRX-DCPC) is presented in [7]. In this algorithm, the power of the transmitters are updated based on DCPC algorithm presented in [9], [10] and the removal is performed based on a predetermined criterion. The presented removal algorithm can be performed in a restricted or non-restricted fashion. In the restricted algorithm known as GRR-DCPC, the user to be removed is selected from the users attaining the maximum power in the power updating procedure. Whereas, in the non-restricted algorithm (GRN-DCPC), the user to be removed is selceted from the all active users. The removal criterion can be based on SMIRA or some other presented alternatives in that work. GRX-DCPC can be performed in a distributed fashion in which a user is removed with a certain probability in each iteration. In addition, this algorithm is capable of removing a multiple users at each iteration. The simulation results show that GRN-DCPC (centralized non-restricted) outperforms other mentioned schemes in [7].

In this paper, we exploit the relationship between the maximum achievable SINR and the PF-eigenvalue of some non-negative matrices, presented in [1], to develop a suboptimal algorithm for the problem of user removal. The algorithm is proposed for different constraints on power. cases including (i) when there is no constraint on the transmit power, (ii) when the power of each user is upper-bounded, and (iii) when the total transmit power is upper-bounded. Simulation results show that the proposed algorithm outperforms the alternative schemes in all cases in terms of the number of active users.

Notation: All boldface letters indicate column vectors (lower case) or matrices (upper case). x_{ij} and $\mathbf{x}_i$ represent the entry (i, j) and the column i of the matrix $\mathbf{X}$, respectively. A matrix $\mathbf{X}_{n\times m}$ is called *non-negative* if $x_{ij} \geq 0$, for all i and j [11]. $\det(\mathbf{X})$, $\mathrm{Tr}(\mathbf{X})$, and $\mathbf{X}'$ denote the determinant, the trace, and the transpose of the matrix $\mathbf{X}$, respectively. $\psi(\mathbf{X}, \mathbf{y}, \mathcal{S})$ is a matrix defined as a function of three parameters, which are respectively a matrix, a vector and a set of indices. It is defined as

$$\psi(\mathbf{X}, \mathbf{y}, \mathcal{S}) = \mathbf{Z} = [\mathbf{z}_j], \quad \mathbf{z}_j = \begin{cases} \mathbf{x}_j + \mathbf{y} & j \in \mathcal{S} \\ \mathbf{x}_j & \text{otherwise} \end{cases}$$

In addition, $\mathbf{X}^{i^-}$ is the matrix $\mathbf{X}$ whose i^{th} column and row is removed. We use a similar notation for a vector whose i^{th} element is removed.

II. System Model and Previous Results

The Gaussian interference channel, including n links (users), is represented by the gain matrix $\mathbf{G} = [g_{ij}]_{n\times n}$ where g_{ij} is the attenuation gain of the power from transmitter j to receiver i. This attenuation can be the result of fading, shadowing, or the processing gain of the CDMA system. A white Gaussian noise with zero mean and variance σ_i^2 is added to the received signal at the receiver i terminal. The SINR of each user, denoted by γ_i, is obtained by

$$\gamma_i = \frac{g_{ii}p_i}{\sigma_i^2 + \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{ij}p_j}, \quad \forall i \in \{1, \dots, n\},$$

where p_i is the power of transmitter i. The users are required to attain a minimum SINR denoted by γ i.e., $\gamma_i \geq \gamma$. We define the normalized gain matrix, $\mathbf{A}$, as

$$\mathbf{A} = [a_{ij}]_{n\times n}, \; a_{ij} = \begin{cases} \dfrac{g_{ij}}{g_{ii}} & i \neq j \\ 0 & i = j \end{cases} \tag{1}$$

Based on this definition, the objective is presented as,

$$\frac{p_i}{\eta_i + \sum_{j=1}^{n} a_{ij} p_j} \geq \gamma, \quad \forall i \in \{1, \ldots, n\}, \tag{2}$$

where $\eta_i = \frac{\sigma_i^2}{g_{ii}}$, $\boldsymbol{\eta} = [\eta_i]_{n\times 1}$. Inequalities in (2) can be reformulated in a matrix form as

$$(\frac{1}{\gamma}\mathbf{I} - \mathbf{A})\mathbf{p} \geq \boldsymbol{\eta}. \tag{3}$$

Characterization of maximum achievable SINR in an intefrence channel is based on the Perron-Frobenius theorem. This theorem states some properties about the eigenvalues of a primitive matrix. A square non-negative matrix $\mathbf{X}$ is said to be primitive if there exists a positive integer k such that $\mathbf{X}^k > \mathbf{0}$ [11].

Theorem 1 *[11] (The Perron-Frobenius Theorem for primitive matrices) Suppose* $\mathbf{X}$ *is an* $m \times m$ *non-negative primitive matrix. Then there exists an eigenvalue* $\lambda^*(\mathbf{X})$ *(Perron-Frobenius eigenvalue or PF-eigenvalue) such that*

(i) $\lambda^*(\mathbf{X}) > 0$ *and it is real.*

(ii) *there is a positive vector* $\mathbf{v}$ *such that* $\mathbf{X}\mathbf{v} = \lambda^*(\mathbf{X})\mathbf{v}$.

(iii) $\lambda^*(\mathbf{X}) > |\lambda(\mathbf{X})|$ *for any eigenvalue* $\lambda(\mathbf{X}) \neq \lambda^*(\mathbf{X})$.

(iv) *If* $\mathbf{X} \geq \mathbf{Y} \geq \mathbf{0}$, *then* $\lambda^*(\mathbf{X}) \geq |\lambda(\mathbf{Y})|$ *for any eigenvalue of* $\mathbf{Y}$.

(v) $\lambda^*(\mathbf{X})$ *is a simple root of the characteristic polynomial of* $\mathbf{X}$.

When there is no constraint on the power vector (rather than the trivial constraint of $\mathbf{p} \geq \mathbf{0}$), the maximum achievable γ in (3), shown as γ^*, is characterized as

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})}, \tag{4}$$

where $\lambda^*(\mathbf{A})$ is the PF-eigenvalue of $\mathbf{A}$. This paradigm was first deployed in [2] for SINR balancing in a satellite network.

For the case that the total power of a subset of users is constrained, the authors showed that the maximum achievable SINR for a system is computed through the following theorem [1].

Theorem 2 *The maximum achievable γ in an interference channel with n links and gain matrix $\mathbf{A}$, with the constraints on power,*

$$\mathbf{p} \geq \mathbf{0}, \quad \sum_{i\in\Omega} p_i \leq \overline{p}_\Omega$$

is equal to

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_\Omega}, \Omega)\big)},$$

where $\Omega \subseteq \{1, 2, \ldots, n\}$ is an arbitrary subset of the users.

As a result of this theorem, when the total power of all users is constrained as $\sum_{i=1}^{n} p_i \leq \overline{p}_t$, the maximum achievable SINR is

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, \ldots, n\})\big)}. \tag{5}$$

We can use Theorem 2 to show that if $p_i \leq \overline{p}_i, \quad \forall i \in \{1, \ldots, n\}$, the maximum achievable SINR is

$$\gamma^* = \min_i \{\frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})\big)}\}. \tag{6}$$

In a congested system, all the users can not satisfy the QoS requirement. Therefore, some of the users should be dropped in order to reduce effective interference on the active users and consequently ameliorate the achievable SINR. As a result, we are interested to find the maximum subset of the users which can meet the minimum required QoS. Unfortunately, this problem is claimed to be NP-complete [7]. In what follows, we propose a suboptimal algorithm for obtaining a subset of the users with maximum cardinality satisfying the rate requirement, based on the equations (4), (5), and (6) .

III. Removal Algorithm

To find the optimal set of active users, satisfying the QoS requirement, we have to examine all the combinations of the users and select the feasible one with the maximum cardinality. Clearly, this scheme is computationally exponential. As a suboptimal alternative scheme, we conjecture that removing the users in a greedy manner yields a result which is very close to the optimum solution. Based on this assumption, we develop an efficient algorithm for user removal. The main idea behind the presented algorithm is as follows. At each step, if the active users do not satisfy the required SINR, one user is removed. This user is the one which provides the highest increase in the maximum achievable SINR if it is removed. We call this user the *worst user*. The proposed algorithm is presented for different sorts of constraints on the transmit powers.

According to (4) and Theorem 2, in general, the maximum γ is equal to the inverse of the PF-eigenvalue of a matrix $\mathbf{X}$, i.e.,

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{X})}.$$

In a system with a large number of users, computing the PF-eigenvalue is computationally extensive. In this case, we use an approximation of the PF-eigenvalue. When a matrix is raised to a power, its eigenvalues are raised to the same power as well [12], i.e.,

$$\lambda(\mathbf{X}^q) = \lambda^q(\mathbf{X}).$$

On the other hand, the trace of a matrix is equal to the summation of the eigenvalues of that matrix [12]; therefore,

$$Tr(\mathbf{X}^q) = \sum_i \lambda_i^q.$$

Since the PF-eigenvalue of a primitive non-negative matrix has the largest norm among all the eigenvalues of that matrix [12], we can approximate $\lambda^{*q}(\mathbf{X})$ with the $Tr(\mathbf{X}^q)$, i.e.,

$$\lambda^{*q}(\mathbf{X}) \approx Tr(\mathbf{X}^q).$$

This approximation is stronger if the power q is larger. However, the simulation results show that $q = 2$ yields a very good approximation of the exact value in our problem. Therefore, we use

$$\gamma^* \approx \frac{1}{\sqrt{Tr(\mathbf{X}^2)}} \tag{7}$$

as an approximate value for γ^*. In what follows, we investigate the problem of user removal for different power constraints and give an efficient algorithm for each case.

Case One: No Power Constraint

Based on the previous discussions on the worst link determination and using (4), when there is no power constraint the index of the user to be removed, $\hat{\imath}$, is obtained as

$$\hat{\imath} = \arg\max_i \{\frac{1}{\lambda^*(\mathbf{A}^{i-})}\}.$$

If this link is removed and still the maximum achievable SINR computed through (4) does not meet the required SINR, additional links are removed in a recursive manner till the remaining users become feasible. This algorithm is called the *Removal Algorithm I-A* throughout this paper.

Algorithm I-A

1) Set $\mathbf{A}$ as in (1), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\mathbf{A})}$.
3) If $\gamma^* \geq \gamma_{th}$, stop.
4) Find the worst link as $\hat{\imath} = \arg\max_i \dfrac{1}{\lambda^*(\mathbf{A}^{i-})}$.
5) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{\imath}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{\imath}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{\imath}-}$, $m \leftarrow m - 1$, and go to step 2.

where $\leftarrow$ is a substitution notation.

To avoid the complexity of computing PF-eigenvalues in each iteration, we present the following algorithm which is an approximate version of algorithm I-A.

According to (4) and (7) for the unconstrained power scenario, we have

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})} \approx \frac{1}{\sqrt{Tr(\mathbf{A}^2)}} = \frac{1}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji}}} \ . \tag{8}$$

We define vector $\mathbf{w}$ as

$$\mathbf{w} = [w_i]_{n\times 1},\ w_i = \sum_{j=1}^{n} a_{ij}a_{ji} \ .$$

Then we have

$$\gamma^* \approx \frac{1}{\sqrt{\sum_{i=1}^{n} w_i}} \ .$$

It is easy to show that by removing user i, $2w_i$ is subtracted from the trace of $\mathbf{A}^2$. An immediate conclusion is that if we want to remove one link to obtain the largest increase in the maximum achievable SINR, the best choice (worst link) is to remove the one with the largest w_i. Therefore, $\hat{i} = \arg\max_i w_i$. Based on this result, an efficient algorithm for gradually removing the users is presented as follows. In each iteration, we find the maximum achievable γ using (4) and if this amount is greater than γ_{th}, all the links can be active. Otherwise, the worst link is determined and removed. This algorithm repeats iteratively until the remaining users satisfy the required threshold.

Algorithm I-B

1) Set $\mathbf{A}$ as in (1), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\mathbf{A})}$.
3) If $\gamma^* \geq \gamma_{th}$, stop.
4) Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = \sum_{j=1}^{m} a_{ij}a_{ji}$.
5) Determine the worst link as $\hat{i} = \arg\max_i w_i$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $m \leftarrow m - 1$, and go to step 2.

Case Two: Constraints on the Power of Individual Transmitters

When the power of each transmitter is subject to an upper-bound constraint, based on (6) we design an efficient suboptimal algorithm to find the maximum cardinality subset of the users satisfying a minimum SINR requirement. We define the matrix $\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ as the matrix $\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ whose i^{th} column and row are removed. Therefore, the worst link is

$$\hat{i} = \arg\max_i \min_{\substack{j \\ j \neq i}} \frac{1}{\lambda^*\big(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}. \tag{9}$$

The users are removed one by one based on (9) until all of the active users satisfy the rate requirement. We call this algorithm the *Removal Algorithm II-A*.

Algorithm II-A

1) Set $\mathbf{A}$ as in (1), $\overline{\mathbf{p}}$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable as SINR $\gamma^* = \min_i \frac{1}{\lambda^*(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, i))}$.
3) If $\gamma^* \geq \gamma_{th}$, stop.
4) Find the worst link as $\hat{i} = \arg\max_i \min_j \frac{1}{\lambda^*(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, j))}$.
5) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}^-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}^-}$, $\overline{\mathbf{p}} \leftarrow \overline{\mathbf{p}}^{\hat{i}^-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}^-}$, and $m \leftarrow m - 1$, and go to step 2.

To reduce the complexity of this algorithm, we use the following approximation scheme. According to (6) and (7), we have

$$\gamma^* = \min_i \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})\big)} \approx \min_i \frac{1}{\sqrt{Tr\big(\psi^2(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})\big)}},$$

which can be rewritten as

$$\gamma^* \approx \min_i \frac{1}{\sqrt{(\frac{\eta_i}{\overline{p}_i})^2 + \sum_{k=1}^{n}\sum_{l=1}^{n} a_{kl}a_{lk} + 2\sum_{k=1}^{n} \frac{\eta_k}{\overline{p}_k} a_{ik}}}.$$

We define the matrix $\mathbf{W}$ as $\mathbf{W} = [w_{ij}]_{n\times n}$,

$$w_{ij} = \begin{cases} (\frac{\eta_i}{\overline{p}_i})^2 + \sum_{\substack{k=1\\k\neq j}}^{n}\sum_{\substack{l=1\\l\neq j}}^{n} a_{kl}a_{lk} + 2\sum_{\substack{k=1\\k\neq j}}^{n}\frac{\eta_k}{\overline{p}_k}a_{ik} & i \neq j \\ 0 & i = j \end{cases}$$

We can show that (9) can be simplified to $\hat{i} = \arg_j \min_j \max_i w_{ij}$. Based on this result, the following algorithm is developed.

Algorithm II-B

1) Set $\mathbf{A}$ as in (1), $\overline{\mathbf{p}}$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable as SINR

$$\gamma^* = \min_i \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})\big)}.$$

3) If $\gamma^* \geq \gamma_{th}$, stop, otherwise go to the next step.
4) Update $\mathbf{W}_{m\times m}$ as

$$w_{ij} = \begin{cases} (\frac{\eta_i}{\overline{p}_i})^2 + \sum_{\substack{k=1\\k\neq j}}^{m}\sum_{\substack{l=1\\l\neq j}}^{m} a_{kl}a_{lk} + 2\sum_{\substack{k=1\\k\neq j}}^{m}\frac{\eta_k}{\overline{p}_k}a_{ik} & i \neq j \\ 0 & i = j \end{cases}.$$

5) Determine the worst link as $\hat{i} = \arg_j \min_j \max_i w_{ij}$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $\overline{\mathbf{p}} \leftarrow \overline{\mathbf{p}}^{\hat{i}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}-}$, and $m \leftarrow m - 1$, and go to step 2.

Case Three: Total Transmit Power Constraint

When the total power is constrained by $\overline{p}_t$, the maximum achievable SINR is computed through (5). In this case, the worst user is determined as

$$\hat{i} = \arg\max_i \{\frac{1}{\lambda^*\big(\psi^{i-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, n\})\big)}\}. \tag{10}$$

We call this algorithm the *Removal Algorithm III-A*. *Algorithm III-A*

1) Set $\mathbf{A}$ as in (1), $\overline{\mathbf{p}}$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, m\}))}$.
3) If $\gamma^* \geq \gamma_{th}$, stop.
4) Find the worst link as $\hat{i} = \arg\max_i \{\dfrac{1}{\lambda^*(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, m\}))}\}$.
5) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}^-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}^-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}^-}$, $m \leftarrow m - 1$, and go to step 2.

To reduce the complexity of the algorithm III-A, we use the following method. According to (5) and (7), we have

$$\gamma^* = \frac{1}{\lambda^*(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, n\}))} \approx \frac{1}{\sqrt{Tr(\psi^2(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, n\}))}}.$$

Therefore, we have

$$\gamma^* \approx \Big(\sum_{i=1}^{n} (\frac{\eta_i}{\overline{p}_t})^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} a_{ji} + 2 \sum_{i=1}^{n} \frac{\eta_i}{\overline{p}_t} \sum_{j=1}^{n} a_{ji} + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\eta_i \eta_j}{\overline{p}_t^{\,2}} \Big)^{-\frac{1}{2}}.$$

We define $\mathbf{w}$ as

$$\begin{aligned} w_i =& (\frac{\eta_i}{\overline{p}_t})^2 + 2 \sum_{j=1}^{n} a_{ij} a_{ji} + 2 \frac{\eta_i}{\overline{p}_t} \sum_{j=1}^{n} a_{ji} \\ &+ 2 \sum_{j=1}^{n} \frac{\eta_j}{\overline{p}_t} a_{ij} + 2 \frac{\eta_i}{\overline{p}_t} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\eta_j}{\overline{p}_t}. \end{aligned}$$

We can show that the worst user can be found by,

$$\hat{i} = \arg\max_i w_i.$$

According to this result, we have the following algorithm.

Algorithm III-B

1) Set $\mathbf{A}$ as in (1), $\overline{p}_t$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, \ldots, m\})\big)}.$$

3) If $\gamma^* \geq \gamma_{th}$, stop; otherwise go to the next step.
4) Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = (\frac{\eta_i}{\overline{p}_t})^2 + 2\sum_{j=1}^{m} a_{ij}a_{ji} + 2\frac{\eta_i}{\overline{p}_t}\sum_{j=1}^{m} a_{ji} + 2\sum_{j=1}^{m}\frac{\eta_j}{\overline{p}_t}a_{ij} + 2\frac{\eta_i}{\overline{p}_t}\sum_{\substack{j=1\\ j\neq i}}^{m}\frac{\eta_j}{\overline{p}_t}$.
5) Determine the worst link as $\hat{i} = \arg\max_{i} w_i$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}-}$, $m \leftarrow m-1$, and go to step 2.

In the following section, we will demonstrate the performance of the proposed algorithms via simulation and compare the results with the performance of the other schemes.

IV. Numerical Results

The simulation results are presented for two environments of cellular networks and Rayleigh fading channels. In each environment, three cases are considered; (i) No constraint on the power, (ii) Constraint on the power of individual users, and (iii) Constraint on the total transmit power. The proposed algorithms are compared with other schemes for the aformentioned environments and constraints on power.

We focus on the uplink ISI-free transmission in a diamond structure cellular network. We consider one channel which is a certain time slot or a frequency interval and discuss the inter-cell interference on the co-channel users in that specific channel. We assume that in each cell there is one user that desires to send data to that cell's base station. The location of each user is uniformly distributed over the assigned cell. We define a cluster as a group of cells with different frequencies in which all the available frequencies are used and no two cells have the same frequency, i.e., the cluster size in Fig.2 and Fig.1 is 4. We used the diamond-shaped (square) clusters in the simulations. In this case, all the co-channel cells are placed symmetrically in a sparse square pattern (Fig. 2). To generate the link gains, we use a simple model which is well accepted in the analysis of cellular networks [5], [7], and [13]. g_{ij} which

1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4

Fig. 1. An 8X8 cellular network with cluster size 4

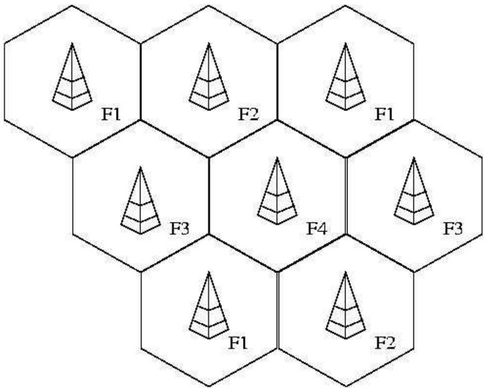

Fig. 2. A hexagonal cellular network with cluster size 4,

is the gain of power from transmitter j to the receiver i is modelled as

$$g_{ij} = \frac{\beta_{ij}}{\nu_{ij}^{\alpha}},$$

where β_{ij} is the shadow fading term which models the irregularities in the terrain, such as mountains, hills, buildings, etc. $1/\nu_{ij}^{\alpha}$ models the large scale propagation loss in which ν_{ij} is the distance between transmitter j and receiver i and α is the propagation constant. For the simulations, we consider the shadow fading term as a log-normal random variable, where

$$E[10 \log \nu_{ij}] = 0 \tag{11}$$

$$Var[10 \log \nu_{ij}] = \varsigma^2. \tag{12}$$

The parameters ς and α depend on the environment and change in the range of $4-10$ dB and $3-5$, respectively. We assume $\varsigma = 6$ dB and $\alpha = 3$ in our simulations. Moreover, the radius of each micro-cell is assumed to be 1 km [7].

In the Rayleigh fading channel, we assume that the parameters g_{ij} follow an exponential distribution with average and variance one for the forward gains, and average 10^{-2} and variance 10^{-4} for the cross gains.

We define *Outage Probability* as the ratio between the number of the inactive users to the total number of the users. This probability shows the percentage of the users that fail to attain the required QoS. We use this function as a metric to compare different algorithms, as it is used in [4], [5].

For the case that there is no constraint on the users' power, the curves of the outage probability for different user removal algorithms are depicted in Fig. 3, Fig. 4, and Fig. 5. Since in SMIRA and SRA algorithms the noise power is considered zero, we assigned a very small value to the noise power to be able to compare all algorithms. As shown in Fig. 5, in Rayleigh fading channel which has strong cross gains and consequently high interference, algorithms I-A and I-B outperform SMIRA and SRA algorithm. In addition, in Fig. 3 and Fig. 4, it is easy to see that algorithms I-A, I-B and SMIRA have a very close-to-optimal outage probability while SRA is very far from the optimal value, compared to the others. Another observation is that the performance of algorithm I-B is very close to that of algorithm I-A, while it enjoys much less operational complexity.

In [7], a number of removal algorithms when the power of transmitters are individually constrained are proposed. We selected centralized GRN-DCPC to compare it with our results since according to [7], it outperforms the other presented algorithms in that work. The simulation results in Fig. 6, Fig. 7, and Fig. 8 show a significant

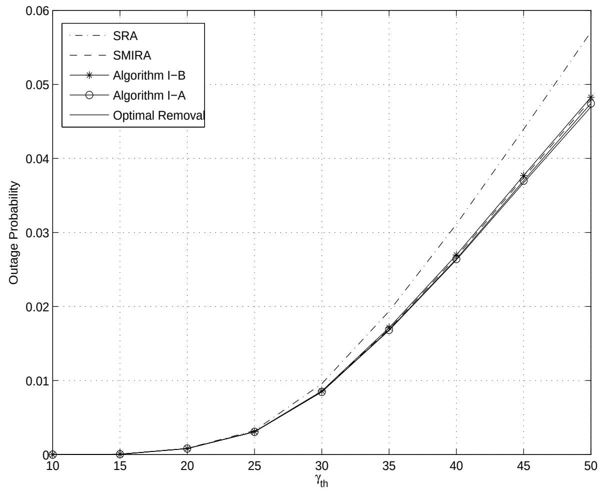

Fig. 3. No Constraint on the Power in an 8×8 Cellular Network with Cluster Size= 4, $n = 16, {\sigma_i}^2 = 10^{-16}\ \forall i$

improvement in the outage probability of the algorithms II-A and II-B compared to GRN-DCPC.

As depicted in Fig. 9, Fig. 10, and Fig. 11, when the total power is bounded, the performance of algorithms III-A and IIII-B is very close to the optimal result. Up to our knowledge, there is no alternative algorithms for the case that the total power is upper-bounded.

V. Conclusion

In this paper, we address the problem of user removal in an interference channel with certain constraints on the power. We utilize the relationship between the maximum achievable SINR and the PF-eigenvalue of some non-negative matrices in bounded power [1] and unbounded power cases to develop a suboptimal algorithm to find the feasible set with maximum number of users. Simulation results show that the proposed algorithm surpasses the available algorithms in the literature.

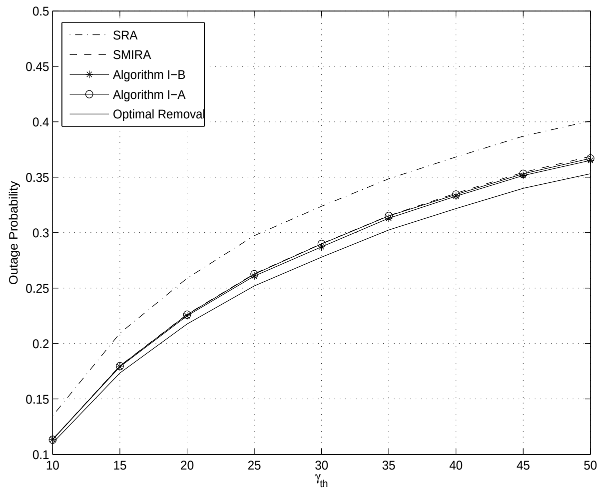

Fig. 4. No Constraint on the Power in a 4×4 Cellular Network with Cluster Size$= 1$, $n = 16, {\sigma_i}^2 = 10^{-16}\ \forall i$

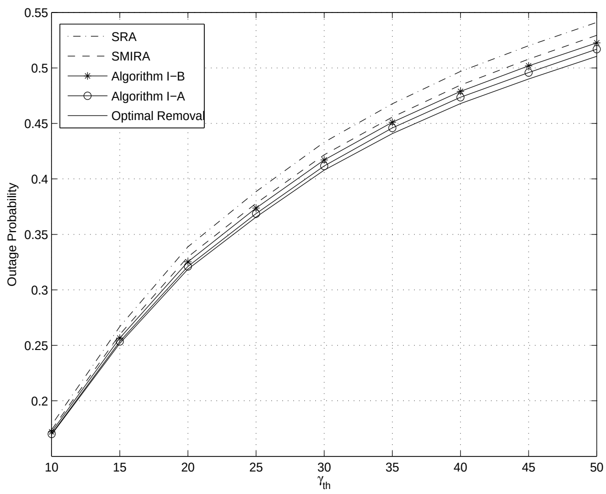

Fig. 5. No Constraint on the Power in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-16}\ \forall i$

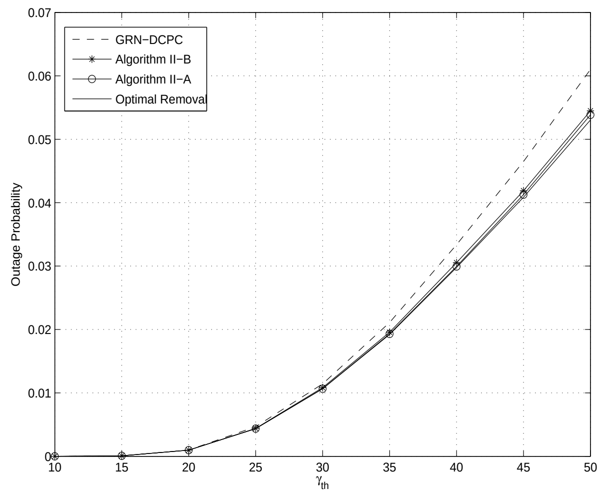

Fig. 6. Constraints on the Power of Individual Transmitters in an 8×8 Cellular Network with Cluster Size= 4, $n = 16, {\sigma_i}^2 = 10^{-12}$, $\overline{p}_i = 1\text{w}\ \forall i$

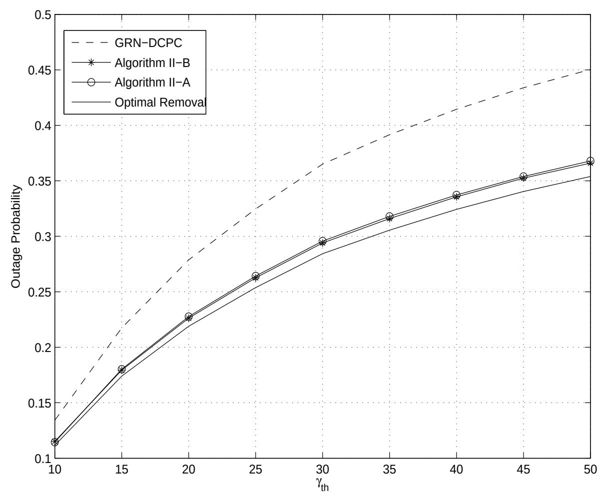

Fig. 7. Constraints on the Power of Individual Transmitters in a 4×4 Cellular Network with Cluster Size= 1, $n = 16, {\sigma_i}^2 = 10^{-12}$, $\overline{p}_i = 1\text{w}\ \forall i$

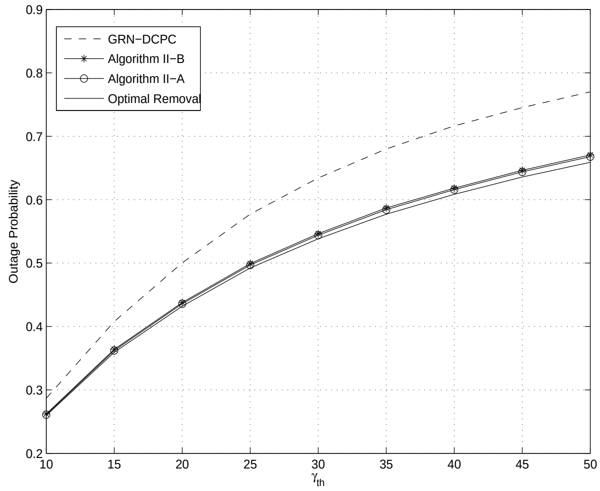

Fig. 8. Constraints on the Power of Individual Transmitters in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-2}$, $\overline{p}_i = 1\text{w}\ \forall i$

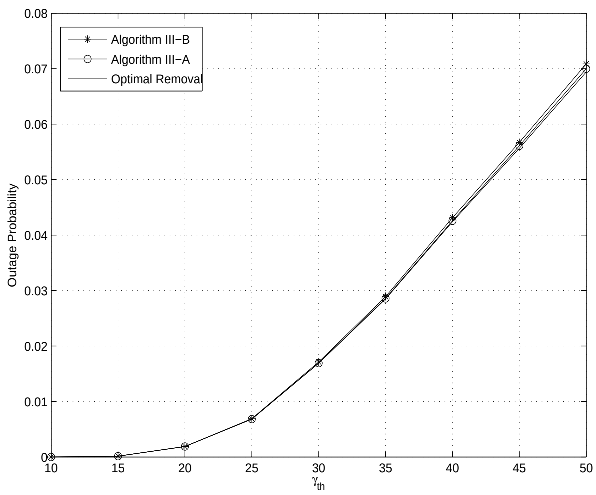

Fig. 9. Constraint on the Total Power in an 8×8 Cellular Network with Cluster Size$= 4$, $n = 16, {\sigma_i}^2 = 10^{-12}\ \forall i$, $\overline{p}_t = 1\text{w}$

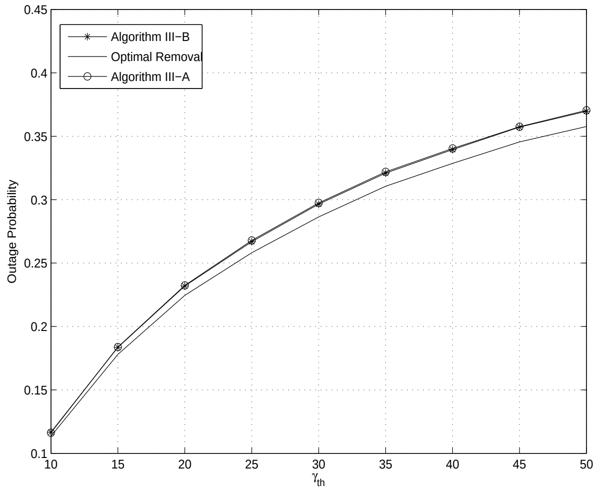

Fig. 10. Constraint on the Total Power in a 4×4 Cellular Network with Cluster Size= 1, $n = 16, {\sigma_i}^2 = 10^{-12}\ \forall i$, $\overline{p}_t = 1$w

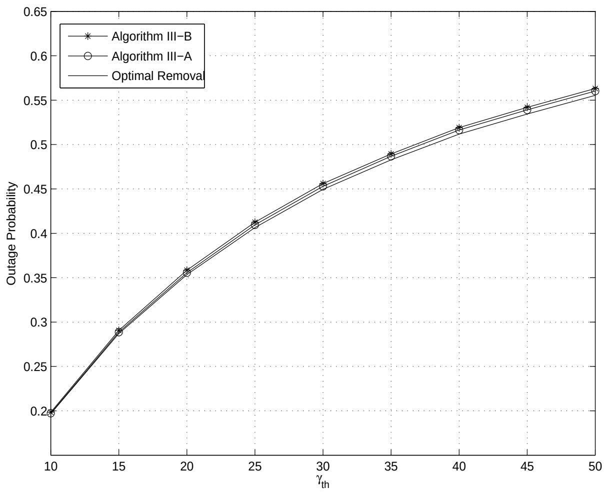

Fig. 11. Total Transmit Power Constraint in a Rayleigh Fading Channel, $n = 8, {\sigma_i}^2 = 10^{-3}\ \forall i$, $\overline{p}_t = 1$w

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