Spectral-Efficient Differential Space-Time Coding Using Non-Full-Diverse Constellations

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Abstract

In this report, a method is proposed to construct spectral-efficient unitary space-time codes for high-rate differential communications over multiple-antenna channels. Unlike most of the known methods which are designed to maximize the diversity product (the minimum determinant distance), our objective is to increase the spectral efficiency. The simulation results indicate that for high spectral efficiency and for more than one receive antenna, the new method significantly outperforms the existing alternatives. In the special case of two transmit antennas, which is the main focus of this report, the relation between the proposed code and the Alamouti scheme helps us to provide an efficient Maximum Likelihood (ML) decoding algorithm. Also, we demonstrate that similar ideas can be applied for designing codes for more than two transmit antennas. As an example, we present a construction for 4 by 4 unitary constellations which has a good performance as compared to the other known codes.

Index Terms: Differential space-time coding, multiple-antenna systems, unitary constellations, Lie groups, diversity sum, Alamouti code.

I. Introduction

Recently, because of the increasing demand for the transmission of video, voice and data in mobile wireless environments, reliable high-rate communications over fading channels has become an important issue. Recent investigations indicate that by using multiple transmit and receive antennas, high rate communications can be achieved [1] [2]. In [2–5], some practical schemes are proposed to realize reliable high-rate communications over multiple-antenna wireless channels. However, these methods require knowledge of the fading coefficients at the receiver. Unfortunately, in many scenarios, such as mobile environments, especially in high-speed vehicles, tracking of the channel coefficients is not feasible.

For noncoherent communications over piecewise constant fading channels, in [6], the authors have proposed the unitary space-time codes that can be used without knowledge of the Channel State Information (CSI) at the receiver. However, in practice, channel coefficients change continuously with time. To deal with this factor, differential space-time modulation schemes (based on unitary matrices) are introduced in [7–9]. These schemes are suitable for mobile communication applications where fading coefficients change rapidly with time. Differential space-time coding is a generalization of the traditional differential phase-shift keying for multiple-antenna communications. In the differential techniques proposed in [8] and [9], the code is full-diverse and the codewords form a group structure which helps in simplifying the encoding. In [10] and [11], full-diverse group constellations for differential space-time coding are further investigated and in [11], all finite fixed-point-free groups (which correspond to full-diverse unitary constellations) are classified. Although some low rate group codes are reported in [11] which have excellent performance, no good full-diverse group constellation have been obtained for very high rate communications. Another attractive approach is based on using orthogonal designs [7] [12] which helps to facilitate the decoding. Nonetheless, differential orthogonal space-time codes do not perform well for very high rates and a large number of receive antennas.

In addition to the previous schemes, due to the importance of the problem, several articles [13–28] have studied the design of good unitary constellations, as well as other aspects of differential space-time coding [29–34]. Some researchers have focused on the case of two transmit antennas which has the most practical importance [15] [18] [24] [17] [25]. Most of their work is founded on improving the differential Alamouti scheme (by using multiple-amplitude elements for the codewords) to construct better full-diverse codes. In [26], a scheme is proposed for constructing full-diverse codes for the general case of differential space-time coding (different numbers of transmit antennas and different rates). The scheme is based on using the Cayley transform to construct unitary matrices.

All the previously mentioned differential space-time codes have been designed to produce full diversity. However, by sacrificing the diversity, we can increase the rate of the space-time modulation [35]. Indeed, in wireless environments, multiple antenna systems can be adopted to achieve diversity or spatial multiplexing. For coherent communications, schemes such as BLAST [2] are proposed to exploit the spatial multiplexing to produce communications with high spectral efficiency over Multiple-Input Multiple-Output (MIMO) channels. Nonetheless, for the noncoherent case, most of the previous works are based on achieving transmit diversity rather than exploiting spatial multiplexing.

Although the performance of MIMO fading systems is determined by the diversity product for very high SNRs, maximizing the diversity product is not appropriate for spectral-efficient communications when the number of receive antennas is greater than one. The latter is similar to the coherent case where for spectral-efficient communications and for a large number of receive antennas, BLAST has a better performance as compared to full-diverse schemes such as the Alamouti code. In some of the previous works [22] [18] [36], there have been attempts to maximize the diversity sum (the minimum Frobenius distance), as well as the diversity product. However, those approaches are useful for only very low rates [18], or are based on an exhaustive search [22] [36] which is not feasible in the case of designing high-rate codes.

This article addresses the problem of constructing unitary constellations which are appropriate for spectral-efficient differential transmission over multiple-antenna channels. The proposed approach relies on sacrificing the transmit diversity and using multiple cosets of a full-diverse (or a partial-diverse) code. Although the main purpose of this paper is to design unitary constellations for two transmit antennas, we show that the same ideas can be applied to construct spectral-efficient codes for more than two antennas. As a specific case, we present a construction for four transmit antennas which, for a large number of receive antennas, considerably outperforms the other known approaches.

In Section II, the system model and some basic concepts of differential space-time coding are introduced. In Section III, the proposed construction for two transmit antennas is presented which is based on relaxing the full-diversity constraint and using different cosets of a Hamiltonian constellation [11]. In Section IV, the decoding of differential space-time codes is considered and a simple decoder for the proposed double-antenna differential code is described. In Section V, we discuss the generalization of the proposed code for more than two antennas where as an example, a construction for four transmit antennas is presented. Finally, the simulation results and comparisons with some of the best known alternatives are presented in Section VI.

II. Differential unitary codes

We consider a multiple antenna system with N_T transmit and N_R receive antennas. The transmitted and the received signals (in N_T consecutive channel uses) can be considered as $\mathbf{X}$ (an $N_T \times N_T$ matrix) and $\mathbf{R}$ (an $N_T \times N_R$ matrix), respectively. These matrices are related by

$$\mathbf{R} = \sqrt{\rho}\mathbf{X}\mathbf{H} + \mathbf{W}$$

where $\mathbf{H}$ is the $N_T \times N_R$ channel matrix, $\mathbf{W}$ is the $N_T \times N_R$ additive noise matrix and ρ is the received Signal to Noise Ratio (SNR). The entries of $\mathbf{H}$ and $\mathbf{W}$ are independent identically distributed (i.i.d.) complex-Gaussian $C(0,1)$. We assume that the transmitted matrix $\mathbf{X}$ has unit energy per time (i.e., $\mathrm{tr}(\mathbf{X}\mathbf{X}^*) = N_T$).

In differential space-time coding with N_T transmit antennas, the identity matrix $\mathbf{X}(0) = \mathbf{I}_{N_T}$ is transmitted in the first N_T channel uses. After that, for the t'th block (which consists of N_T consecutive channel uses), the information is encoded into an $N_T \times N_T$ unitary matrix $\mathbf{S}(t)$, transmitting

$$\mathbf{X}(t) = \mathbf{S}(t)\mathbf{X}(t-1). \tag{1}$$

For differential space-time codes, it is known that the pairwise error probability satisfies the following upper bound [8]:

$$P(\mathbf{S} \longrightarrow \mathbf{S}') \leq \frac{1}{2}\prod_{m=1}^{N_T}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sigma_m^2\left(\mathbf{S}-\mathbf{S}'\right)\right]^{-N_R} \tag{2}$$

where $\sigma_m(.)$ denotes the *m*'th singular value. At a high SNR [8],

$$P(\mathbf{S} \longrightarrow \mathbf{S}') \leq \frac{1}{2}\left(\frac{8}{\rho}\right)^{N_T N_R} \frac{1}{|\det(\mathbf{S}-\mathbf{S}')|^{2N_R}}. \tag{3}$$

Thus, similar to the case of coherent space-time communication, at very high SNR, the performance of the code is related to the *minimum determinant distance* or the *diversity product* [8]:

$$\zeta = \frac{1}{2}\min_{l \neq l'} |\det(\mathbf{S}_l - \mathbf{S}_{l'})|^{\frac{1}{N_T}}. \tag{4}$$

Based on this fact, almost all the design schemes have focused on finding constellations of unitary matrices with large diversity products [8] [9] [7] [11] [10] [37] [13], or an averaged determinant distance [26]. However, for a large number of receive antennas, practical SNR values and reasonable bit error rates, the low-SNR approximation of (2) is more appropriate (using the first order approximation):

$$Pr(\mathbf{S} \longrightarrow \mathbf{S}') \leq \frac{1}{2}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sum_{m=1}^{N_T}\sigma_m^2(\mathbf{S}-\mathbf{S}')\right]^{-N_R}. \tag{5}$$

Therefore, for a large number of receive antennas, maximizing the minimum Frobenius distance (the diversity sum [18]) is more useful:

$$d_{min} = \min \|\mathbf{S}-\mathbf{S}'\|_F \tag{6}$$

where $\|\mathbf{S}-\mathbf{S}'\|_F = \mathrm{tr}\left[(\mathbf{S}-\mathbf{S}')(\mathbf{S}-\mathbf{S}')^*\right]$.

III. CODE DESIGN FOR TWO TRANSMIT ANTENNAS

The set of $N_T \times N_T$ complex unitary matrices form a manifold, the Stiefel manifold [38], with N_T^2 real dimensions. As a result, the unitary matrices can be identified by N_T^2 real parameters. In order to construct good unitary constellations with high spectral-efficiency, we must exploit all of these degrees of freedom. For this purpose, we can use parametrization methods for unitary matrices. We require simple parametrization methods to easily construct a family of unitary matrices with certain distance properties. In the full-diverse case, the Cayley transform [26] is an attractive solution for this problem because computing the transform and its inverse are simple. In addition, the resulting unitary constellation is full-diverse if and only if the original constellation (which consists of quasi Hermitian matrices) is full-diverse. Thus, the Cayley transform is very useful to design high-rate full-diverse unitary codes which are suitable for one receiver antenna. However, for more than one receiver antenna, maximizing the minimum determinant distance (or some form of averaged determinant distance as used in [26]) is no longer appropriate.

Other parametrization methods, such as the exponatiation of quasi Hermitian matrices or the multiplication of Givens rotations or Householder matrices [39], can be used to construct unitary matrices. However, there is no feasible decoding method for unitary codes obtained by such parameterizations.

Here, we consider the especial case of two transmit antennas. Every 2×2 complex unitary matrix **A** can be represented by

$$\mathbf{A} = \begin{bmatrix} ae^{j(\alpha+\gamma)} & be^{j\beta} \\ -b^* e^{j(-\beta+\gamma)} & a^* e^{-j\alpha} \end{bmatrix} \tag{7}$$

where $|a|^2 + |b|^2 = 1$. The resulting set can be seen as a union of the cosets of a full-diverse subset, consisting of the following matrices:

$$\begin{bmatrix} ae^{j\alpha} & be^{j\beta} \\ -b^* e^{-j\beta} & a^* e^{-j\alpha} \end{bmatrix}, |a|^2 + |b|^2 = 1 \tag{8}$$

which is the same as the Hamiltonian constellation, mentioned in [11]. Indeed, the codewords of Hamiltonian constellations are unit-determinant, whereas for a fixed γ, the unitary matrices given by (7) have a determinant equal to $e^{j\gamma}$.

As a similar parameterization for 2 by 2 complex unitary matrices, we can write

$$e^{j\varphi}\begin{bmatrix} ae^{j\alpha} & be^{j\beta} \\ -b^*e^{-j\beta} & a^*e^{-j\alpha} \end{bmatrix}, \; |a|^2+|b|^2=1, \; 0 \le \varphi < \pi \tag{9}$$

where for a fixed φ, the resulting unitary matrices have a determinant equal to $e^{2j\varphi}$.

If (7) is used for the parameterization and if we choose $e^{j\gamma}$ from a K-element subset of the unit circle (with the minimum squared Euclidean distance d^2), the minimum Frobenius distance among the codewords from different cosets is (see Appendix A),

$$d^2 \ge d_{min-inter-coset} \ge \frac{1}{2}d^2 \tag{10}$$

no matter how we choose the full-diverse subcode. Therefore, to design the code, we must design the full-diverse subcode, and then choose K such that $d_{min-inter-coset}$ is not less than the minimum distance of the subcode. To maximize d^2, $e^{j\gamma}$ must be chosen from a PSK constellation. In this case, from (10),

$$4\sin^2(\pi/K) \ge d_{min-inter-coset} \ge 2\sin^2(\pi/K). \tag{11}$$

The full-diverse subcode is a subset of $SU(2)$, the Lie group of 2 by 2 unit-determinant complex unitary matrices, which is one of the only two full-diverse Lie groups. In [37], the authors show that only two infinite Lie groups are fixed-point-free, the group of unit-norm scalars and the group of 2×2 unit-determinant unitary matrices. The full-diverse subcode corresponds to a set of points on S_3 (the unit sphere in R^4). The pairwise Euclidean distances of the points on S_3 are directly related to the pairwise determinant distances, as well as to the pairwise Frobenius distances, of the corresponding codewords [11]. Therefore, to maximize the minimum distance (the Frobenius distance as well as the determinant distance) among the codewords of the subcode, in general, we must find a good packing in S_3.

However, to simplify both the encoding and the decoding processes, we impose a restriction on the constellation, and assume that for each *a* and *b*, α and β are independent from each other. With this restriction, for every *a* and *b*, the Alamouti decoder can be used to find α and β. Similar approaches are presented in [19], [25] to improve the unitary Alamouti code.

If *a* and *b* are chosen from a set of possibilities $\{(a_1, b_1), .., (a_n, b_n)\}$, the full-diverse subcode consists of *n* subsets where for the codewords of the *i*'th subset, $a = a_i$ and $b = b_i$. For the *i*'th subset, $i = 1, ..., n$, the minimum determinant distance (diversity product) and the minimum Frobenius distance are respectively equal to

$$\zeta(i) = \min\left\{\frac{1}{2}d_i^{(1)}, \frac{1}{2}d_i^{(2)}\right\}$$

$$d_{min-intrasubset}(i) = 8\zeta^2(i)$$

where $d_i^{(1)}$ and $d_i^{(2)}$ are the minimum Euclidean distances of the constellations that are related to α_i and β_i (α and β for the *i*'th subset), $i = 1, ..., n$. For the fixed sizes of these constellations, $d_i^{(1)}$ and $d_i^{(2)}$ are maximized if α_i and β_i are chosen from a PSK constellation. In this case,

$$\zeta(i) = \min\left\{|a_i|\sin\left(\frac{\pi}{P_i}\right), |b_i|\sin\left(\frac{\pi}{Q_i}\right)\right\}$$

$$d_{min-intrasubset}(i) = 8\zeta^2(i) \tag{12}$$

where P_i and Q_i are the sizes of the PSK constellations which are related to α_i and β_i, for the *i*'th subset. Also, for the minimum distance between the *i*'th and *j*'th subsets, we have the following formula:

$$d_{min-intersubset}(i, j) = \min|a_i e^{j\alpha_i} - a_j e^{j\alpha_j}|^2 + \min|b_i e^{j\beta_i} - b_j e^{j\beta_j}|^2 \tag{13}$$

where α_i and β_i are from the constellations which correspond to the *i*'th subset and α_j and β_j are from the constellations which correspond to the *j*'th subset.

Due to the practical considerations, $n = 1$ or $n = 2$ is the appropriate choice for the number of the subsets. For $n = 1$, the full-diverse subcode is the Alamouti code. For the case of $n = 2$, for the sake of simplicity, we assign $(a_1, b_1) = (\eta, \xi)$ and $(a_2, b_2) = (\xi e^{j\delta}, \eta e^{j\delta})$, where

$0 < \eta < \frac{1}{\sqrt{2}} < \xi$ and $\eta^2 + \xi^2 = 1$ (as it is depicted in figure 1). To have the largest minimum distance between the two subsets, we must maximize the minimum phase difference among the points of the two PSK constellations corresponding to η and ξ. If PSK constellations are selected with sizes $P_1 = Q_2 = M_1$ and $P_2 = Q_1 = M_2$, corresponding to the amplitudes η and ξ, then the minimum phase difference is maximized when $\delta = \frac{\pi}{l.c.m(M_1,M_2)}$ where $l.c.m(M_1, M_2)$ is the least common multiplier of M_1 and M_2 (see Appendix B). In this case, using (12) and (13):

$$d_{min-intersubset} = 2\xi^2 \sin^2 \delta + 2(\xi \cos \delta - \eta)^2,$$

$$d_{min-intrasubset} = \min \left\{ 8\eta^2 \sin^2(\pi/M_1), 8\xi^2 \sin^2(\pi/M_2) \right\}.$$

For the fixed M_1 and M_2, when η increases, $d_{min-intersubset}$ and $8\xi^2 \sin^2(\pi/M_2)$ decrease and $8\eta^2 \sin^2(\pi/M_1)$ increases. Thus, to maximize d_{min}, we must choose η such that

$$2\xi^2 \sin^2 \delta + 2(\xi \cos \delta - \eta)^2 = 8\eta^2 \sin^2(\pi/M_1) \tag{14}$$

or

$$8\xi^2 \sin^2(\pi/M_2) = 8\eta^2 \sin^2(\pi/M_1). \tag{15}$$

For $n = 2$, by using these formulas, for each choice of the constellation sizes (i.e., K, M_1, M_2), we can compute the other parameters of the code (i.e., η, ξ, δ). To choose the size of the constellations, corresponding to α, β and γ, we must balance inter-coset, inter-subset and intra-subset distances. This situation is similar to the rate assignment for the different levels of a multilevel code or a trellis coded modulation scheme, to attain balanced distances [40].

IV. DECODING

In a differential space-time transmission system, for the t'th block (consisting of N_T consecutive channel uses), we transmit the multiplication of the current codeword (which is a

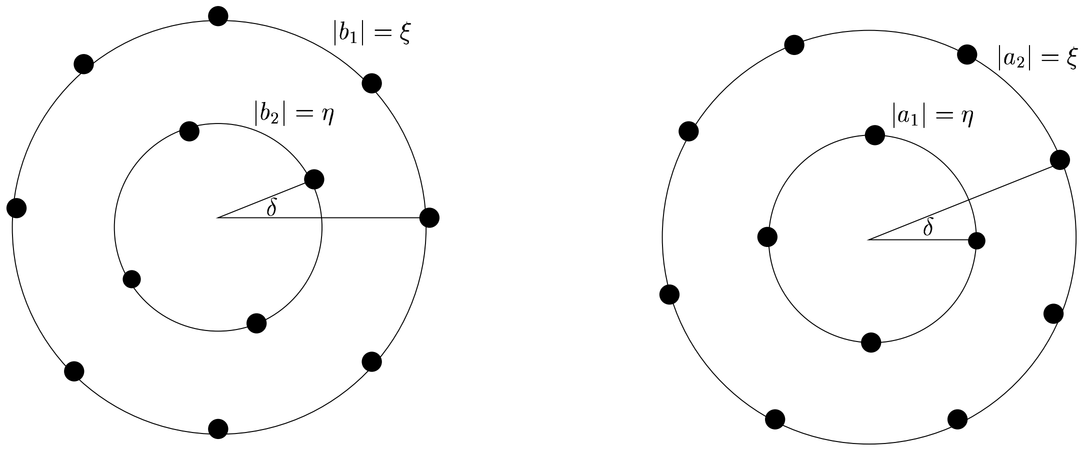

Fig. 1. Two subsets of the full-diverse subcode with $M_1 = 4$ and $M_2 = 8$.

unitary matrix) and the previous transmitted signal. Thus, the transmitted signal can be represented by

$$\mathbf{X}(t) = \mathbf{S}(t)\mathbf{S}(t-1)...\mathbf{S}(1)\mathbf{S}(0) \tag{16}$$

where $\mathbf{S}(0)$ is the initial matrix and can be any unitary matrix (with the proper scaling). At the receiver, we have

$$\mathbf{R}(t) = \mathbf{S}(t)\mathbf{S}(t-1)...\mathbf{S}(1)\mathbf{S}(0)\mathbf{H} + \mathbf{W}(t) \tag{17}$$

where $\mathbf{W}(t)$ is the additive noise at the receiver and $\mathbf{H}_{N_T \times N_R}$ is the matrix of the fading coefficients. The received signal can be written as

$$\mathbf{R}(t) = \mathbf{S}(t)\mathbf{R}(t-1) + \mathbf{W}(t) - \mathbf{S}(t)\mathbf{W}(t-1). \tag{18}$$

In this case, if the elements of the noise matrices are i.i.d. with $CN(0, \sigma^2)$ distribution, the elements of $\mathbf{S}(t)\mathbf{W}(t-1)$ are i.i.d. with the same distribution (because $\mathbf{S}(t)$ is unitary). Thus, $\mathbf{W}' = \mathbf{W}(t) - \mathbf{S}(t)\mathbf{W}(t-1)$ has the following i.i.d. complex Gaussian elements with the variance $2\sigma^2$:

$$\mathbf{R}(t) = \mathbf{S}(t)\mathbf{R}(t-1) + \mathbf{W}'. \tag{19}$$

To decode the received signal, the receiver must find $\hat{\mathbf{S}}$ such that $\hat{\mathbf{S}}\mathbf{R}_(t-1)$ has the minimum Euclidean distance to $\mathbf{R}(t)$.

For the proposed code, for the fixed values of γ and a and b, we can easily use a decoding method that is similar to the Alamouti scheme [41]:

$$\hat{\alpha} = \arg\min_{\alpha} |\mathbf{r}_1(t)\mathbf{r}_1^*(t-1)e^{j\gamma} - \mathbf{r}_2(t)\mathbf{r}_2^*(t-1) - \|\mathbf{R}(t-1)\|ae^{j\alpha}|$$

$$\hat{\beta} = \arg\min_{\beta} |\mathbf{r}_1(t)\mathbf{r}_2^*(t-1) + \mathbf{r}_2(t)\mathbf{r}_1^*(t-1)e^{j\gamma} - \|\mathbf{R}(t-1)\|be^{j\beta}|,$$

where $\mathbf{r}_i(t)$ is the i'th row of $\mathbf{R}(t)$. By using this method, the decoding of the proposed code is equivalent to nK parallel Alamouti decoders for PSK signals.

For very high rates and large values of n and K, it is more appropriate to use bucket algorithms [42] to find a, b and γ (in the process of decoding). This approach helps us to reduce the number of parallel Alamouti decoders. In general, bucket algorithms operate on the data which are partitioned into d-dimensional hyper-rectangles, called cells or buckets. For the decoding of the proposed code, the use of a bucket algorithm is similar to the Kannan strategy to find the closest lattice point [43]. In fact, we can restrict the search to a neighborhood of the starting point. The complexity of these algorithms is essentially independent of K and n (indeed, independent of the rate), because the number of the points in the search neighborhood is almost independent of the overall size of the constellation [43]. However, for the practical rates and practical values of K and n, the first approach is quite simple and bucket algorithms are not helpful.

V. Differential codes for more than two transmit antennas

To generalize the proposed double-antenna system for more than two transmit antennas, we can consider the cosets of a good full-diverse (or a code which has a partial diversity) for more than two transmit antennas. Also, we can employ the proposed double-antenna construction instead of the Alamouti code in the codes which use the Alamouti scheme as a building block.

As a straight-forward generalization of the proposed code for more than two transmit antennas, we can use a union of cosets of a subcode, obtained from orthogonal designs. The resulting codewords are the same as the transmitted codewords in the branches of the trellis which is presented in [41]. However, instead of using a trellis structure and trying to have full diversity, we use all the cosets of the orthogonal code at the same time, and the cosets are independent of the previous codewords. Using an orthogonal code as the subcode simplifies the decoding. Nonetheless, due to the rate constraints for the codes from orthogonal designs, it is more appropriate to use other structures which are more spectral-efficient.

As an example, for 4 transmit antennas, we can use the cosets of a code from $Sp(2)$ (the Lie group consisting of 4 by 4 symplectic unitary matrices). The set of $2N$ by $2N$ symplectic matrices consists of the matrices which can be presented by following:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\bar{\mathbf{B}} & \bar{\mathbf{A}} \end{bmatrix},$$

where $\mathbf{A}$ and $\mathbf{B}$ are two N by N matrices. $SU(2)$, which was mentioned earlier, can be seen as the group consisting of 2 by 2 symplectic unitary matrices, $Sp(1)$. Therefore, codes from $Sp(2)$ can be regarded as extensions of the Alamouti code or the Hamiltonian constellation. The set of $2N$ by $2N$ symplectic matrices is closed under the matrix addition and subtraction [13]. Therefore, when $\mathbf{S}$ and $\mathbf{S}'$ belong to $Sp(2)$, then $\mathbf{S} - \mathbf{S}'$ is also a symplectic matrix (not necessarily unitary). It is easy to see that the codes obtained from $Sp(2)$ have a diversity of the order 2 [13].

It is shown that a matrix $\mathbf{S}$ belongs to $Sp(2)$ if and only if there exist 2 by 2 unitary matrices $\mathbf{U}$ and $\mathbf{V}$ and diagonal matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ such that [13]:

$$\mathbf{S} = \begin{bmatrix} \mathbf{U}\mathbf{D}_1\mathbf{V} & \mathbf{U}\mathbf{D}_2\bar{\mathbf{V}} \\ -\bar{\mathbf{U}}\mathbf{D}_2\mathbf{V} & \bar{\mathbf{U}}\mathbf{D}_1\bar{\mathbf{V}} \end{bmatrix} \tag{20}$$

where $\mathbf{D}_1\mathbf{D}_1^* + \mathbf{D}_2\mathbf{D}_2^* = \mathbf{I}_2$ and $\bar{\mathbf{U}}$ means the complex conjugate of $\mathbf{U}$. In [13], the authors have chosen $\mathbf{U}$ and $\mathbf{V}$ which have Alamouti structures with PSK signals and they have considered $\mathbf{D}_1$ and $\mathbf{D}_2$ to be the identity matrix. Also, the size of the PSK constellations is chosen such that the resulting code is full-diverse (the constellation sizes must be relatively prime). Instead, we can exploit more degrees of freedom to construct the subcode from $Sp(2)$ in order to obtain codes with a greater spectral effi ciency. For this purpose, we consider $\mathbf{U}$ and $\mathbf{V}$ to be two independent 2 by 2 unitary matrices. These matrices can be constructed by the method which we used for 2 transmit antennas. As another parametrization for $Sp(2)$, we can use the following structure (by using (20) and (9)):

$$\begin{bmatrix} e^{j\varphi_1}\mathbf{U}\mathbf{D}_1\mathbf{V} & e^{j\varphi_2}\mathbf{U}\mathbf{D}_2\bar{\mathbf{V}} \\ -e^{-j\varphi_2}\bar{\mathbf{U}}\mathbf{D}_2\mathbf{V} & e^{-j\varphi_1}\bar{\mathbf{U}}\mathbf{D}_1\bar{\mathbf{V}} \end{bmatrix} \tag{21}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices from $SU(2)$ (the Hamiltonian constellation [11]). The problem is that these two parameterizations are not one-to-one correspondences. Therefore, to construct codewords by using these parameterizations, we must impose some restrictions to avoid the overlapping of the codewords.

When (21) is used to construct the subcode and if only equal-norm elements are considered, $\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{I}$ (the identity matrix) and $\mathbf{U}$ and $\mathbf{V}$ have the Alamouti structure:

$$\mathbf{U} = \begin{bmatrix} e^{j\varphi_3} & e^{j\varphi_4} \\ -e^{-j\varphi_4} & e^{-j\varphi_3} \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} e^{j\varphi_5} & e^{j\varphi_6} \\ -e^{-j\varphi_6} & e^{-j\varphi_5} \end{bmatrix}.$$

If we assume that $0 \leq \varphi_1, \varphi_2, \varphi_3, ...\varphi_6 < \pi$ (where $\varphi_3, ...\varphi_6$ are the parameters of the Alamouti structures, $\mathbf{U}$ and $\mathbf{V}$) and $\varphi_3 \pm \varphi_5 \neq \varphi_4 \mp \varphi_6$, then the resulting codewords from (21) are distinct and we can use them as the codewords of the basic subcode (see Appendix C).

The cosets of this subcode are obtained by multiplying the fi rst two columns of the codewords of the subcode by arbitrary unit-norm scalars.

VI. SIMULATION RESULTS

Figure 2 compares the proposed unitary space-time code and the differential methods that are based on orthogonal designs [7], the Cayley transform [26] and the TAST code [27]. It is evident that for the same spectral efficiency (the same as that in [26] and [27]), the proposed method has a considerably better performance. Also, exploiting the maximum degrees of freedom (for example, using two choices for *a* and *b* in this case, i. e. $n = 2$) can be very useful in high rates. For the proposed code, 4, 4 and 3 bits are transmitted by α, β and γ respectively and one bit corresponds to the choice of the layer (choosing *a* and *b*). ML decoding of the proposed codes is reasonably simple (only $16 = 2 \times 8$ linear processing) and compared to TAST, we have about 2 dB improvement with a simpler decoder (even as compared to the suboptimal decoder for TAST which is based on the sphere decoder). In this case, $K = 8$ and $M_1 = M_2 = 16$. We have used $\delta = \pi/l.c.m(16, 16) = \pi/16$ and based on (14), we have chosen $\eta = 0.895$, $\xi = 1.095$ to have relatively balance distances (among $d_{min-intersubset}$, $d_{min-intrasubset}$ and $d_{min-intercoset}$).

Fig. 3 shows the performance of the proposed code for a lower rate. In this case, $n = 1$ and all the codewords have equal-norm elements. Indeed, the code is the union of four cosets of an Alamouti code with PSK signals.

Figures 4, 5, and 6 represent the performance of the proposed code for 4 transmit antennas for 1, 2 and 4 receive antennas, compared to the performance of the code presented in [13]. For the proposed code, we have used (21) with the Alamouti structure for **U** and **V**. To have a fair comparison, we have considered a rate of 3.25 bits per channel use for the proposed code which is more than the rate for the code in [13] which is 3.13 bits per channel use. To transmit at the rate of 3.25 bits per channel use, 13 bits per matrix must be transmitted. We have considered $\varphi_1, \varphi_2, \varphi_4, \varphi_5, \varphi_6 \in \{0, \pi/4, 2\pi/4, 3\pi/4\}$ and $\varphi_3 \in \{\pi/8, 3\pi/8, 5\pi/8, 7\pi/8\}$, to transmit 12 bits by choosing the codeword from the subcode. These sets are chosen such that the overlap among the codewordes is avoided (i.e. $\varphi_3 \pm \varphi_5 \neq \varphi_4 \mp \varphi_6$ and $0 \leq \varphi_1, \varphi_2, \varphi_3, ...\varphi_6 < \pi$). To have two cosets to transmit one extra bit, the first two columns of the unitary codewords are multiplied by $e^{j\varphi_7}$ where $\varphi_7 \in \{0, \pi\}$. The resulting code still has a diversity of the order two. It is observed that for two and four receive antennas, the proposed code significantly outperforms the full-diverse code in [13]. Also, for reasonable probability of errors, the proposed code has a better performance, even for one receive antenna.

Figure 7 compares the performance of the proposed code, the full-diverse code based on $Sp(2)$ [13] and the modified diagonal code [36] for the rates around 2 bits per channel use[1]. The modified diagonal code of [36] is designed to have a larger diversity sum as compared to the original diagonal code [8] [9], to be more appropriate for a large number of receive antennas. We see that the proposed code has a better performance, even as compared to the modified diagonal code of [36] which is obtained by computer search over various diagonal codes. It should be noted that for higher rates, the approaches based on computer exhaustive search (such as [36], [22]) are not feasible.

VII. CONCLUSIONS

A new method to construct unitary space-time codes have been presented. Instead of having the maximum diversity, these codes are designed to have high rates with an appropriate Euclidean distance. For two transmit antennas, the proposed structure allows a simple addressing and encoding method and facilitates an efficient ML decoding. We see that by relaxing the full diversity restriction, there is a substantial improvement over the best differential schemes in the literature, for more than one receive antenna. Also, a similar structure is proposed for four transmit antennas. The simulation results prove that this structure can be very useful to construct spectral-efficient differential space-time codes.

[1] Due to the size constraints for the underlying PSK constellations in the code in [13], comparison with exactly the same rate is not possible. To have a fare comparison, we have chosen a higher rate for the proposed code

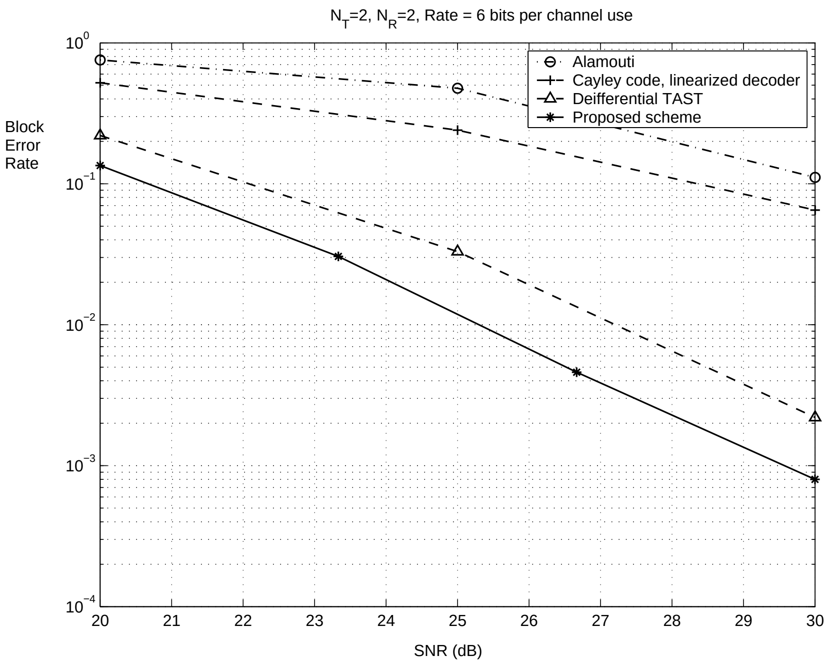

Fig. 2. Block Error Rate of the proposed codes, orthogonal code, Cayley code and TAST code for $N_T = 2$ tranmit and $N_R = 2$ receive antennas with rate=6 bits per channel use.

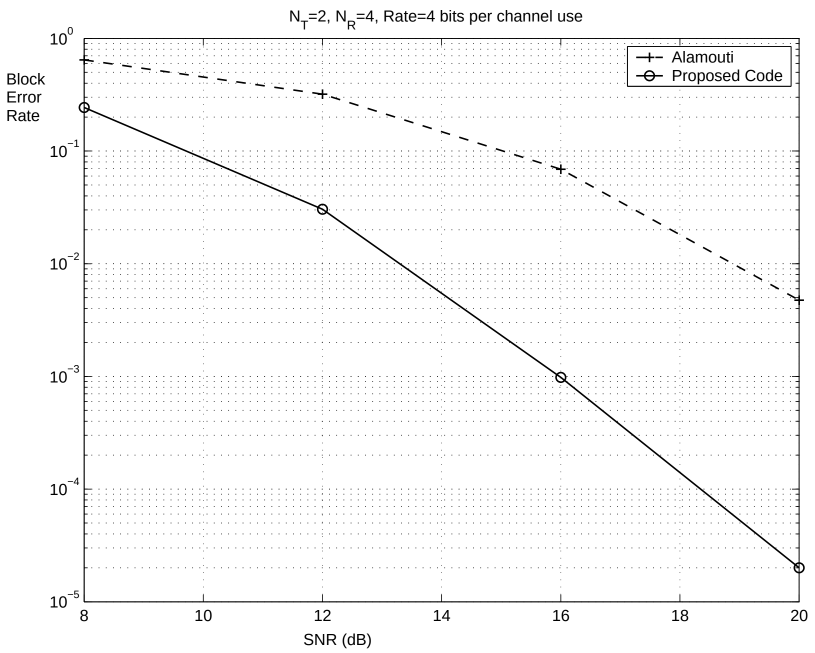

Fig. 3. Block Error Rate of the proposed code and the orthogonal code for $N_T = 2$ transmit and $N_R = 4$ receive antennas with rate $R = 4$ bits per channel use.

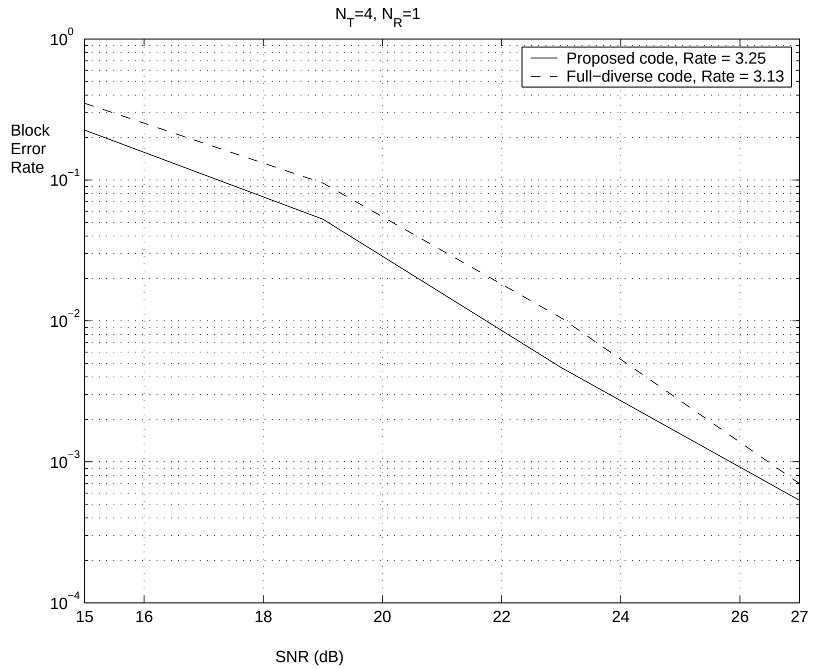

Fig. 4. Block Error Rate of the proposed code and the full-diverse code, based on $Sp(2)$ [13], for $N_T = 4$ transmit antennas and $N_R = 1$ receive antenna.

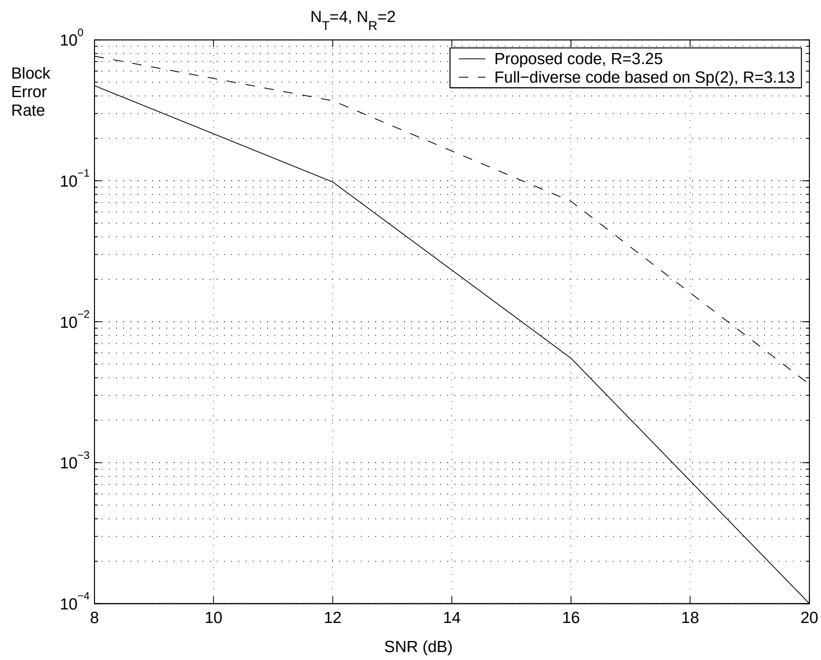

Fig. 5. Block Error Rate of the proposed code and the full-diverse code, based on $Sp(2)$ [13], for $N_T = 4$ transmit and $N_R = 2$ receive antennas.

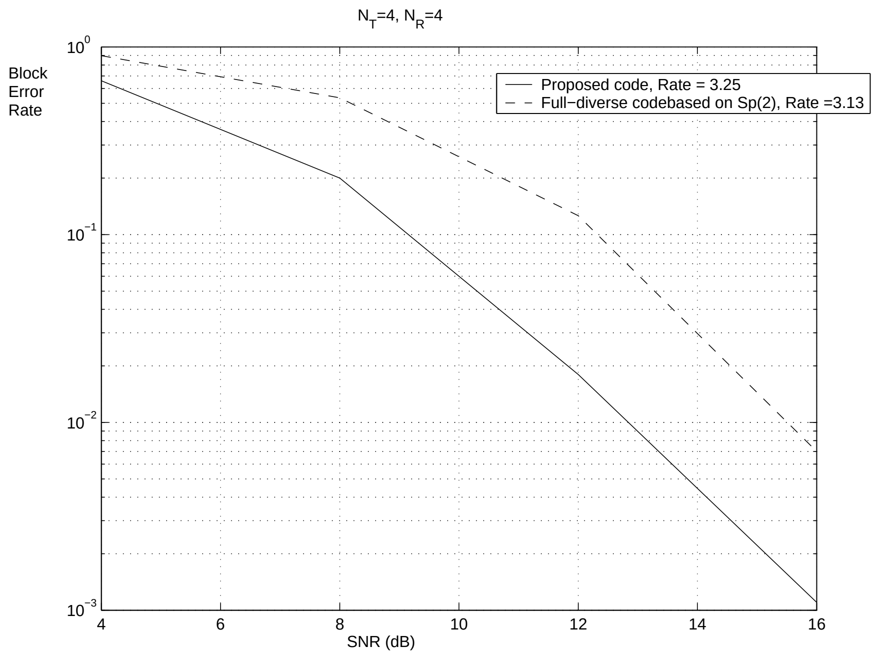

Fig. 6. Block Error Rate of the proposed code and the full-diverse code, based on $Sp(2)$ [13], for $N_T = 4$ transmit and $N_R = 4$ receive antennas.

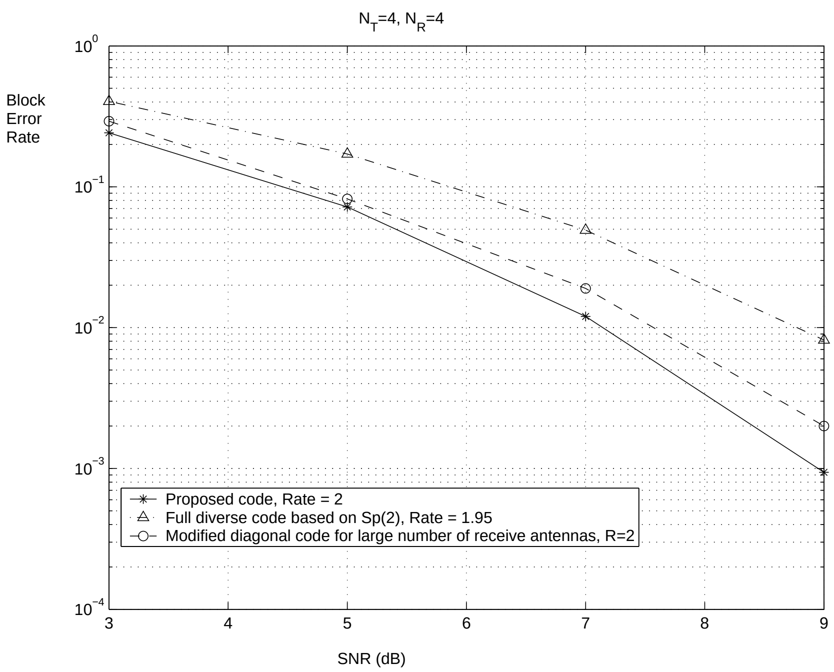

Fig. 7. Block Error Rate of the proposed code, the full-diverse code (based on $Sp(2)$ [13]) and the modified diagonal code (modified for large number of receive antennas) for $N_T = 4$ transmit and $N_R = 4$ receive antennas.

APPENDIX A

We show that in the construction proposed for two transmit antennas, if we choose $e^{j\gamma}$ from a subset of the unit circle (with the minimum squared Euclidean distance d^2), the minimum Frobenius distance among the codewords from different cosets, is

$$d^2 \geq d_{min-inter-coset} \geq \frac{1}{2}d^2$$

Proof: Assume that $\mathbf{A}$ and $\mathbf{A}'$ are two codewords from two different cosets ($\gamma \neq \gamma'$):

$$\mathbf{A} = \begin{bmatrix} e^{j\gamma}x & y \\ -e^{-j\gamma}y^* & x^* \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} e^{j\gamma'}x' & y' \\ -e^{-j\gamma'}y'^* & x'^* \end{bmatrix}$$

Then,

$$\begin{aligned}
\|\mathbf{A} - \mathbf{A}'\|_F &= |e^{j\gamma}x - e^{j\gamma'}x'|^2 + |e^{j\gamma}y^* - e^{j\gamma'}y'^*|^2 \\
&+|y - y'|^2 + |x^* - x'^*|^2 \\
&= |(e^{j\gamma} - e^{j\gamma'})x + e^{j\gamma'}(x - x')|^2 + |x^* - x'^*|^2 + \\
&|(e^{j\gamma} - e^{j\gamma'})y^* + e^{j\gamma'}(y^* - y'^*)|^2 + |y - y'|^2 \\
&= |(e^{j\gamma} - e^{j\gamma'})x + e^{j\gamma'}(x - x')|^2 +^{j\gamma'} (x - x')|^2 + \\
&|(e^{j\gamma} - e^{j\gamma'})y^* + e^{j\gamma'}(y^* - y'^*)|^2 + (|e^{j\gamma'}y^* - y'^*)|^2 \\
&\geq \frac{1}{2}|(e^{j\gamma} - e^{j\gamma'})x + e^{j\gamma'}(x - x') - e^{j\gamma'}(x - x')|^2 \\
&+\frac{1}{2}|(e^{j\gamma} - e^{j\gamma'})y^* + e^{j\gamma'}(y^* - y'^*) - e^{j\gamma'}y^* - y'^*)|^2 \\
&= \frac{1}{2}|(e^{j\gamma} - e^{j\gamma'})x|^2 + \frac{1}{2}|(e^{j\gamma} - e^{j\gamma'})y^*|^2 \\
&= \frac{1}{2}|e^{j\gamma} - e^{j\gamma'}|^2(|x|^2 + |y^*|^2) \\
&= \frac{1}{2}|e^{j\gamma} - e^{j\gamma'}|^2 \geq \frac{1}{2}d^2.
\end{aligned}$$

As the upper bound for $d_{min-intercoset}$, we consider $\mathbf{A}$ and $\mathbf{A}'$, with the $x = x'$, $y = y'$ and $|e^{j\gamma'} - e^{j\gamma}|^2 = d^2$. Now,

$$d_{min-intercoset} \leq \|\mathbf{A} - \mathbf{A}'\|_F$$

$$= |e^{j\gamma} - e^{j\gamma'}|^2(|x|^2 + |y^*|^2) = d^2.$$

APPENDIX B

Consider two sets of points, with phases $A = \{2\pi/M_1, 4\pi/M_1, ..., (2M_1 - 2)\pi/M_1\}$ and $B = \{2\pi/M_2 + \delta, 4\pi/M_2 + \delta, ..., (2M_2 - 2)\pi/M_2 + \delta\}$. We show that the minimum phase difference between the two sets is maximized when $\delta = \frac{\pi}{l.c.m(M_1,M_2)}$.

Proof: Consider $\theta = \frac{2\pi}{l.c.m(M_1,M_2)}$,

$$X = A/\theta = \left\{\frac{l.c.m(M_1, M_2)}{M_1}, 2\frac{l.c.m(M_1, M_2)}{M_1}, ..., (M_1 - 1)\frac{l.c.m(M_1, M_2)}{M_1}\right\}$$

and [2]

$$Y = (B - \delta)/\theta = \left\{\frac{l.c.m(M_1, M_2)}{M_2}, 2\frac{l.c.m(M_1, M_2)}{M_2}, ..., (M_2 - 1)\frac{l.c.m(M_1, M_2)}{M_2}\right\}.$$

Now, $\frac{l.c.m(M_1,M_2)}{M_1}$ and $\frac{l.c.m(M_1,M_2)}{M_2}$ are relatively prime, therefore, the set of differences among the elements of X and Y modulo M_1 is $\{0, 1, 2, ..., M_1 - 1\}$ [44]. Now, if we choose $\delta = \frac{\pi}{l.c.m(M_1,M_2)}$, the minimum difference among the elements of A and B will be $\delta = \frac{\pi}{l.c.m(M_1,M_2)}$. In general, $\delta = k\theta + \phi$ where k is an integer and $-\theta/2 < \phi \leq \theta/2$. The minimum difference will be $|\phi| \leq \theta/2 = \frac{\pi}{l.c.m(M_1,M_2)}$. Thus, the minimum difference can not be greater than $\delta = \frac{\pi}{l.c.m(M_1,M_2)}$.

APPENDIX C

In this appendix, we show that if $\mathbf{U}$ and $\mathbf{V}$ are Alamouti matrices (with parameters φ_3, φ_4 and φ_5, φ_6):

[2] We use the notation $X = A/\theta$, to show that elements of X are obtained by dividing all the elements of A by θ (also, a similar notation for subtraction).

$$\mathbf{U} = \begin{bmatrix} e^{j\varphi_3} & e^{j\varphi_4} \\ -e^{-j\varphi_4} & e^{-j\varphi_3} \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} e^{j\varphi_5} & e^{j\varphi_6} \\ -e^{-j\varphi_6} & e^{-j\varphi_5} \end{bmatrix}.$$

Then, by using (21), we obtain distinct codewords as long as $0 \leq \varphi_1, \varphi_2, \varphi_3, ..., \varphi_6 < \pi$ and $\varphi_3 \pm \varphi_5 \neq \varphi_4 \mp \varphi_6$.

Proof: assume that $\mathbf{A}$ and $\mathbf{A}'$ are the matrices constructed by (21), using parameters $\varphi_1, \varphi_2, ..., \varphi_6$ and $\varphi_1', \varphi_2', ..., \varphi_6'$. We show that if $\mathbf{A} = \mathbf{A}'$, then $\varphi_i = \varphi_i'$ for $i = 1, ..., 6$.

$$\mathbf{A} = \mathbf{A}' \Longrightarrow e^{j\varphi_1}\mathbf{UV} = e^{j\varphi_1'}\mathbf{U}'\mathbf{V}'$$

$$\Longrightarrow \det(e^{j\varphi_1}\mathbf{UV}) = \det(e^{j\varphi_1'}\mathbf{U}'\mathbf{V}')$$

$$\Longrightarrow e^{2j\varphi_1}\det(\mathbf{UV}) = e^{2j\varphi_1'}\det(\mathbf{U}'\mathbf{V}')$$

$$\Longrightarrow e^{2j\varphi_1}\det\mathbf{U}\det\mathbf{V} = e^{2j\varphi_1'}\det\mathbf{U}'\det\mathbf{V}'$$

$$\Longrightarrow e^{2j\varphi_1} = e^{j\varphi_1'} \Longrightarrow \varphi_1 = \varphi_1'$$

Similarly, using $e^{j\varphi_2}\mathbf{U}\overline{\mathbf{V}} = e^{j\varphi_2'}U'\overline{\mathbf{V}'}$, we conclude that $\varphi_2 = \varphi_2'$. Now,

$$\mathbf{A} = \mathbf{A}' \Longrightarrow e^{j\varphi_1}\mathbf{UV} = e^{j\varphi_1}\mathbf{U}'\mathbf{V}'$$

$$\Longrightarrow \begin{bmatrix} e^{j\varphi_1}\left(e^{j(\varphi_3+\varphi_5)} - e^{j(\varphi_4-\varphi_6)}\right) & e^{j\varphi_1}\left(e^{j(\varphi_3+\varphi_6)} + e^{j(\varphi_4-\varphi_5)}\right) \\ -e^{j\varphi_1}\left(e^{-j(\varphi_3+\varphi_6)} + e^{-j(\varphi_4-\varphi_5)}\right) & e^{j\varphi_1}\left(e^{-j(\varphi_3+\varphi_5)} - e^{-j(\varphi_4-\varphi_6)}\right) \end{bmatrix} =$$

$$\begin{bmatrix} e^{j\varphi_1}\left(e^{j(\varphi_3'+\varphi_5')} - e^{j(\varphi_4'-\varphi_6')}\right) & e^{j\varphi_1}\left(e^{j(\varphi_3'+\varphi_6')} + e^{j(\varphi_4'-\varphi_5')}\right) \\ -e^{j\varphi_1}\left(e^{-j(\varphi_3'+\varphi_6')} + e^{-j(\varphi_4'-\varphi_5')}\right) & e^{j\varphi_1}\left(e^{-j(\varphi_3'+\varphi_5')} - e^{-j(\varphi_4'-\varphi_6')}\right) \end{bmatrix}$$

$$\Longrightarrow e^{j\varphi_1}\left(e^{j(\varphi_3+\varphi_5)} - e^{j(\varphi_4-\varphi_6)}\right) = e^{j\varphi_1}\left(e^{j(\varphi_3'+\varphi_5')} - e^{j(\varphi_4'-\varphi_6')}\right)$$

$$\Longrightarrow e^{j(\varphi_3+\varphi_5)} - e^{j(\varphi_4-\varphi_6)} = e^{j(\varphi_3'+\varphi_5')} - e^{j(\varphi_4'-\varphi_6')}.$$

Since $\varphi_3 + \varphi_5 \neq \varphi_4 - \varphi_6$, $e^{j(\varphi_3+\varphi_5)} - e^{j(\varphi_4-\varphi_6)} \neq 0$. Therefore, $e^{j(\varphi_3+\varphi_5)} = e^{j(\varphi_3'+\varphi_5')}$ and $e^{j(\varphi_4-\varphi_6)} = e^{j(\varphi_4'-\varphi_6')}$. Simlarly, using $\mathbf{U}\overline{\mathbf{V}} = \mathbf{U}'\overline{\mathbf{V}'}$, we conclude that $e^{j(\varphi_3-\varphi_5)} = e^{j(\varphi_3'-\varphi_5')}$ and $e^{j(\varphi_4+\varphi_6)} = e^{j(\varphi_4'+\varphi_6')}$. Now,

$$(\varphi_3 + \varphi_5 = \varphi_3' + \varphi_5',\ \varphi_3 - \varphi_5 = \varphi_3' - \varphi_5') \Longrightarrow \varphi_3 = \varphi_3', \ \ \varphi_5 = \varphi_5'$$

Similarly, $\varphi_4 = \varphi_4'$ and $\varphi_6 = \varphi_6'$.

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