Reconstruction of Multi-Stage Vector Quantized Sources Over Noisy Channels-Applications to MELP Codec

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Technical Report UW-E&CE#2004-15

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May 2004

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Abstract

Design of source decoders that exploit the residual redundancy at the source coder output is an interesting research direction in the joint source channel coding framework. Such decoders are expected to replace the traditionally heuristic error concealment units that are elements of most multimedia communication systems. In this work, we consider the reconstruction of signals encoded with a Multi-Stage Vector Quantizer and transmitted over a noisy channel. The MSVQ maintains a moderate complexity and, due to its successive refinement feature, is a suitable choice for the design of layered (progressive) source codes. An approximate MMSE source decoder for MSVQ is presented and its application to the reconstruction of the LPC parameters in MELP is analyzed. Numerical results demonstrate the effectiveness of the proposed schemes.

*This work is funded in part by the Natural Sciences and Engineering Research Council of Canada. This work has been presented in part at the International Conference on Acoustics, Speech, and Signal Processing, Montreal, QC, Canada, 2004. The authors are affiliated with the Coding & Signal Transmission Lab., Dept. of E&CE, University of Waterloo, Waterloo, ON, N2L 3G1, Canada, Email: (farshad, khandani)@cst.uwaterloo.ca.

1 Introduction

Recently, methods to exploit the *residual redundancy* [1] in the output of a source coder for improved reconstruction of a signal transmitted over error prone channels has found increasing attention, e.g., [1]-[21]. As a method of joint source channel coding, researchers have used the residual redundancy for enhanced channel decoding, for effective source decoding, or for iterative source and channel decoding. The residual redundancy is often modeled by Markov models.

Source decoders that exploit the residual redundancy provide effective new solutions for concealment of errors in multimedia communications. In this direction, reconstruction of sources encoded with a memoryless vector quantizer transmitted over a memoryless noisy channel is studied in [2][3][4]. The case of transmission over a noisy channel with memory is considered in [5][6]. A sequence MMSE decoder for reconstruction of sources encoded with a predictive quantizer is proposed in [7]. Reconstruction of variable length coded sources has attracted many researchers as reported in, e.g., [8]-[13]. Applications to decoding of compressed speech are presented in, e.g., [14][15]. Applications to robust transmission of digital images over noisy channels are presented in, e.g., [16][17]. Error concealment for error resilient video transmission using the residual redundancies is considered in, e.g., [18].

In this work, we investigate the redundancy at the output of a Multi-Stage Vector Quantizer and, present an approximate minimum mean squared error technique for reconstruction of MSVQ-encoded sources transmitted over a noisy channel. Numerical results are presented for the application of the proposed techniques to the reconstruction of the Linear Predictive Coefficients (LPC) in the Mixed Excitation Linear Prediction (MELP)[22] speech codec. The MELP is the US Federal speech coding Standard established in 1997. It compresses speech at a low rate of 2.4 kbps and has a variety of applications. Specifically, MELP is considered for voice over IP in [23][24].

Due to its structure, Multi-Stage Vector Quantizer benefits from a moderate level of complexity and a reduced codebook size at the cost of a suboptimal performance, when compared to a full-search vector quantizer. One important characteristic of the Multi-Stage Vector Quantizer, also known as the Residual Vector Quantizer, is its (additive) successive refinement feature. This characteristic makes it attractive for designing progressive source codes for effective multimedia communications, in presence of error/loss and delay. For a comprehensive review of the applications of MSVQ in image coding refer to [25]. The MSVQ is part of the MELP speech coding standard [22] and it is applied to video coding in [26].

The rest of this article is organized as follows. In section 2, notations are introduced and the MSVQ structure is briefly described. An MMSE-based algorithm for the reconstruction of MSVQ encoded signals over noisy channels is presented in section 3. Applications to Multi-Stage Vector Quantization of LSF parameters in MELP and numerical results are presented in section 4. We also draw comparisons to an MSVQ decoder presented by Gadkari and Rose in [32].

2 Multi-Stage Vector Quantization

2.1 Preliminaries

In this paper, capital letters, e.g. I, represent random variables, while small letters, e.g. i, is a realization. For simplicity $P(I = i)$ is represented by $P(I)$. Vectors are shown bold faced, e.g. $\mathbf{X}$. Lower index indicate time instant. Upper index in parenthesis indicate components of a vector. The sequence of random variables $(I_{n_1}, \ldots, I_{n_2})$ over time is represented by $\underline{I}_{n_2}^{n_1}$, and when $n_1 = 1$ simply by $\underline{I}_{n_2}$.

Figure 1 depicts the block diagram of the system. The source coder is an MSVQ, that is composed of η stages. We refer to each stage of the MSVQ as an *Stage-VQ (SVQ)*. The input to the system at time n is the signal $\mathbf{X}_n \in \mathcal{R}^N$.

The structure of the SVQ is presented in Figure 2. At time instant n, the input to the k'th SVQ, $1 \leq k \leq \eta$, is the signal $\mathbf{Y}_n^{(k)} \in \mathcal{R}^N$, that following a vector quantization operation, is mapped to two outputs: (i) the index $I_n^{(k)}$ within the finite index set $\mathcal{J}^{(k)}$ of $M^{(k)}$ elements, which corresponds to the codeword $\tilde{\mathbf{Y}}_n^{(k)}$ of the quantizer $Q^{(k)}$, and (ii) the quantization error signal $\mathbf{Y}_n^{(k+1)} \in \mathcal{R}^N$. This signal is the input to the subsequent SVQ. We have

$$\begin{aligned}
\mathbf{Y}_n^{(1)} &= \mathbf{X}_n, && (1) \\
\mathbf{Y}_n^{(k)} &= \mathbf{X}_n - \sum_{i=1}^{k-1} \tilde{\mathbf{Y}}_n^{(i)}, \quad 1 < k \leq \eta. && (2)
\end{aligned}$$

The source coder output is the symbol

$$\mathbf{I}_n = [I_n^{(1)}, I_n^{(2)}, \ldots, I_n^{(\eta)}],$$

composed of the output indices of different Stage-VQs. To search the MSVQ codebook, we use the so-called M-best search as described in [27]. Corresponding to the MSVQ input signal $\mathbf{X}_n$, the MSVQ output quantized signal is given by

$$\tilde{\mathbf{X}}_n = \sum_{k=1}^{\eta} \tilde{\mathbf{Y}}_n^{(k)}. \quad (3)$$

The bitrate of the k'th SVQ, $r^{(k)}$, is given by $\lceil(\log_2 M^{(k)})\rceil$ bits per symbol or $\lceil(\log_2 M^{(k)})\rceil/N$ bits per dimension. The MSVQ bitrate is given by $r = \sum_{k=1}^{\eta} r^{(k)}$.

At the receiver, for each transmitted r-bit symbol $\mathbf{I}_n$, a vector $\mathbf{J}_n$ with r components is received. The source decoder maps this information to an output sample

$\hat{\mathbf{X}}_n$. This is depicted in Figure 1.

The noisy channel together with the channel encoder and decoder is replaced by a channel model. We assume that the equivalent channel between $I_n^{(k)}$ and $J_n^{(k)}$ is memoryless, and the probability $P(J_n^{(k)}|I_n^{(k)})$ is available at the source decoder. For simulations in section 4, we assume a BPSK modulation and an AWGN channel, which produces soft outputs.

2.2 Redundancy in MSVQ

For a Multi-Stage Vector Quantizer residual redundancy could exist in different forms. Since the k'th stage of the MSVQ quantizes a quantization error signal, to analyze the MSVQ one requires to know the statistical behavior of the quantization error. Lee and Neuhoff presented a high resolution analysis of the error density of vector quantization in [28]. For stationary sources, they showed that, the marginals of the multidimensional error density of an optimal vector quantizer with large dimension are approximately i.i.d. Gaussian. In another line of works on analysis of uniform scalar quantization, it is shown that only under certain strict conditions, the quantization noise is uniform, independently distributed and uncorrelated with the quantizer input [29][30]. As a result, in general, the residual redundancy in a MSVQ could exist in the form of non-uniform symbol probability, and dependency between different stage quantizer outputs. A correlated MSVQ input signal results in residual redundancy in the sequence of output symbols over time.

3 Reconstruction of MSVQ-encoded Signals

As discussed, a Multi-Stage Vector Quantizer, like many other practical source coders, leave some level of redundancy in its output stream. Our objective is to design a source decoder that exploits this residual redundancy and produces the minimum mean squared error estimate of the source sample $\mathbf{X}_n$, given the received sequence $\underline{\mathbf{J}}_n = [\underline{J}_n^{(1)}, \underline{J}_n^{(2)}, \ldots, \underline{J}_n^{(\eta)}]$. Based on the fundamental theorem of estimation, this is given by $\hat{\mathbf{x}}_n = E[\mathbf{X}_n|\underline{\mathbf{J}}_n]$, and equivalently

$$\hat{\mathbf{x}}_n = \sum_{\underline{\mathbf{I}}_n} E[\mathbf{X}_n|\underline{\mathbf{I}}_n]P(\underline{\mathbf{I}}_n|\underline{\mathbf{J}}_n). \tag{4}$$

In equation (4), the decoder codebook $E[\mathbf{X}_n|\underline{\mathbf{I}}_n]$ provides a finer reconstruction of the source, when compared to the quantized signal at the encoder. This comes at the cost of extra memory requirement. To maintain a reasonable level of complexity, we choose to use the same encoder codebook at the decoder. This is equivalent to assuming $E[\mathbf{X}_n|\underline{\mathbf{I}}_n] \approx \tilde{\mathbf{X}}_n$. Now, using equation (3) and given that $\tilde{\mathbf{Y}}_n^{(k)}$ is specified by $I_n^{(k)}$, the equation (4) is simplified to

$$\hat{\mathbf{x}}_n \approx \sum_{k=1}^{\eta} \sum_{I_n^{(k)} \in \mathcal{J}^{(k)}} \tilde{\mathbf{Y}}_n^{(k)} P(I_n^{(k)}|\underline{\mathbf{J}}_n). \tag{5}$$

This indicates a source decoder that has the same structure as depicted in figure 1, i.e., a reconstructor followed by a summation unit. Note that, in traditional MSVQ decoding the reconstructor is simply a mapping of the received indices to corresponding codewords. In equation (5), the a posteriori probability

$$P(I_n^{(k)}|\underline{\mathbf{J}}_n) = P(I_n^{(k)}|\underline{J}_n^{(1)}, \ldots, \underline{J}_n^{(\eta)})$$

encapsulates the dependencies of symbol $I_n^{(k)}$ with other MSVQ symbol stages at time instant n (intersymbol intraframe), as well as MSVQ output symbols of previous time instants (interframe). Methods to calculate similar a posteriori probabilities under various assumptions for the redundancy model and using different approximations or formulations are discussed in [15] [19] [20].

To maintain a low level of complexity, we devise the following approximation:

$$\hat{\mathbf{x}}_n \approx \sum_{k=1}^{\eta} \sum_{I_n^{(k)}} \tilde{\mathbf{Y}}_n^{(k)} P(I_n^{(k)} | \underline{J}_n^{(k)}, J_n^{(k')}, J_n^{(k'')}), \tag{6}$$

which exploits the dependency of each symbol, $I_n^{(k)}$, with only two other symbols, $I_n^{(k')}$ and $I_n^{(k'')}$, $k', k'' \neq k \in \{1, \ldots, \eta\}$, at the same time instant n and also with the same symbol at the previous time instant. The parameters k' and k'' are selected for each k, based on the particular application scenario and the available intersymbol intraframe dependencies. Assuming a first-order Markov model to capture both the interframe and intraframe dependencies and a memoryless channel, the required a posteriori probability in equation (6) is calculated by (see Appendix for details)

$$\begin{aligned} &P(I_n^{(k)} | \underline{J}_n^{(k)}, J_n^{(k')}, J_n^{(k'')}) \approx \\ &\quad C.P(I_n^{(k)} | \underline{J}_n^{(k)}).P(J_n^{(k')} | I_n^{(k)}).P(J_n^{(k'')} | I_n^{(k)}), \end{aligned} \tag{7}$$

where

$$\begin{aligned} &P(I_n^{(k)} | \underline{J}_n^{(k)}) = C'.P(J_n^{(k)} | I_n^{(k)}) \,. \\ &\quad \sum_{I_{n-1}^{(k)} \in \mathcal{J}^{(k)}} P(I_n^{(k)} | I_{n-1}^{(k)}) P(I_{n-1}^{(k)} | \underline{J}_{n-1}^{(k)}) \end{aligned} \tag{8}$$

is computed recursively over time, and,

$$\begin{aligned} &P(J_n^{(k')} | I_n^{(k)}) = \\ &\quad \sum_{I_n^{(k')} \in \mathcal{J}^{(k')}} P(J_n^{(k')} | I_n^{(k')}) P(I_n^{(k')} | I_n^{(k)}). \end{aligned} \tag{9}$$

In equations (7) and (8), C and C' are terms that normalize the sum of the probabilities to one. As well, the transition probabilities $P(I_n^{(k')}|I_n^{(k)})$ and $P(I_n^{(k)}|I_{n-1}^{(k)})$, respectively represent the dependency between two MSVQ stage indices at each time instant and the dependency of an MSVQ index over time. These probabilities are stored at the decoder.

4 Performance Analysis

In this section, we consider the application of the presented MMSE-based decoder for MSVQ-encoded signals to the reconstruction of the Linear Predictive Coefficients in MELP [22]. We use a training database of $175,726$ speech frames (20ms frame). This database contains a combination of clean speech and speech with background noise from a number of male and female speakers. Another outside test database of $30,000$ frames of recorded clean speech is used to test the performance of the proposed decoders[1]. The spectral distortion measure [27] (in the frequency range of 60 Hz to 3500 Hz), and the signal to noise ratio (dB) are used to evaluate the objective quality of the reconstructed LPC coefficients.

4.1 MSVQ for Quantization of LPC Parameters

In the 2.4 kb/s Mixed Excitation Linear Prediction speech codec [22], that is selected as a U.S. Federal Standard in 1997, a Multi-Stage Vector Quantizer is used for quantization of speech Linear Prediction Coefficients in the Line Spectral Frequency (LSF) representation [31]. The linear prediction order is 10 and therefore, the number of LSF parameters in each frame and subsequently, the VQ dimension

[1]The speech databases used in this work are provided by Nortel Networks.

in MSVQ is also 10. The MSVQ in this standard consists of 4 stages, with bit-rates of 7,6,6,6 bits for an overall rate of 25 bpf. An 8-best search is used to find the nearest codewords.

In this work, we use a slightly different structure for the Multi-Stage Vector Quantization of LSF parameters. This structure of MSVQ consists of 4 stages of 64 codevectors (6 bits) each, for an overall bitrate of 24 bpf. For code-book search a 2-best search is used. In fact, this configuration is recognized in [27] as "one of the best [MSVQ] configurations in terms of the trade-off between complexity and performance".

Table 1 presents the mutual information of different MSVQ stages for quantization of LSF parameters in the training database. Noticeable dependency is observed, particularly between the first stage and other stages of the MSVQ. Note that the diagonal terms represent the symbol entropy.

Table 2 presents the mutual information of different MSVQ stages across consecutive time instants, for quantization of LSF parameters in the training database. This indicates the interframe dependency of MSVQ output symbols. As seen noticeable dependency exists, and specially there is more than 2 bits of redundancy between the first MSVQ stages over time. This is due to the high time correlation of LSF parameters.

4.2 Numerical Results

We evaluate six decoders for the reconstruction of LSF parameters encoded with the described MSVQ.

The Maximum Likelihood (ML) decoder is simulated and provides a baseline for comparisons. The decoder MS0, often referred to as the *Basic MMSE* decoder, given by

$$\hat{\mathbf{x}}_n = \sum_{k=1}^{\eta} \sum_{I_n^{(k)}} \tilde{\mathbf{Y}}_n^{(k)} P(J_n^{(k)} | I_n^{(k)}) \tag{10}$$

is also considered. The MS0 decoder does not exploit any residual redundancy. A variant of this decoder, which exploits the residual redundancy in the form of non-uniform distribution of the symbols, is suggested for decoding of MSVQ encoded signals in [32]. This decoder is referred to as MS1 in this work, and is given by

$$\hat{\mathbf{x}}_n = \sum_{k=1}^{\eta} \sum_{I_n^{(k)}} \tilde{\mathbf{Y}}_n^{(k)} P(J_n^{(k)} | I_n^{(k)}) P(I_n^{(k)}) \tag{11}$$

The decoder MS2 is given in equation (6), and exploits both intraframe and inter-frame dependencies. Examining the intersymbol intraframe dependencies of LSF parameters, provided in Table 1, we selected the values of k' and k'' according to Table 3 to incorporate a high level of residual redundancy.

In some applications, the successive refinement feature of the MSVQ is utilized in the source coder design. In such cases, the decoder should also have the same characteristic, i.e., decoding each stage of the MSVQ must be independent of future stages. This is not the case in MS2. Therefore, we present decoder MS3 as a progressive version of the decoder MS2. That is, in reconstruction of each MSVQ stage only the intraframe dependency with prior stages are exploited. Although, this feature of the MSVQ is not utilized in MELP, however, the numerical results provide an insight into the incurred level of performance degradation with respect to MS2, to facilitate the progressive feature of the decoder.

The decoder MS4 is also a variant of the decoder MS2, which exploits only the intraframe dependencies, but ignores the interframe dependencies. This decoder is given by

$$\hat{\mathbf{x}}_n = C.\sum_{k=1}^{\eta}\sum_{I_n^{(k)}} \tilde{\mathbf{Y}}_n^{(k)} P(I_n^{(k)}|J_n^{(k)}).P(J_n^{(k')}|I_n^{(k)}).P(J_n^{(k'')}|I_n^{(k)}). \tag{12}$$

Table 4 presents the performance of the decoders MS0, MS1, MS2, MS3, and MS4 for reconstruction of the LSF test database encoded with the 24 bpf MSVQ. The MSVQ output bits are transmitted using a BPSK modulation, over an AWGN channel, which provides soft outputs to the MSVQ decoder.

As seen from Table 4, the decoders that exploit the residual redundancy achieve noticeable gains, in comparison with the basic MMSE decoder MS0. Specifically, using decoder MS2 the spectral distortion in the reconstructed signal is reduced by more than 1dB, for very noisy channels. This is followed, rather closely, by the progressive decoder MS3. Examining the performance of the decoder MS4, it is observed that about 40% of the gain of MS2 is due to the intraframe residual redundancy.

Figure 3 demonstrates the performance of the presented decoders for the reconstruction of different LSF parameters over a noisy channel with SNR = 2 dB (BER=0.0377). In this Figure, the performance measure is the signal to noise ratio. It is clear that the performance improvements are achieved almost uniformly across different vector components (LSF parameters), and amount to more than 4 dB, when comparing the MS2 decoder with the ML decoder. The performance of the MS1 decoder is very similar to that of the MS0 decoder, which as reflected in Table 1, is due to the limited available residual redundancy in the form of non-uniform symbol distribution.

Figure 4 presents the performance of the presented decoders, in terms of the reconstructed signal SNR (dB), for transmission of the LSF vectors in the test database over a noisy channel. It is clear that substantial performance improvement is achieved, even when the channel SNR is as high as 5 dB (BER = 0.0059). This gain is further increased for lower channel SNRs.

5 Conclusions

In this work, we consider the reconstruction of the signals encoded with a Multi-Stage Vector Quantizer and transmitted over a noisy channel. An approximate MMSE source decoder for MSVQ is presented that exploits the residual redundancy. Another MMSE-based decoder is also proposed that conforms with the progressive (successively refinable) feature of the MSVQ. The application to the reconstruction of the MSVQ-encoded LPC parameters in MELP is analyzed. Numerical results demonstrate the effectiveness of the proposed decoders and indicate noticeable gains. An interesting future step is to also exploit the dependency with the following samples in time for improved reconstruction. The results in [4], for the case of (full-search) VQ-encoded signals, suggest that interesting gains could be achieved depending on the level of available residual redundancy in time, and the delay permitted by the application.

Appendix

The symbol a posteriori probabilities, described in section 3, are calculated as detailed below.

$$
\begin{aligned}
P(I_n^{(k)}|\underline{\mathbf{J}}_n) &\approx P(I_n^{(k)}|\underline{J}_n^{(k)}, J_n^{(k')}, J_n^{(k'')}) \\
&= C_1.P(I_n^{(k)}, \underline{J}_n^{(k)}, J_n^{(k')}, J_n^{(k'')}) \\
&= C.P(I_n^{(k)}|\underline{J}_n^{(k)}).P(J_n^{(k')}|I_n^{(k)}, \underline{J}_n^{(k)}).P(J_n^{(k'')}|I_n^{(k)}, \underline{J}_n^{(k)}, J_n^{(k')}) \\
&\approx C.P(I_n^{(k)}|\underline{J}_n^{(k)}).P(J_n^{(k')}|I_n^{(k)}).P(J_n^{(k'')}|I_n^{(k)}) \qquad (13)
\end{aligned}
$$

where $C_1 = 1/P(\underline{J}_n^{(k)}, J_n^{(k')}, J_n^{(k'')})$ and $C = C_1.P(\underline{J}_n^{(k)})$. The second approximation in equation (13) relies on a Markov model for the dependencies between the symbols, i.e., given $I_n^{(k)}$, the symbols $I_n^{(k')}$ and $I_n^{(k'')}$ are independent of other observations. The memoryless assumption of the channel is also utilized. Equation (13) shows that the desired symbol a posteriori probability is composed of three multiplying terms. The first term (forward recursive term) exploits the residual redundancy in time, while the other two terms, as given in equation (9), exploit the dependency of the symbol $I_n^{(k)}$ with the two symbols $I_n^{(k')}$ and $I_n^{(k'')}$ with the highest level of dependency among the set of source coder symbols $[I_n^{(1)}, \ldots, I_n^{(\eta)}]$ at time instant n.

The forward term is given by (for simplicity we drop the superscript k)

$$
\begin{aligned}
P(I_n|\underline{J}_n) &= C_1'.P(I_n, \underline{J}_n) \\
&= C_1'.P(\underline{J}_{n-1}).P(I_n|\underline{J}_{n-1}).P(J_n|I_n, \underline{J}_{n-1}) \\
&= C'.P(J_n|I_n).P(I_n|\underline{J}_{n-1}) \\
&= C'.P(J_n|I_n).\sum_{I_{n-1}} P(I_n|I_{n-1}, \underline{J}_{n-1}).P(I_{n-1}|\underline{J}_{n-1}) \\
&= C'.P(J_n|I_n).\sum_{I_{n-1}} P(I_n|I_{n-1}).P(I_{n-1}|\underline{J}_{n-1}), \qquad (14)
\end{aligned}
$$

in which $C_1' = 1/P(\underline{J}_n)$ and $C' = P(\underline{J}_{n-1})/P(\underline{J}_n)$. The above derivations rely on the memoryless assumption of the channel and a first-order Markov model for the symbol dependency over time.

References

[1] K. Sayood, J. C. Brokenhagen, "Use of residual redundancy in the design of joint source/channel coders," *IEEE Trans. Commun.,* vol.39, No.6, pp. 838-845, 1991.

[2] N. Phamdo and N. Farvardin, "Optimal detection of discrete Markov sources over discrete memoryless channels- Applications to combined-source channel coding," *IEEE Trans. Inform. Theory,* vol. 40, pp. 186-103, 1994.

[3] D. J. Miller and M. Park, "A sequence-based approximate MMSE decoder for source coding over noisy channels using discrete hidden Markov models," *IEEE Trans. Commun.,* vol.46, No.2, pp. 222-231, 1998.

[4] F. Lahouti and A. K. Khandani, "An efficient MMSE source decoder for noisy channels," *Proc. Int. Symp. Telecommun.,* pp. 784-787, Tehran, Iran, Sept. 2001.

[5] N. Phamdo, F. Alajaji and N. Farvardin, "Quantization of memoryless and Gauss-Markov sources over binary Markov channels," *IEEE Trans. Commun.,* vol. 45, No. 6, pp. 668-675, June 1997.

[6] M. Skoglund, "Soft decoding for vector quantization over noisy channels with memory," *IEEE Trans. Inform. Theory,* vol. 45, No. 4, pp. 1293-1307, 1999.

[7] F. Lahouti and A. K. Khandani, “Sequence MMSE source decoding over noisy channels using the residual redundancies”, *Annual Allerton Conference on Communication, Control and Computing,* Illinois, Urbana-Champaign, October 2001.

[8] K. Sayood, H. H. Otu, and N. Demir, ”Joint source/channel coding for variable length codes,” *IEEE Trans. Commun.,* Vol. 48, No. 5, pp. 787-794, 2000.

[9] M. Park, and D. J. Miller, ”Joint source-channel decoding for variable-length encoded data by exact and approximate MAP sequence estimation,” *IEEE Trans. Commun.,* Vol. 48, No. 1, pp. 1-6, 2000.

[10] K. P. Subbalakshmi and J. Vaisey, ”On the joint source-channel decoding of variable-length encoded sources: The BSC case,” *IEEE Trans. Commun.,* Vol. 49, No. 12, pp. 2052-2055, 2001.

[11] Rainer Bauer and Joachim Hagenauer, “Symbol-by-Symbol MAP decoding of variable length codes,” *3rd ITG Conference Source and Channel Coding,* Munich, Germany, pp. 111-116, Jan. 2000.

[12] A. Murad and T. Fuja, “Joint source-channel decoding of variable-length encoded sources,” *IEEE Information Theory Workshop,* Killarney, Ireland, June 1998.

[13] R. Thobaben and J. Kliewer, ”Robust decoding of variable-length encoded Markov sources using a three-dimensional trellis,” *IEEE Commun. Letters,* Vol. 7, No. 7, pp. 320-322, 2003.

[14] T. Fingscheidt and P. Vary, “Softbit speech decoding: A new approach to error concealment,” *IEEE Trans. on Speech and Audio Proc.,* vol. 9, No. 3, Mar. 2001.

[15] F. Lahouti and A. K. Khandani, "Approximating and exploiting the residual redundancies- Applications to efficient reconstruction of speech over noisy channels," *Proc. IEEE Int. Confs. Acoust., Speech and Sig. Proc.,* vol.2, May 2001.

[16] R. Link and S. Kallel, "Optimal use of Markov models for DPCM picture transmission over noisy channels." *IEEE Trans. Image Proc.,* vol. 48, No. 10, pp. 1702-1711, 2000.

[17] T. Guionnet and C. Guillemot, "Soft decoding and synchronization of arithmetic codes: Application to image transmission over noisy channels," *IEEE Trans. Image Process.,* Vol. 12, No. 12, pp. 1599-1609, 2003.

[18] Y. Zhang, and K. Ma, "Error concealment for video transmission with dual multiscale Markov random field modeling," *IEEE Trans. Image Process.,* Vol. 12, No. 2, pp. 236-242, 2003.

[19] M. Adrat, J. Spittka, S. Heinen and P. Vary, "Error concealment by near optimum MMSE-estimation of source codec parameters," *IEEE Workshop on Speech Coding,* pp. 84-86, 2000.

[20] T. Fingscheidt, T. Hindelang, R. V. Cox, and N. Seshadri, "Joint source-channel (de-)coding for mobile communications," *IEEE Trans. Commun.,* vol. 50, No. 2, pp. 200-212, 2002.

[21] F. I. Alajaji, N. Phamdo and T. E. Fuja, "Channel codes that exploit the residual redundancy in CELP-encoded speech," *IEEE Trans. Speech and Audio Proc.,* vol. 4, No. 5, 1996.

[22] A. V. McCree and T. P. Barnwell, III, “A 2.4 kbits/s MELP coder candidate for the new U.S. federal standard,” *Proc. Int. Conf. Acoust.,Speech, Signal Process.,* pp. 200-203, 1996.

[23] E. J. Daniel and K. A. Teague, ”Sensitivity of MIL-STD-3005 MELP to packet loss on IP networks,” *Proc. 45th IEEE Midwest Symp. Circuits and Systems,* vol. 3 , pp. III-77 - III-80, 2002.

[24] E. J. Daniel and K. A. Teague, ”Federal standard 2.4 kbps MELP over IP,” *Proc. 43rd IEEE Midwest Symp. Circuits and Systems,* vol. 2 , pp. 568 - 571, 2000.

[25] C. F. Barnes, S. A. Rizvi, and N. M. Nasrabadi, “Advances in residual vector quantization: a review,” *IEEE Trans. Image Proc.,* vol. 5, No. 2, pp. 226-262, 1996.

[26] H. Khalil and K. Rose, ”Asymptotic closed-loop design of predictive multi-stage vector quantizers,” *Proc. Int. Conf. Image Proc.,* vol. 2 , pp. 199 -202.

[27] W. F. LeBlanc, B. Bhattacharya, S. A. Mahmoud and V. Cuperman, ”Efficient search and design procedures for robust multi-stage VQ of LPC parameters for 4 kb/s speech coding,” *IEEE Trans. Speech and Audio Process.*, vol. 1, no. 4, pp. 373-385, 1993.

[28] D. H. Lee and D. L. Neuhoff, ”Asymptotic distribution of the errors in scalar and vector quantizers,” *IEEE Trans. Inform. Theory,* vol. 42, pp. 446-460, Mar.1996.

[29] A. B. Sripad and D. L. Snyder, “A necessary and suffcient condition for quantization errors to be uniform and white,” *IEEE Trans. Acoustics, Speech, and Signal Proc.,* vol.25, No.5, pp. 442- 448, 1977.

[30] B. Widrow, I. Kollr, and M.- C. Liu “Statistical theory of quantization,” *IEEE Trans. on Instrumentation and Measurement,* 45(2), pp. 353- 361, 1996.

[31] F. Itakura, ”Line spectrum representation of linear predictive coefficients of speech signals,” *Journal of Acoustical Society of America,* vol.57, p. 535, Apr.1975.

[32] S. Gadkari and K. Rose, “Unequally protected multistage vector quantization for time-varying CDMA channels,” *IEEE Trans. Commun.*, vol. 49, no. 6 , pp. 1045-1054, 2001.

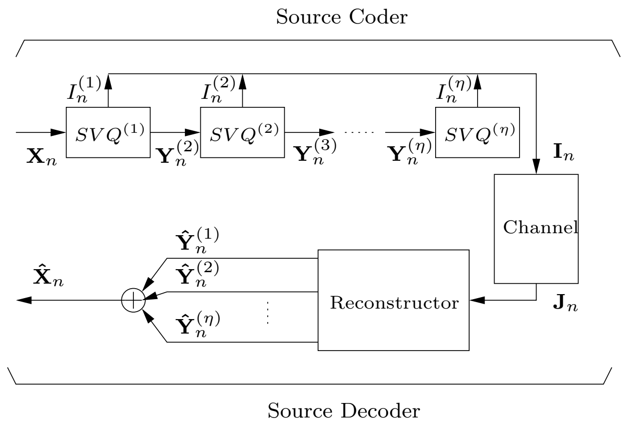

Figure 1: Overview of the system

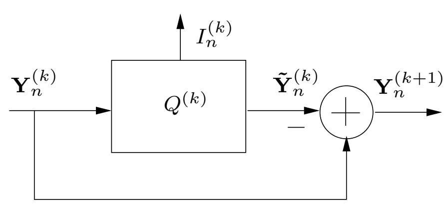

Figure 2: The kth stage (Stage-VQ) of the MSVQ ($SVQ^{(k)}$)

Stage No.	1	2	3	4
1	5.77	0.66	0.41	0.35
2	0.66	5.84	0.31	0.24
3	0.41	0.31	5.89	0.26
4	0.35	0.24	0.26	5.92

Table 1: Mutual information of different MSVQ stages within a frame in bits, diagonal elements of the table represent symbol entropy; 4-stage Multi-Stage Vector Quantization of LSF parameters at 24 bpf.

Stage No.	1	2	3	4
1	2.05	0.46	0.28	0.22
2	0.46	0.57	0.23	0.17
3	0.28	0.23	0.28	0.17
4	0.22	0.17	0.17	0.20

Table 2: Mutual information of MSVQ stages between successive frames in bits; 4-stage Multi-Stage Vector Quantization of LSF parameters at 24 bpf.

k	k'	k''	k	k'	k''
1	2	3	3	1	2
2	1	3	4	1	3

Table 3: The values of parameters k' and k'' in equation (6) for reconstruction of LSF parameters using decoder MS2.

Channel SNR (dB)	BER	MS0	MS1	MS2	MS3	MS4
1.00	0.0560	3.68	3.56	2.57	2.65	3.27
2.00	0.0370	3.05	2.94	2.15	2.22	2.68
3.00	0.0220	2.41	2.33	1.78	1.82	2.13
4.00	0.0120	1.85	1.80	1.47	1.50	1.67
5.00	0.0059	1.45	1.41	1.26	1.27	1.35
6.00	0.0023	1.20	1.19	1.14	1.14	1.16
7.00	0.0008	1.08	1.08	1.08	1.08	1.08

Table 4: Average spectral distortion (dB) of the test LSF database reconstructed using five MMSE-based decoding schemes for transmission over an AWGN channel with soft outputs and BPSK modulation.

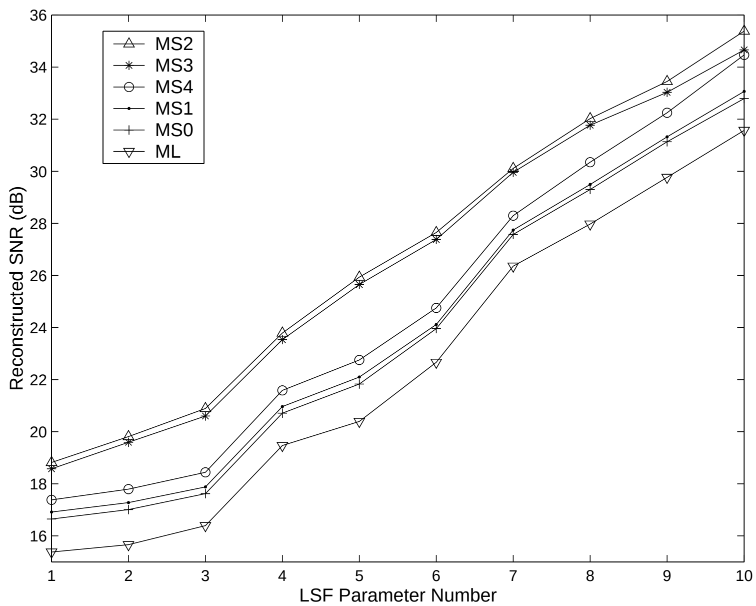

Figure 3: Reconstructed SNR (dB) for transmission of MSVQ-encoded LSF parameters over a noisy channel with SNR=2 dB, BER=0.0377, using different decoding schemes.

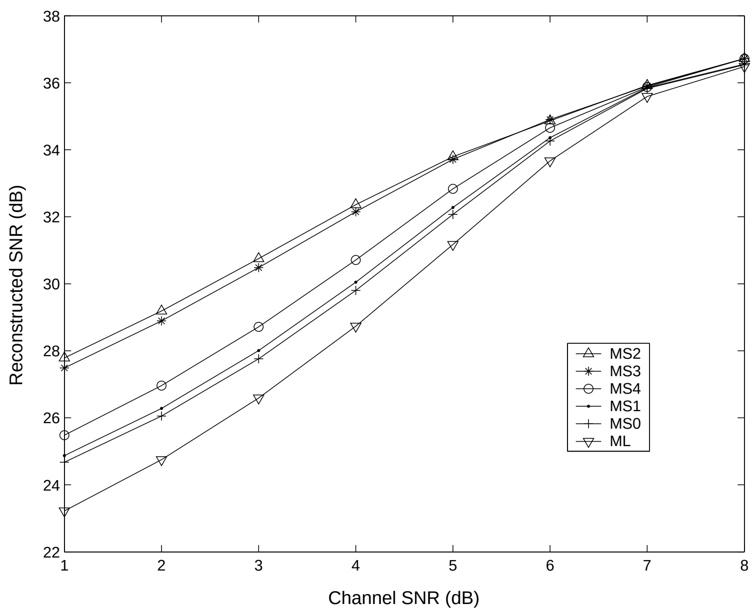

Figure 4: Average reconstructed SNR (dB) for transmission of MSVQ-encoded LSF parameters over a noisy channel.