Efficient Reconstruction of Channel State Information for Low-Rate Feedback Schemes

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Abstract

The closed-loop transmit diversity technique is used to increase the capacity of the downlink channel in multiple-input-multiple-output (MIMO) communication systems. The WCDMA standard [1] endorsed by 3GPP [2] adopts two modes of downlink closed-loop schemes based on partial channel information that is fed back from the mobile unit to the base station through a low-rate uncoded feedback bit stream. In this article, some soft reconstruction techniques are introduced to improve the performance of Mode 1 of 3GPP, by taking advantage of the redundancy available in the channel information. We propose some algorithms for reconstruction of beamforming weights in the base station.

The performance is examined in a simulated 3GPP framework in the presence of different feedback error rates at various mobile speeds. It is demonstrated that the proposed algorithms have substantial gain over the conventional approach for low to high mobile speeds.

Index Terms

Closed-loop transmit diversity, channel feedback, channel state information (CSI), downlink communication, FDD WCDMA, mode 1 of 3GPP, joint source-channel coding

I. Introduction

The increasing demand for internet and wireless services such as voice, video and data highlights the need for an increase in the system capacity, which is aimed at in the third generation of mobile communication. In particular, the 3rd Generation Partnership Project (3GPP) [2] and the 3rd Generation Partnership Project Two (3GPP2) [3] have developed the Wideband Code-Division Multiple Access (WCDMA) technologies and CDMA2000, respectively. The improvement of the downlink capacity is one of the main challenges of the 3G systems, because many of the proposed services are expected to be downlink-intensive. By exploiting the available spatial diversity, multiple antenna techniques are known to enhance the capacity and quality of wireless communication especially in fading channels [4], and for downlink applications, *transmit antenna diversity* is typically suitable [5].

In the 3G evolution, both open-loop (without channel feedback) and closed-loop transmit diversity schemes (with channel feedback) have been considered: Open-loop techniques such as Orthogonal Transmit Diversity [5] and Space-Time Transmit Diversity [6], and closed-loop techniques such as Switched Transmit Diversity and Beamforming (also called Transmit Adaptive Array) [5]. Alamouti Space-Time Coding [7], and Beamforming are parts of the 3GPP standard of WCDMA FDD (Frequency Division Duplex) downlink system [8]. The closed-loop scheme enables the transmit array to optimally beamform the transmit signal to a particular channel state. Using the closed-loop communication for its high capacity achievement has been standardized in 3GPP, and is known to be effective for low-speed mobile users, whereas it fails at high mobile speeds [5].

Assume that the message is intended for a single receiver. When there is little (or no) channel state information at the transmitter, the diversity schemes are optimal, whereas when there is adequate channel state information available at the transmitter, beamforming strategy is optimal and provides much higher capacity for the system [9]. With beamforming, the transmissions from different antenna elements at the base station add constructively at the receiver, which results in some enhancement in the received SNR. However, this improvement requires that the transmitter has a fairly accurate knowledge of the parameters of the channel to the intended receiver. This is difficult to achieve when the parameters are time-varying. Furthermore, in a practical communication system, feedback data is subject to imperfections such as quantization [10], feedback error [10], [11] and feedback delay [12].

Main object of this article is to mitigate the problems of mode 1 of 3GPP with feedback error, and inefficiency of its weight reconstruction algorithm. Mode 1 is the main close-loop algorithm of 3GPP which is designed for higher mobile speeds. There are so many works that propose new limited-rate feedback schemes for 3G systems and beyond, but there are few works to address the problems of mode 1 (and mode 2) of the 3GPP standard, like [13]. In this article, our focus is to enhance the performance of mode 1 of 3GPP in presence of feedback error, for different mobile speeds.

The structure of the rest of this article is as follows: Section I-A includes a general description of a closed-loop system, as well as our notations and channel model. In Section I-B, some conventional channel feedback schemes are examined. Section II describes the algorithm of mode 1 of the 3GPP standard, and its problems and challenges in presence of feedback error. Section III explains our approach to reconstruct the beamforming weights, and elaborates on the proposed methods and algorithms. Finally, Section IV shows the simulation results and compares the performance of our algorithms with the standard algorithm in different conditions.

A. Closed-loop Systems

In this article, our assumptions and simulation parameters are consistent with WCDMA FDD closed-loop modes, especially mode 1. Fig. 1 is a functional diagram of the closed-loop modes of downlink 3GPP standard [8], and a general description of the feedback system follows.

After the channel is estimated in the receiver (the mobile unit) by using the transmitted pilots, the channel state information is quantized and sent to the transmitter. The transmitter computes the required beamforming coefficients and applies them to the transmit antennas. The more precise the channel estimation is, the better the total performance will be. However, the feedback channel has a low rate. Furthermore, there is delay and error from the receiver to the transmitter. In spite of these error sources, the closed-loop schemes perform significantly better than the open-loop schemes, at low mobile speeds [5].

Consider a system with M transmit antennas and one receive antenna. As a

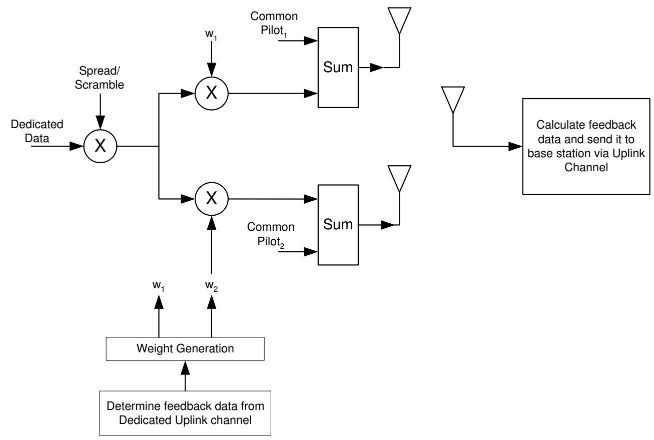

Fig. 1. Closed-loop mode of 3G systems

channel model, a flat fading channel is considered from each transmit antenna to the receive antenna. Dropping the time indices for simplicity, we write

$$r = \mathbf{h}^T \mathbf{x} + \eta, \tag{1}$$

where r is the received signal at the receiver,

$$\mathbf{h} = [h^{(1)} \cdots h^{(M)}]^T \in \mathbb{C}^M \tag{2}$$

is our channel coefficients complex vector, where $h^{(m)}$ represents the channel between the m-th transmit antenna and the receive antenna,

$$\mathbf{x} = [x^{(1)} \cdots x^{(M)}]^T \in \mathbb{C}^M \tag{3}$$

represents the channel input vector. η is a complex circularly symmetric AWGN with the variance N_0, $\eta \sim \mathcal{N}(0, N_0)$.

There is a constraint on the total transmit power, $E[\|\mathbf{x}\|^2] = \sum_{m=1}^{M} E[|x^{(m)}|^2] \leq M$.

For the channel model, we consider a Rayleigh fading model, and so $h^{(m)}$, $m = 1, \cdots, M$, are zero-mean independent, identically distributed, circularly symmetric

Gaussian random variables. Each coefficient is expressed as

$$h^{(m)} = \alpha^{(m)} e^{j\phi^{(m)}}, \tag{4}$$

where $\alpha^{(m)}$ and $\phi^{(m)}$ are the amplitude and the phase, respectively. It is well-known that $\alpha^{(m)}$ has a Rayleigh distribution [14]

$$p(\alpha^{(m)} = a) = a\, e^{-a^2/2}, \qquad a \geq 0 \tag{5}$$

and $\phi^{(m)}$ has a uniform distribution,

$$p(\phi^{(m)} = \theta) = \frac{1}{2\,\pi}, \qquad 0 \leq \theta < 2\,\pi. \tag{6}$$

It is also known that the autocorrelation function of the fading for two-dimensional isotropic scattering and an omni-directional receiving antenna is given by [15]

$$R(t, t-\delta) = \frac{E[h(t)h^*(t-\delta)]}{\sigma_h^2} = J_0(2\pi f_d \delta), \tag{7}$$

where $J_0(\cdot)$ is the first-kind Bessel function of the zero order, f_d is the doppler frequency, and δ is the time difference.

For simulating the mobile fading channel, some models have been proposed based on the properties (5 - 7). In one popular implementation, known as Jakes Model [16], some low-frequency oscillators are utilized to generate the fading. Several derivations of Jakes model have been developed to mitigate its flaws [15]. We use a modified Jakes fading generator suggested in [15] that generates a stationary signal.

In our analysis, we assume that all the elements of the channel vector $\mathbf{h}$ are completely known at the receiver. In practice, these parameters need to be estimated from the received signal, but this assumption allows a better comparison between different schemes, regardless of the channel estimation errors at the receiver. It is noteworthy that in the 3GPP systems, there is a separate Common Pilot Channel (CPICH), with a relatively large proportion of the base station power, which allows each receiver to accurately estimate its channel coefficients [17], and this supports the above assumption.

B. Beamforming

In a closed-loop system, channel input $\mathbf{x}$ should be appropriately selected according to the channel state [18]. Controlling the channel input can be accomplished with a conventional beamformer which applies some weights on the transmitted signal for each antenna, which can be expressed as

$$\mathbf{x} = \mathbf{w}s, \tag{8}$$

where

$$\mathbf{w} = [w^{(1)} \cdots w^{(M)}]^T \in \mathbb{C}^M. \tag{9}$$

Note that without losing the generality, we assume that $\|\mathbf{w}\|^2 = 1$, signifying that the beamformer does not change the total transmit power. Therefore,

$$r = \mathbf{h}^T\mathbf{w}s + n = (\sum_{m=1}^{M} h^{(m)}w^{(m)})s + \eta. \tag{10}$$

It is also notable that with this presentation, an open-loop system can be represented as

$$w^{(m)} = \frac{1}{\sqrt{M}}; \qquad m = 1, \cdots, M. \tag{11}$$

Having one receiver, the received signal, r, is a complex number which is the superposition of the signals from different channels, as well as the noise. Relating r

to the output, the combining variable, $v \in \mathbb{C}$, is introduced and applied as [14]

$$z = v^H r = v^H \mathbf{h}^T \mathbf{w} s + v^H \eta. \tag{12}$$

For an optimum solution, $\mathbf{w}$ and v should be selected, simultaneously. It has been shown [19] that to minimize the average probability of error and/or to maximize the capacity in a MIMO system, $\mathbf{w}$ and v should be selected to maximize the SNR. Therefore,

$$\text{SNR}_{inst} = \frac{\left|v^H \mathbf{h}^T \mathbf{w} s\right|^2}{\left|v^H \eta\right|^2} = \left|\mathbf{h}^T \mathbf{w}\right|^2 \frac{E_s}{N_0} \tag{13}$$

where $E_s = E[|s|^2]$. So for one receive antenna, v has no effect on the performance, and we select it to normalize the coefficient of s in (12), then we have

$$z = s + \frac{(\mathbf{h}^T \mathbf{w})^H}{|\mathbf{h}^T \mathbf{w}|^2} \eta. \tag{14}$$

About the effect of different combining schemes on WCDMA systems, refer to [20].

1) Ideal Feedback: Maximizing the instantaneous SNR in (13), the optimum weights are found as

$$\mathbf{w}^{ideal} = \arg \max_{\mathbf{w}} \left|\mathbf{h}^T \mathbf{w}\right|^2, \tag{15}$$

subject to the constraint $\|\mathbf{w}\|^2 = 1$, which results in [21]

$$\mathbf{w}^{ideal} = \frac{\mathbf{h}^*}{\|\mathbf{h}\|}. \tag{16}$$

2) Co-Phase Feedback: The Ideal feedback requires phase and amplitude information of the beamforming vector. A Co-phase feedback scheme corrects the phases of the received signals from different channels that they add coherently together, and it does not need the amplitude information of the channel. The Co-phase feedback algorithm may be shown as

$$\begin{cases} w^{(1)} = \frac{1}{\sqrt{M}} \\ \\ w^{(m)} = \frac{1}{\sqrt{M}}\, e^{-j(\phi^{(m)}-\phi^{(1)})}, \qquad m = 2, \cdots, M \end{cases} \tag{17}$$

It has been shown that Ideal feedback and Co-phase feedback schemes enhance the (average) received SNR by a factor of M and $1+(M-1)\frac{\pi}{4}$, respectively, regards to the transmit SNR [12], [21]. This SNR gain is used as a figure of merit for an uncoded system. For $M = 2$, the SNR gains are 3.0 dB and 2.5 dB, respectively. Therefore, by using the phase-only information, we are losing only 0.5 dB of SNR (and at maximum 1.0 dB for large number of transmit antennas). This confirms that the channel phase information is usually more important than the amplitude information.

3) Quantized Feedback: In practice, usually a limited capacity is available for the feedback channel. Therefore, for quantization of the channel state information, an efficient source-coding scheme should be used. There are a variety of works on quantization of feedback data, like [18], [22]–[25].

WCDMA standard suggests two closed-loop modes using quantized feedback data. Mode 1 which is essentially a quantized Co-phase feedback scheme is described in the next section. Mode 2 incorporates the channel amplitude information in the feedback data as well, and it uses 3 bits of phase information and 1 bit of amplitude information to calculate the beamforming weight. Mode 2 is intended to provide a high data rate for very low speed users, while mode 1 is usually used to provide the service for low to moderate mobile speeds [26]. Therefore, the performance of mode

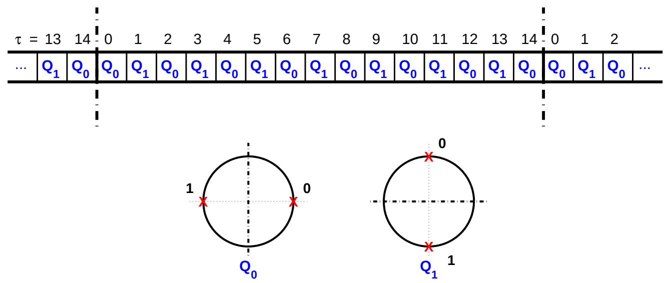

Fig. 2. Framing structure and quantization scheme of mode 1 of 3GPP

1 is more important, and it is challenged at high mobile speeds.

II. Closed-Loop Mode 1 of 3GPP

In the standard of 3GPP [8], it is assumed that $M = 2$ transmit antennas, and so $\mathbf{h}_n = [h_n^{(1)}, h_n^{(2)}]$ and $\mathbf{w}_n = [w_n^{(1)}, w_n^{(2)}]$, where n is the time index. It is also assumed that in base station, the first beamforming weight is constant,

$$w_n^{(1)} = \frac{1}{\sqrt{2}}, \tag{18}$$

and $w_n^{(2)}$ is constructed from the feedback data. $\phi_n = \angle h_n^{(2)} - \angle h_n^{(1)}$ is the co-channel phase which is quantized and fed back from mobile to the base station. The quantization of the phase is subject to a special framing structure as follows:

Fig. 2 depicts the the slot structure of 3GPP. Each (uplink) frame has a duration of 10 msec and includes 15 slots, and each slot contains a number of data symbols depending on the data rate. We use a framed time index, $\tau = n \bmod 15$, to show the relative place of each slot or feedback bit in a frame, i.e., $\tau = 0, 1, \cdots, 14$, as shown in Fig. 2.

For each slot, 1 bit of feedback data is sent from mobile unit which makes a feedback stream of 1500 bits per seconds. The feedback bit is determined by $Q_0 =$

$\{0, \pi\}$ or $Q_1 = \{\pi/2, -\pi/2\}$ one-bit quantizers, as 0 and 1 for the first and the second phases in each set, respectively.

The received feedback data bits (in the base station) are converted to $\tilde{\phi}_n$ stream, so as Fig. 2 shows, each sample takes a value from the respective subset:

$$\tilde{\phi}_n \in \begin{cases} Q_0, & \tau = 0, 2, \cdots, 14 \\ \\ Q_1, & \tau = 1, 3, \cdots, 13 \end{cases} \tag{19}$$

$w_n^{(2)}$ is constructed linearly from the two recent phase information $e^{j\tilde{\phi}_n}$, one from an Q_0 slot and one from an Q_1 slot. It can be shown as

$$w_n^{(2)} = \begin{cases} \frac{1}{2}(e^{j\tilde{\phi}_n} + e^{j\tilde{\phi}_{n-1}}), & \tau \neq 0 \\ \\ \frac{1}{2}(e^{j\tilde{\phi}_n} + e^{j\tilde{\phi}_{n-2}}), & \tau = 0 \end{cases} \tag{20}$$

Note that in each time, (20) selects the $w_n^{(2)}$ from a set of 4 predefined weights, by just one bit of feedback data. This construction also guarantees that always $\left|w_n^{(2)}\right| = \frac{1}{\sqrt{2}}$ for all n.

A. Effect of Feedback Error on the Performance

One of the major problems for feedback-based downlink beamforming techniques is the presence of feedback errors in the uncoded feedback stream. If a command for the base station becomes corrupted during the transmission in the uplink slot, an incorrect antenna weight vector will be applied in the subsequent downlink slot [11].

There are two consequences for such an error. First, the received signal power is smaller, because a non-optimum weight is applied. However, our simulations show that the performance degradation due to this effect is rather small, and the second consequence is much more serious. Each time a feedback error occurs, the mobile station does not know the correct antenna weight vector that is applied at the base station. Since the mobile station obtains the dedicated channel estimate by combining the estimates for individual antennas from common pilots with the assumed weight vector used at the base station, this leads to serious dedicated channel estimation error. Later we will see that this causes some error floor in error curves. Because each mismatch can potentially result in in an error in the decoding process, hence an error floor proportional to the feedback error is imposed on the performances.

The loss of the closed-loop gain because of feedback error (the first effect) has been analyzed in [10]. To minimize the effect of the second problem, a technique called Antenna Weight Verification (AV) [27] can be utilized, which is addressed in the next section.

B. Antenna Weight Verification

In this algorithm, the mobile station considers the possibility of an erroneous feedback transmission by comparing the Common-Pilot-based channel estimate, with the one obtained from the few training symbols in the dedicated channel which include the effect of $\hat{\mathbf{w}}$ applied in transmitter. An AV algorithm for mode 1 of 3GPP has been suggested in the annex to [8], and the performance of the AV algorithm has been analyzed in [28]. Also a trellis-based AV algorithm has been proposed in [17] to improve the performance of the verification process.

But this technique has some drawbacks. Applying an AV algorithm requires extra calculations at the mobile unit, and also requires special dedicated preamble bits to be transmitted to all users. Also the structure of an AV algorithm is based on the beamforming scheme in the transmitter, which means AV algorithm is different for each beamforming algorithm. This complexity in the mobile unit could be limiting for complicated adaptive beamforming schemes. Therefore, there has been a tendency to find a substitute technique for AV, for example see [29].

In the next section, we will introduce a new approach to improve the performance of the beamforming scheme, especially in presence of feedback error. In this approach, the redundancies available in the feedback data are exploited to find the best estimate of the antenna weight. Our approach helps to solve both problems formerly explained, and the performance is usually good enough that an AV algorithm is no longer needed, although, an AV could be used along with our algorithms if higher performances are required. Furthermore, our proposed algorithms are accomplished mostly at the base station.

III. Efficient Reconstruction of Beamforming Weights

In the sequel, we preserve the framing structure and the quantization scheme of mode 1 of the 3GPP standard, described in Section (II). Also we do not consider the effect of feedback delay here.

A. Proposed Approach

It is assumed that $\left|w_n^{(2)}\right| = \frac{1}{\sqrt{2}}$, which comes from the fact that we want the transmit power to be constant and so we are working in the Co-phase feedback framework, and the related schemes are optimized by controlling the phase information of $w_n^{(2)}$. Mode 1 suggests a linear combination to produce $w_n^{(2)}$, as is shown in (20). However, we attempt to make better reconstruction algorithms.

Fig. 3 shows the quantization process in the receiver and the de-quantization process in the transmitter. $\tilde{\phi}_n$ is the quantized co-phase, which is introduced in (19), and I_n is the respective index, which attains the values of $I_n \in \{0, 1, 2, 3\}$ corresponding to $\hat{\phi}_n \in \{-\pi/2, 0, \pi/2, \pi\}$, respectively. Also $\hat{\phi}_n$ is the estimated quantized co-phase, and similarly, J_n is the respected index. Note that for an error-free feedback channel, $\hat{\phi}_n = \tilde{\phi}_n$ and $I_n = J_n$.

The fundamental theorem of estimation states that given the received sequence $\underline{J}_n = [J_n, J_{n-1}, \cdots, J_2, J_1]$,

$$\breve{w}_n^{(2)} = E\left[w_n^{(2)} \middle| \underline{J}_n\right] \tag{21}$$

is the minimum mean-squared error (MMSE) estimate of the weight $w_n^{(2)}$. It has been shown in [?] that the formula can be approximated by the following feasible form:

$$\breve{w}_n^{(2)} = \sum E\left[w_n^{(2)} \middle| \underline{I}_n^{n-\mu+1}\right] P\left(\underline{I}_n^{n-\mu+1} \middle| \underline{J}_n\right), \tag{22}$$

where the summation is over all the possible μ-fold sequences of $\underline{I}_n^{n-\mu+1} = [I_n \cdots I_{n-\mu+1}]$, and the formula is asymptotically optimum for sufficiently large values of μ.

We are dealing with estimation of a complex variable with a constant amplitude, as $\left|\hat{w}_n^{(2)}\right| = \frac{1}{\sqrt{2}}$. But there is no control on the amplitude of $\breve{w}_n^{(2)}$ in (21) and (22).

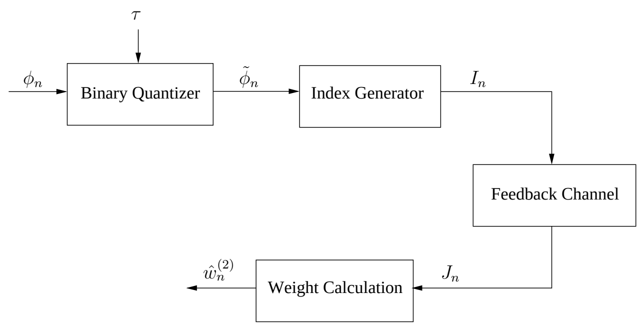

Fig. 3. Block diagram of encoding and decoding of the quantizer system

So we need an MMSE estimator with a constant amplitude, which is introduced in Lemma 1 (see Appendix I). According to Lemma 1, we can always calculate the needed antenna weight using the MMSE solution of (21) or (22) as

$$\hat{w}_n^{(2)} = \frac{1}{\sqrt{2}} \overline{E}\left[w_n^{(2)} \middle| \underline{J}_n\right] = \frac{1}{\sqrt{2}} \frac{\check{w}_n^{(2)}}{\left|\check{w}_n^{(2)}\right|}. \tag{23}$$

B. Error-Free Feedback Case

When there is no error in the feedback channel, the probability term of (22) disappears and

$$\check{w}_n^{(2)} = E[w_n^{(2)} | \underline{I}_n^{n-\mu+1}] \tag{24}$$

Equation (24) is a codebook definition where μ indices is needed to specify each codeword. The codeword associated with the specified $\underline{i}_n^{n-\mu+1}$ is constructed as

$$w_{CB}^{(2)}(k) = \frac{1}{\sqrt{2}} \overline{E}\left[w_n^{(2)} \middle| \underline{I}_n^{n-\mu+1} = \underline{i}_n^{n-\mu+1}\right], \tag{25}$$

where $w_n^{(2)}$ is the weight calculated from Co-phase feedback algorithm in (17). With a sufficient amount of training data, the expectation of (25) can be implemented by empirical averaging. The codebook is constructed for $k = 1, \cdots, N_{CB}$, where N_{CB} is the codebook size (which is the number of possible sequences of $\underline{I}_n^{n-\mu+1}$). It is shown

μ	1	2	3	4
4^{μ}	4	16	64	256
N_{CB}	4	12	32	80

TABLE I

CODEBOOK SIZE

in Appendix II that for our framework, the codebook size is $N_{CB} = (\mu + 1)\, 2^{\mu}$, which is less than 4^{μ} possible codewords. That is because there are some constraints on the value of I_n in each time slot subject to the framing structure shown before. Table I shows the codebook size for some typical values of μ.

C. *Erroneous Feedback Case*

In Section III-B, the error-free case was considered. For the erroneous feedback channel, which is almost always the practical case, we should consider the probability part of (22). This probability depends on the erroneous sequence of J_n, and can help in optimizing the feedback scheme using the redundancy in the sequence. One approach to the problem is to design a trellis structure and use a reconstruction algorithm [?] to find the best weight.

1) Trellis Structure: It is assumed that we have a Markov source of order γ. A trellis structure is used to exploit the Markov model, and the states are defined as follows

$$S_n = \underline{I}_n^{n-\gamma+1} \tag{26}$$

It has been shown in Appendix II that there are $N_{states} = (\gamma + 1)\, 2^{\gamma}$ possible states in each time.

To find a proper value for γ, we examine the redundancies by measuring the conditional entropy $H(I_n|S_{n-1})$ [30], entropy of each symbol given the previous state. Table II shows typical entropies for values of $\gamma = 1, 2, 3, 4$, for different mobile speeds in the range of our interest. It is expected and observed that entropies are smaller for larger memory depths. But at low speeds, for memory depths $\gamma \geq 3$, entropies do not decrease significantly, which means that there is no more redundancies after that depth. This threshold is smaller for higher mobile speeds. Hence we will use $\gamma = 3$ in our simulations, which is a fairly small value of memory depth but can capture most of the redundancies.

This trellis will be used by some of our algorithms, in association with other reconstruction criteria.

D. Proposed Methods

Its notable that our algorithms are all real-time algorithms, and decide on the needed antenna weight without any delay. In other words, receiving the most recent feedback symbol J_n, the algorithm will calculate $\hat{w}_n^{(2)}$.

1) Codebook Algorithm: In codebook algorithm, the received feedback sequence is taken as the estimate of the transmitted sequence,

$$\hat{\underline{i}}_n = \underline{j}_n, \tag{27}$$

γ	0	1	2	3	4	5
$v = 1$	2.00	1.28	0.44	0.43	0.41	0.41
$v = 10$	2.00	1.28	0.68	0.67	0.65	0.65
$v = 25$	2.00	1.28	0.92	0.92	0.89	0.89
$v = 100$	2.00	1.29	1.27	1.27	1.25	1.24

TABLE II

$H(I_n|S_{n-1})$ FOR DIFFERENT MEMORY DEPTHS γ, AND DIFFERENT MOBILE SPEEDS

which is used to generate the index for the codeword to be applied as the current weight,

$$\hat{w}_n^{(2)} = w_{CB}^{(2)}\left(\hat{\underline{i}}_n^{n-\mu+1}\right). \tag{28}$$

This algorithm will reveal the strength of the codebook, which is calculated by the nonlinear estimator of (25), and is asymptotically optimum for the error-free feedback case.

2) MMSE Solution: Implementing (22), we need the probabilities of the all possible states of the trellis in each time. We have assumed a Markov source and a memoryless feedback channel, so from the BCJR algorithm [31] we can find the probability of each state recursively [?]

$$P(S_n|\underline{J}_n) = C\,P(J_n|I_n) \sum_{S_{n-1}\to S_n} P(I_n|S_{n-1})\,P(S_{n-1}|\underline{J}_n) \tag{29}$$

where the summation is over all possible transitions from the states of time $n-1$ to S_n, and C is the normalizing factor so that $\sum_{S_n} P(S_n|\underline{J}_n) = 1$. For simplicity, we assume that $\mu \leq \gamma$, then the weight could be calculated using (29) and (22) and there is no need to calculate the backward term of BCJR algorithm.

3) Normalized-MMSE Algorithm: Assuming a constant amplitude weight, Lemma 1 provides the solution using the normalized MMSE estimation, as in (33). This algorithm is used instead of the conventional MMSE solution to keep the transmit power constant.

4) SMAP Algorithm: The sequence MAP (SMAP) decoder receives the sequence $\underline{J}_n$ and determines the most probable transmitted sequence as follows

$$\hat{\underline{i}}_n = \arg\max P(\underline{I}_n|\underline{J}_n), \tag{30}$$

using the trellis of Section III-C.1. With some mathematical manipulation [30], the following branch metric can be obtained for the respective trellis,

$$m\Big(J_n, S_{n-1} \overset{I_n}{\longrightarrow} S_n\Big) = \log\Big\{P(J_n|I_n)\, P(I_n|S_{n-1})\Big\}, \tag{31}$$

where $P(I_n|S_{n-1})$'s are the *a priori* information, and $P(J_n|I_n)$'s are the channel transition probabilities. $P(I_n|S_{n-1})$ is a codebook of probabilities calculated by using a sufficient amount of training data. Assuming a binary symmetric feedback channel

(BSC) with the error probability of p_e,

$$P(J_n|I_n) = \begin{cases} 1 - p_e, & \text{if } J_n = I_n \\ p_e, & \text{else if they are in the same set} \\ 0, & \text{else} \end{cases} \tag{32}$$

Performing the Viterbi algorithm [32] on the trellis provides the solution for (30).

Note that SMAP algorithm is not directly resulted from the MMSE approach, because it only uses the best path in the trellis. In fact, it can exploit the trellis with a lower computational complexity in comparison to MMSE algorithm.

5) Soft-Output Methods: So far, we assumed the quantized feedback information. However, the use of soft feedback data can potentially improve the performance of the aforementioned algorithms, and this technique is well-known in the literature. It is assumed that instead of hard-decided bits of feedback data, soft-output (noisy) feedback symbols are available. Equivalently, instead of a BSC channel, an AWGN channel is assumed for the feedback data. The noise power is selected to keep the hard-decision feedback error probability the same if there was a decision unit after the AWGN channel.

E. Implementation and Complexity

The weight-codebook and probability-codebook are required to be calculated once. Because they depend on the channel model, the optimal codebooks should be calculated for each mobile speed. For Jakes fading, the codebooks could be estimated with a sufficient amount of training data. It will be shown that with a limited number of codebooks, the algorithms are applicable to all mobile speeds. For other channel models, the optimal codebooks could be estimated similarly. In the severe cases that the channel model is changing rapidly, a tracking approach could be used.

For selecting the proper codebook, an estimate of the mobile speed is required. There are various algorithms suggested for mobile speed estimation, like [33], and in particular for 3G systems, like [34], [35], that could be used. Our simulations show that for the estimation precisions reported in the references, the performance of our algorithms does not change significantly.

About the computational complexity, our algorithms use known methods like Viterbi algorithm and forward-BCJR, with very small memory depths. Therefore, the needed complexity is not high. Furthermore, these algorithms are implemented at the base station where complexity is not a great concern. Although, we introduce an approach to decrease the complexity of our trellis-based algorithms in the next section.

1) Time-Dependent Trellis Structure: As we saw in Section II, the framing structure of the closed-loop mode 1 of 3GPP which is preserved here, is *time-dependent*. In each slot, I_n can adopt one of the four values $\{0, 1, 2, 3\}$, which represents a 2-bit value. One of these bits is the information bit, but the other bit is specified by the slot number. In other words, given the time slot, I_n attains one of two possible values.

In our trellis, we use the $P(I_n|S_{n-1})$ codebook to specify the trellis structure. There are $(\gamma + 1)2^{\gamma}$ possible states; some states occur only at the frame-boundaries,

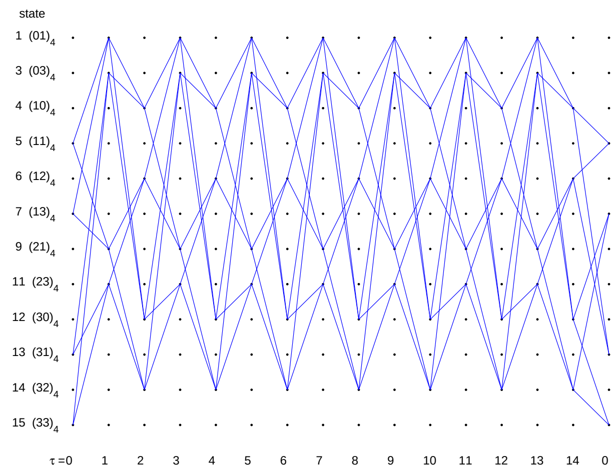

Fig. 4. Time-based trellis for the case $\gamma = 2$

whereas the others can occur at middle of a frame or close to the frame boundaries (see Fig. 10). For 2^γ states, $P(I_n|S_{n-1})$ have positive probabilities for all the four possible I_n's, indicating that $P(I_n|S_{n-1})$ carry an average of the middle-frame effects and boundary effects. Since the slot number is known, these states can be split into two categories, subject to the different framing effects. Therefore, its appropriate to calculate and use the $P(I_n|S_{n-1}, \tau_n)$ for $\tau_n = 0, 1, 2, \cdots, 14$. Fig. 4 shows a typical trellis made by the time-based approach for $\gamma = 2$. Because there are less possible paths on the trellis, this approach can decrease the complexity of our trellis-based algorithms by a factor of $2\,(\gamma + 1)$.

IV. Results

A. Simulation Parameters

For testing and comparison of the algorithms, we have established a communication system similar to the downlink of FDD WCDMA [1]. Fig. 5 represents our complete system, including the channel coding and the details of the feedback system.

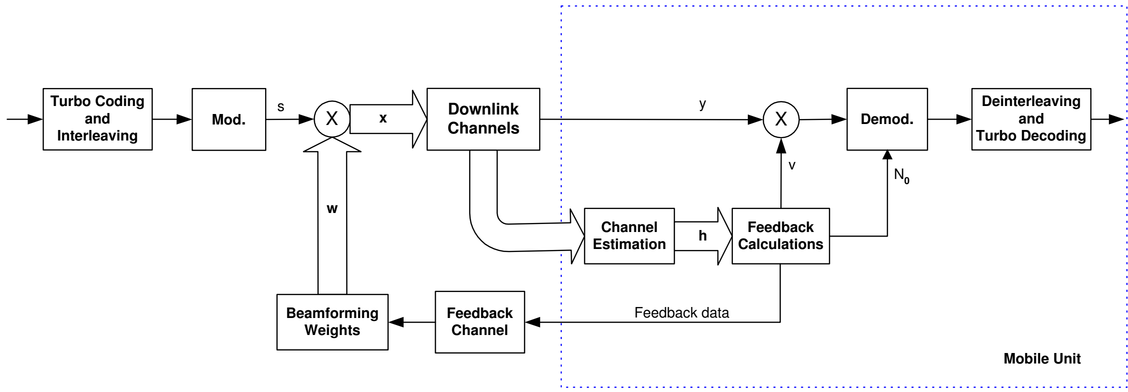

Fig. 5. Block diagram of our feedback system

For channel coding, there are two options in 3GPP systems: Convolutional coding, and Turbo coding. A Turbo code [36] is a good channel coding option because its performance tightly approaches the theoretical limit, so it is used in our system. The parameters of the Turbo code structure suggested by 3GPP standard [37] are used in our simulations.

For the channel interleaving, we apply a randomly-generated interleaver with the same length as that of the frame, suggested by 3GPP. Because of this interleaver, the coding scheme faces a channel which is closer to an i.i.d. channel, respect to the case without any channel interleaving.

In 3GPP systems, there is a power control scheme intended to eliminate the slow fading. Therefore, we assume that slow fading is compensated by power control. Table III is a summary of the parameters of our simulations. The other parameters, that may change in different simulations, are specified in the simulation results. It is also assumed that there is no antenna verification, and transmitter and receiver have to calculate their own beamforming weights and other parameters they need.

Carrier Frequency	2.15 GHz
Modulation	QPSK
Transmitter	2 Antennas
Receiver	1 Antenna
Data Rate	15000 bps
Feedback Rate	1500 bps
Channel Model	Modified Jakes
Channel Coding	Turbo Code
Code Rate	1/3
Frame Length	300 (20 mSec)
Bit Interleaving	One Frame
Channel Estimation	Ideal
Power Control	No

TABLE III

Simulation parameters

B. Simulation Results

As a measure of performance, we use FER (frame error rate), which is an appropriate performance measure for the whole system in presence of channel coding and interleaving. It is expected that FER is decreasing with increasing the mobile speed which is the result of the increased fading diversity due to the channel coding and interleaving. The results are shown for the following algorithms: standard (Mode 1 of the 3GPP), codebook algorithm, SMAP, NMMSE (Normalized-MMSE), Soft-SMAP (Soft-output SMAP), and Soft-NMMSE (Soft-output Normalized-MMSE) which are explained before.

Observing the effect of feedback error, we will show the results for a 5 and 10 percent of feedback error. Also it is assumed that $\gamma = 3$ and $\mu = 3$.

Fig. 6 shows the effect of feedback error on the FER performance of the algorithms versus transmit SNR (E_b/N_0), for 5 percent of feedback error, for the mobile speeds of $V = 1, 5, 25$ and 100 kmph. Fig. 7 similarly shows FER's for 10 percent of feedback error.

At low mobile speeds, there is a significant redundancy in feedback data stream, therefore there are strong codebooks for beamforming weights and the transition probabilities of the trellis. It can be observed that trellis-based algorithms perform almost similarly, because more or less they can exploit the redundancies well. Increasing the speed, the algorithms start to show their differences. For v=5 kmph, the algorithms have appeared in order of their performances; first the standard algorithm, then our hard-decision algorithms, and then our soft-decision algorithms.

Codebook algorithm has some gain over the standard which comes from using better weight-codewords (calculated by a nonlinear estimator) than the linear codewords of mode 1. SMAP algorithm tries to correct some of errors in the feedback stream, which results in higher gains in low to moderate mobile speeds. MMSE algorithm considers different possible feedback symbols from the received feedback data, and combines the respective codewords proportional to the probabilities. This approach gives some more gain in comparison to SMAP. Also soft versions of SMAP and NMMSE have some gains over them as expected.

At high mobile speeds, there is no major redundancy in feedback data stream and probability-codebooks are not so powerful, therefore, the codebook algorithm, SMAP and NMMSE act almost the same, which is the effect of weight-codebooks. It is seen that Soft-NMMSE algorithm always is better that other algorithms for all mobile speeds.

Figs. 8 and 9 show the needed SNR for a target FER, versus the mobile speed, for 5 and 10 percent of feedback error, respectively. The plots include the standard algorithm with an ideal AV, i.e., assume that receiver knows the exact beamforming weights applied in the transmitter. It is observed that for example NMMSE algorithms act better than or similar to the standard-IAV bound for low to moderate mobile speeds. The difference at high mobile speeds come from the fact that channel is changing rapidly, and even an AV scheme can not easily catch the IAV bound. It is observed from Fig. 8 that our approach can decrease the transmit power by about 2 dB for low to high mobile speeds. This gain is even higher for a smaller target FER, or for a higher feedback error rate.

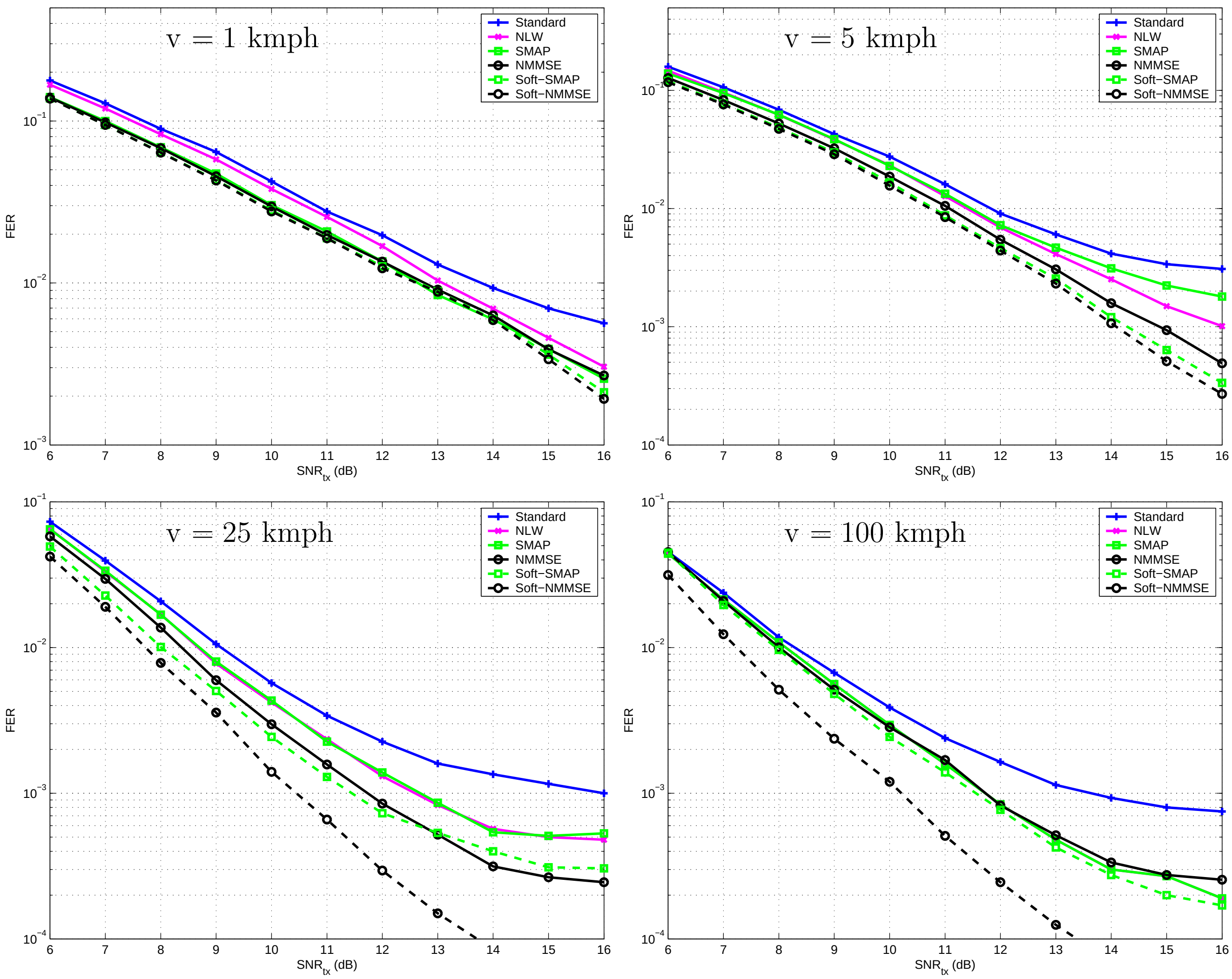

Fig. 6. FER of the feedback schemes in 5 percent feedback error for mobile speeds of 1,5,25 and 100 kmph

C. Summary and Conclusion

The closed-loop transmit diversity, which uses a combination of transmit diversity and channel feedback, is recognized as a promising scheme to achieve high data rates in mobile communications. Because of rate limit, error in the feedback channel in 3GPP systems, the performance is limited. In this article, we will try to improve

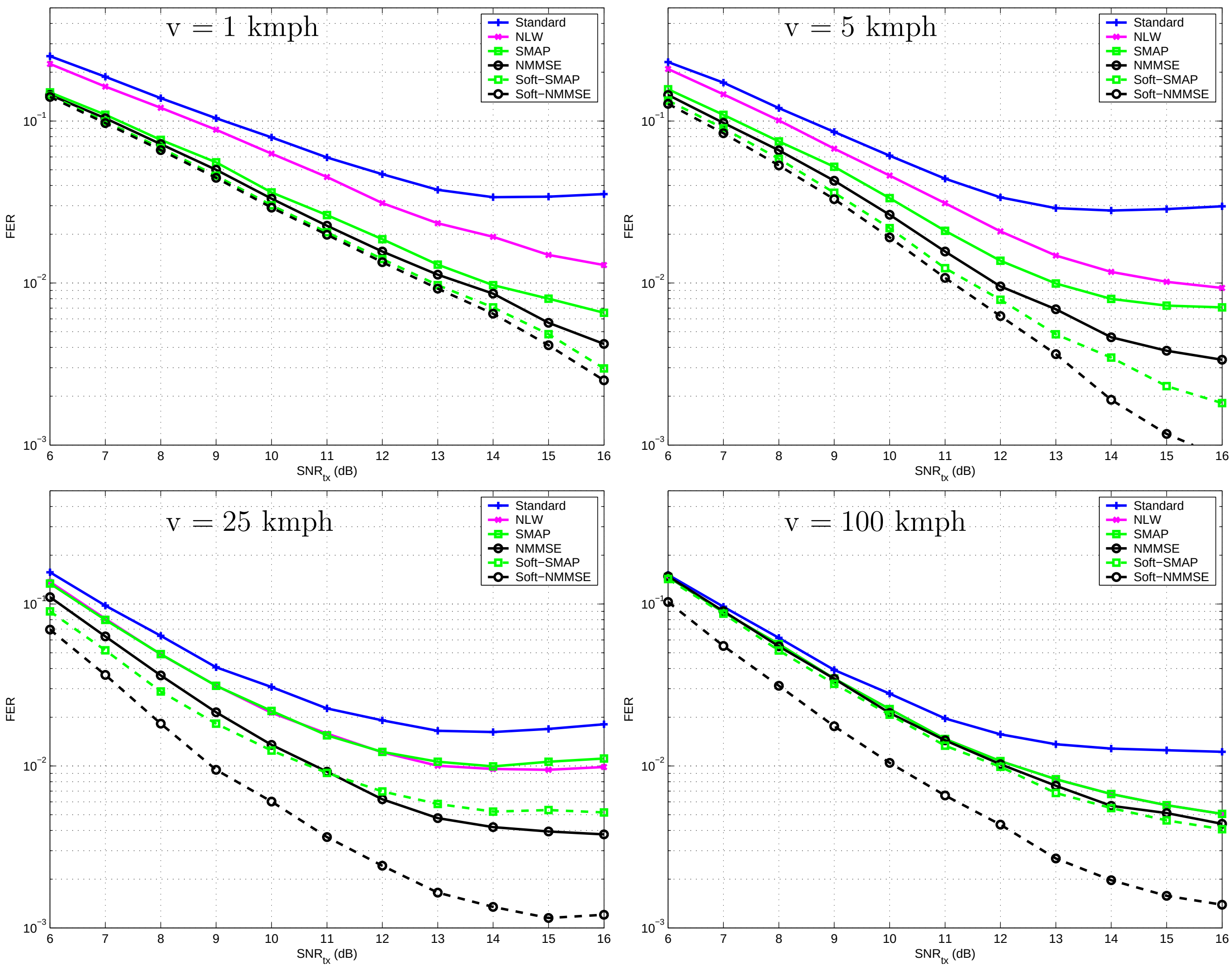

Fig. 7. FER of the feedback schemes in 10 percent feedback error for mobile speeds of 1,5,25 and 100 kmph

the performance of the closed-loop system. We have proposed some algorithms to improve the performance of mode 1 of 3GPP. It has been shown that our algorithms can provide significant gains over the standard algorithm, in low, moderate and high mobile speeds and they always outperform the standard algorithm.

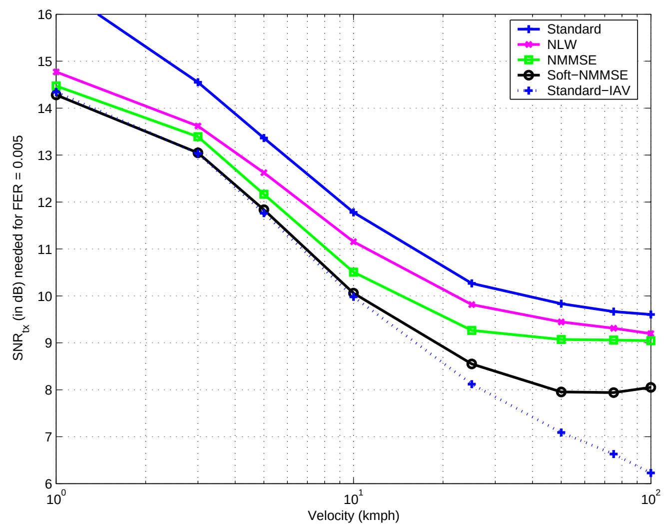

Fig. 8. Needed SNR for FER=5e-3 for 5 percent feedback error

Appendices

Appendix I: Lemma 1

Lemma 1 *If $\hat{w}$ is the MMSE estimation of a complex variable w, given $|w| = \beta$ and $|\hat{w}| = \alpha$, then*

$$\hat{w} = \alpha\,\overline{E}[w], \tag{33}$$

where $\overline{E}[w] = \frac{E[w]}{|E[w]|}$ is the normalized MMSE solution.

Proof: We should estimate the $\hat{w} = \alpha e^{j\hat{\phi}}$ using the random variable $w = \beta e^{j\phi}$, where α and β are real positive constants. In the MMSE sense, it turns out to minimizing the following criterion

$$E\left[\,|w - \hat{w}|^2\,\right] \tag{34}$$

$$= E\left[\,\left|\beta e^{j\phi} - \alpha e^{j\hat{\phi}}\right|^2\,\right] \tag{35}$$

$$= \alpha^2 + \beta^2 - 2\alpha\beta\, E\left[\,\cos(\phi - \hat{\phi})\right] \tag{36}$$

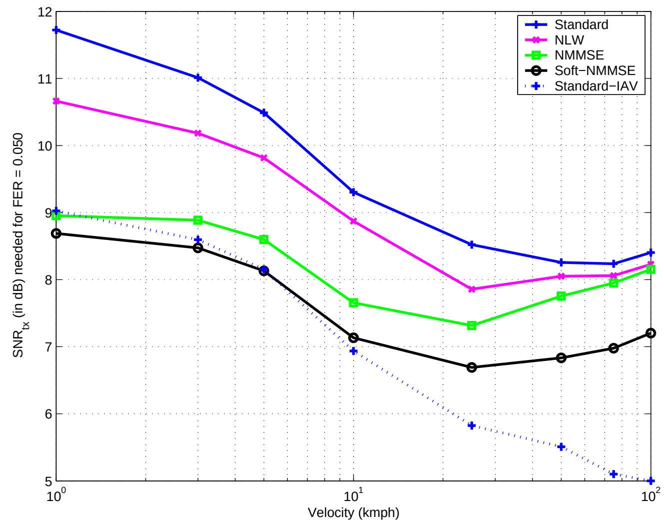

Fig. 9. Needed SNR for FER=5e-2 for 10 percent feedback error

or maximizing

$$E\left[\cos\phi\cos\hat{\phi}+\sin\phi\sin\hat{\phi}\right] \tag{37}$$

$$= E[\cos\phi]\cos\hat{\phi}+E[\sin\phi]\sin\hat{\phi} \tag{38}$$

which results in

$$\tan\hat{\phi}=\frac{E[\sin\phi]}{E[\cos\phi]}, \tag{39}$$

where

$$\begin{cases} E[\cos\phi]\geq 0 \quad \Rightarrow \quad -\frac{\pi}{2}<\hat{\phi}\leq\frac{\pi}{2} \\ E[\cos\phi]<0 \quad \Rightarrow \quad \frac{\pi}{2}<\hat{\phi}\leq 3\frac{\pi}{2} \end{cases} \tag{40}$$

It is easy to show that in both cases,

$$e^{j\hat{\phi}}=\frac{E[e^{j\phi}]}{|E[e^{j\phi}]|}. \tag{41}$$

Therefore,

$$\hat{w} = \alpha \frac{E[w]}{|E[w]|} = \alpha \overline{E}[w]. \tag{42}$$

■

Appendix II: Calculation of the Number of States

Consider the framing structure of mode 1 of 3GPP, depicted in Fig. 10. The values shown in each slot is the subset index of possible symbol values; $Q_0 = \{0, \pi\}$ and $Q_1 = \{\pi/2, -\pi/2\}$, according to Section II. Correspondingly, $\mathcal{I}_{Q_0} = \{1, 3\}$ and $\mathcal{I}_{Q_1} = \{2, 0\}$.

At time slot n, the current state is composed of γ symbols, from the starting slot of $n_{win} = n - \gamma + 1$ to the slot number n, which is shown by dashed sliding windows in the figure. The windows shown in the figure correspond to the case $\gamma = 3$, but our discussion is for any value of $0 < \gamma < 15$. For a given window, which is specified by n_{win}, there are 2^γ possible states.

Case 1: For $n_{win} = 0, 1, \cdots, 15 - \gamma$, the window is within the same frame, and the symbols are selected from Q_0 and Q_1, consecutively. Therefore, for all of the odd n_{win}'s, sequence of possible subsets are the same. This is also the case for all even n_{win}'s. Hence, there is two possible sequence of subsets for this range of n_{win}, resulting in $2 \times 2^\gamma$ different states.

Case 2: For $n_{win} = 16 - \gamma, \cdots, 14$, the window is in two neighbor frames. The symbols are also selected from Q_0 and Q_1 consecutively, except for the slots at the boundary of two frames where symbols are selected from two consecutive Q_0. Therefore, for each n_{win} in the range, the sequences of possible subsets are unique,

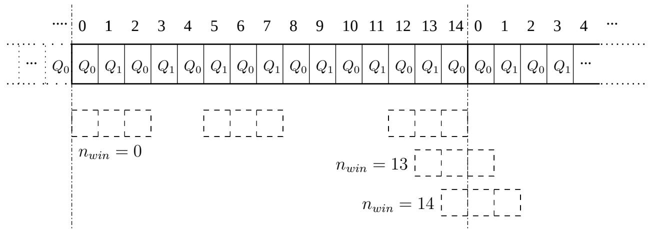

Fig. 10. Calculation of the number of states

and different from case 1. Hence, there are $14-(16-\gamma)+1=\gamma-1$ possible sequences of subsets for this range of n_{win}, resulting in $(\gamma-1)\,2^{\gamma}$ different states.

Adding up the possible states of the two cases, it leads to $(\gamma+1)\,2^{\gamma}$ possible states for the structure of mode 1 of 3GPP.

Note that a similar deduction could be done to calculate the codebook size by replacing γ with μ, which results in $N_{CB}=(\mu+1)\,2^{\mu}$.

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