An Efficient Adaptive Distributed Space Time Coding Scheme for Cooperative Relaying

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***Abstract*— A non-regenerative dual-hop wireless system based on a distributed space-time-coding strategy is considered. It is assumed that each relay retransmits an appropriately scaled space-time coded version of received signals. The main goal of this paper is to investigate a power allocation strategy in relay stations using analytical and simulation arguments to satisfy the quality of service requirements. In the high signal-to-noise ratio regime for the relay-destination link, it is shown that the optimum power allocation strategy in each relay which minimizes the outage probability is to remain silent, if its channel gain with the source is less than a prespecified threshold level. The Monte-Carlo simulations show that the near-optimal power allocation scheme in each relay in order to minimize the outage probability or the frame-error rate is the threshold-based on-off power scheme. Also, the numerical results demonstrate a dramatic improvement in the system performance by using this scheme compared to the case that the relay stations forward their received signals with full power. Finally, a hybrid amplify-and-forward/detect-and-forward scheme is numerically evaluated.**

I. Introduction

The main challenge in wireless networks is to mitigate multipath-induced channel fading that degrades the network performance. Exploiting diversity techniques such as time, frequency and space diversity are the most effective methods to combat the channel fading. Since, it is difficult to install multiple antenna in mobile stations (due to size and cost limitations), using a collection of distributed antennas belonging to multiple users improve the network performance. This new diversity approach, referred to as the *cooperative relaying*, has attracted considerable attention in wireless networks in recent years [1]–[3]. In the cooperative relaying, multiple relay nodes collaborate with each other through exploiting different independent fading channels in order to achieve a better performance. Cooperative relaying has been addressed from various perspectives; including capacity and outage probability analysis [1], [4], resource allocation [5], coding schemes [6], etc. Central to the study of the cooperative relaying is the problem of using the *distributed space-time coding* (DSTC) technique and the power allocation for regenerative and non-regenerative relay networks [4], [7]–[12].

The first study of using DSTC schemes in cooperative relaying was framed in [4]; several relay nodes transmit jointly to the same receiver in order to achieve full spatial diversity. They analyze the outage capacity in the high signal-to-noise ratio (SNR) regime. Nabar *et al.* [7] analyze the pairwise-error-probability (PEP) of a single-relay fading system which relies on a DSTC scheme operating in the amplify-and-forward (AF) mode. In [8], the authors investigate the high SNR bit-error-rate (BER) of a two-relay system with a Rayleigh fading model and by using a switching scheme. The performance of a DSTC scheme in regenerative relay networks is analyzed in [10] by exploiting alternative cooperation and decoding strategies. The average BER of single and dual-hop non-regenerative fading systems with DSTC has been analyzed in [11]. They investigate different transmissions policies in order to maximize the end-to-end SNR. The performance of regenerative relay networks by using the DSTC scheme and the optimum power allocation over non-identical Ricean channels is considered in [13]. Zhao *et al.* [5] introduce two optimum power allocation algorithms for AF mode in order to minimize the outage probability without DSTC scheme.

In this work, we address the question: How do we deploy an efficient DSTC scheme and the optimum power control strategy among relays in order to satisfy the quality-of-service (QoS)? To motivate our proposed idea, we consider a two-hop wireless network consisting of the source, two parallel relay stations (RS) and the destination. In this setup, RSs forward the space-time-coded (STC) version of their received signals from the source to destination. However, if the instantaneous received SNR of the relays are unbalanced, the performance of the cooperative relaying degrades substantially. To overcome this problem, an adaptive distributed space-time-coding (AD-STC) method is proposed, in which instead of transmitting the noisy signal at each relay with full power, the RS retransmits an appropriately scaled STC version of the received signals.

The main goal of the proposed scheme is to optimize the scaling factor based on the channel-state information (CSI) provided at each RS in order to minimize the outage probability. Our scheme is different from the power allocation algorithms in [5], in which no DSTC scheme is used. In the high SNR regime for the relay-destination link, it is shown that the optimum power allocation strategy in each relay that minimizes the outage probability is to remain silent, if its channel gain with the source is less than a prespecified threshold level. The Monte-Carlo simulations show that the near-optimal power allocation scheme in each relay in order to minimize the outage probability or the frame-error rate

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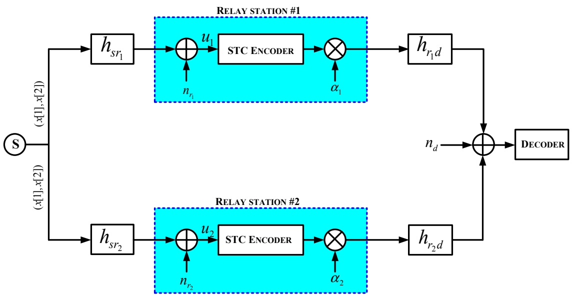


Fig. 1. A discrete-time baseband model of dual-hop wireless network.

(FER) is the threshold-based on-off power scheme (i.e., the relay which its channel gain with the source is above a certain threshold transmits at full power). Also, the numerical results show a significant improvement in the system performance by using this simple scheme in comparison with the case that the relay stations forward their received signals with full power. In addition, the FER of the proposed scheme using convolutional and turbo codes is numerically investigated. Finally, a hybrid amplify-and-forward/detect-and-forward scheme is numerically evaluated.

The rest of the paper is organized as follows. In Section II, the system model and objectives are described. The performance of the system is analyzed in Section III. In Section IV, the simulation results are presented. Finally, conclusions are drawn in Section V.

Throughout this paper, we use boldface lower case letters to denote vectors, boldface capital letters to denote matrices. The conjugate, transposition and conjugated transposition of a complex matrix $\mathbf{A}$ are denoted by $\mathbf{A}^*$, $\mathbf{A}^T$ and $\mathbf{A}^\dagger$, respectively. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$. A circularly symmetric complex Gaussian random variable is a random variable $Z = X + jY \sim \mathcal{CN}(0, \sigma^2)$, where X and Y are independent identically distributed (i.i.d.) $\mathcal{N}(0, \frac{\sigma^2}{2})$. Also, $A \rightarrow B$ denotes the link from node A to node B.

II. System Model and Objectives

In this work, we consider a non-regenerative dual-hop wireless system depicted in Fig. 1. The model consists of the source (S), the destination (D) and two parallel relay stations denoted by RS_1 and RS_2. We assume that all the nodes are equipped with a single antenna. Also, it is assumed that no information is exchanged between the relays. The background noise at each receiver is assumed to be additive white Gaussian noise (AWGN). The channel model is assumed to be frequency-flat block Rayleigh fading. The link $\text{S} \rightarrow \text{RS}_i$ is represented by the channel gain $g_{sr_i} \triangleq |h_{sr_i}|^2$, where the complex random variables h_{sr_i}'s are the channel coefficients. Similarly, the link $\text{RS}_i \rightarrow \text{D}$ is represented by the channel gain $g_{r_id} \triangleq |h_{r_id}|^2$. Under the Rayleigh fading channel model, g_{sr_i}'s and g_{r_id}'s are exponentially distributed with unit mean. We also assume that the channel-state information, i.e., the link channel gains, is perfectly known at the receivers. Moreover, it is assumed that the destination node has perfect knowledge of the equivalent links $\text{S} \rightarrow \text{RS}_i \rightarrow \text{D}$, $i = 1, 2$. For this model, the communication process between S and D is performed based on the following steps.

i) Data Transmission: The data transmission process between S and D is performed over two phases. In the first phase, the source node broadcasts the symbols $x[1]$ and $x[2]$ to relay nodes during two consecutive time slots and over one frequency subchannel. Let us assume that the channel gains remain constant over two successive symbol transmissions. In this case, the received discrete-time baseband equivalent signals at RS_1 and RS_2 are given by

$$u_1[k] = h_{sr_1} x[k] + n_{r_1}[k], \quad k = 1, 2, \tag{1}$$

$$u_2[k] = h_{sr_2} x[k] + n_{r_2}[k], \quad k = 1, 2, \tag{2}$$

respectively, where k represents the transmission time index and $n_{r_i}[k] \sim \mathcal{CN}(0, \sigma_r^2)$, for $i = 1, 2$. Here, we assume that the background noise and the interference from other transmitters at each relay are represented by $n_{r_i}[k]$ and with power σ_r^2. Letting $\mathbf{x} \triangleq \left[x[1], x[2]\right]^T$, we assume that the covariance matrix $\mathbf{R_{xx}} \triangleq \mathbb{E}[\mathbf{xx}^\dagger] = \mathbf{I}_2$. Thus, the energy of the transmitted signals denoted by $E_s \triangleq \mathbb{E}\left[|x[k]|^2\right]$ are unity. In order to balance the energies of the received signals at relays, we normalize $u_i[k]$ with $\sqrt{\mathbb{E}\left[|u_i[k]|^2 \middle| h_{sr_i}\right]}$, $i = 1, 2$. Thus, (1) and (2) can be written as

$$y_1[k] = \frac{1}{\sqrt{g_{sr_1} + \sigma_r^2}} \left(h_{sr_1} x[k] + n_{r_1}[k]\right), \tag{3}$$

$$y_2[k] = \frac{1}{\sqrt{g_{sr_2} + \sigma_r^2}} \left(h_{sr_2} x[k] + n_{r_2}[k]\right). \tag{4}$$

for $k = 1, 2$. In the second phase, the relay nodes cooperate with each other and forward the space-time-coded version of their received noisy signals to the destination over another frequency subchannel[1] (Table I). It should be noted that the channel phase of the link $\text{S} \rightarrow \text{RS}_i$, denoted by θ_{sr_i}, is compensated at RS_i through multiplying the received signal by the factor $e^{-j\theta_{sr_i}} = h_{sr_i}^*/|h_{sr_i}|$. Denoting the average signal-to-noise ratios of $\text{S} \rightarrow \text{RS}_i$ and $\text{RS}_i \rightarrow \text{D}$ by $\text{SNR}_{sr} \triangleq E_s/\sigma_r^2$ and $\text{SNR}_{rd} \triangleq E_s/\sigma_d^2$, respectively, RS_i multiplies $y_i[.]$ by the scaling factor $\sqrt{\alpha_i}$, where $0 \leq \alpha_i \leq 1$ is a function of g_{sr_i}, SNR_{sr} and SNR_{rd}. Here, we assume that SNR_{sr} and SNR_{rd} are known at RSs. Assuming fixed SNR_{sr}, the main goal is to choose the optimum factor α_i for every value of SNR_{rd} and g_{sr_i} such that the outage probability or the BER is minimized.

In each time slot, the destination receives a superposition of the transmitted signals by RS's. To this end, the received signals at D in the first and the second time slots are given by

$$\begin{aligned} r_d[1] &= \sqrt{\alpha_1} e^{-j\theta_{sr_1}} y_1[1] h_{r_1d} + \sqrt{\alpha_2} e^{-j\theta_{sr_2}} y_2[2] h_{r_2d} \\ &+ n_d[1], \\ r_d[2] &= -\sqrt{\alpha_1} \left(e^{-j\theta_{sr_1}} y_1[2]\right)^* h_{r_1d} \\ &+ \sqrt{\alpha_2} \left(e^{-j\theta_{sr_2}} y_2[1]\right)^* h_{r_2d} + n_d[2], \end{aligned}$$

respectively, where $n_d[k] \sim \mathcal{CN}(0, \sigma_d^2)$ represents the noise plus the interference from the other transmitters at destination.

[1] It is assumed that the relay nodes have the same configuration and use the same transmission policy.

TABLE I
TRANSMISSION POLICY IN RELAY STATIONS

	RS_1 antenna	RS_2 antenna
1^{st} time slot	$\sqrt{\alpha_1}e^{-j\theta_{sr_1}}y_1[1]$	$\sqrt{\alpha_2}e^{-j\theta_{sr_2}}y_2[2]$
2^{nd} time slot	$-\sqrt{\alpha_1}\left(e^{-j\theta_{sr_1}}y_1[2]\right)^*$	$\sqrt{\alpha_2}\left(e^{-j\theta_{sr_2}}y_2[1]\right)^*$

During the cooperation phase, we assume that the signals transmitted by relays in each time slot arrive at the same time at the destination node. It should be noted that due to the long distance between S and D or due to the strong shadowing, we ignore the received signal of the direct link S $\rightarrow$ D. Substituting (3) and (4) in the above equations yields

$$\begin{aligned} r_d[1] &= h_1^{'}x[1] + h_2^{'}x[2] + z_d[1], && (5)\\ r_d[2] &= h_2^{'}x^*[1] - h_1^{'}x^*[2] + z_d^*[2], && (6)\end{aligned}$$

where

$$h_i^{'} = h_{r_id}\sqrt{\frac{\alpha_i g_{sr_i}}{g_{sr_i}+\sigma_r^2}} \quad ; \quad i = 1,2, \tag{7}$$

and

$$\begin{aligned} z_d[1] &\triangleq \frac{h_1^{'}e^{-j\theta_{sr_1}}}{|h_{sr_1}|}n_{r_1}[1] + \frac{h_2^{'}e^{-j\theta_{sr_2}}}{|h_{sr_2}|}n_{r_2}[2] + n_d[1],\\ z_d[2] &\triangleq -\frac{h_1^{'*}e^{-j\theta_{sr_1}}}{|h_{sr_1}|}n_{r_1}[2] + \frac{h_2^{'*}e^{-j\theta_{sr_2}}}{|h_{sr_2}|}n_{r_2}[1] + n_d^*[2].\end{aligned}$$

It is observed that $\mathbb{E}\left[|z_d[1]|^2\middle|\mathbf{h}\right] = \mathbb{E}\left[|z_d[2]|^2\middle|\mathbf{h}\right] = \sigma^2$, where $\mathbf{h} \triangleq [h_{sr_1}, h_{sr_2}, h_{r_1d}, h_{r_2d}]$ and

$$\sigma^2 \triangleq \left(\frac{|h_1^{'}|^2}{g_{sr_1}} + \frac{|h_2^{'}|^2}{g_{sr_2}}\right)\sigma_r^2 + \sigma_d^2. \tag{8}$$

ii) Decoding Process: According to Alamouti's scheme, the maximum likelihood (ML) decoding process is performed as follows[2]:

$$\begin{bmatrix} r_d[1] \\ r_d^*[2]\end{bmatrix} = \begin{bmatrix} h_1^{'} & h_2^{'} \\ h_2^{'*} & -h_1^{'*}\end{bmatrix}\begin{bmatrix} x[1] \\ x[2]\end{bmatrix} + \begin{bmatrix} z_d[1] \\ z_d[2]\end{bmatrix},$$

or equivalently

$$\mathbf{r}_d = \mathbf{H}\mathbf{x} + \mathbf{z}_d. \tag{9}$$

To this end, the input of the ML decoder is

$$\begin{aligned} \tilde{\mathbf{r}}_d &= \mathbf{H}^\dagger\mathbf{r}_d\\ &= \mathbf{H}^\dagger\mathbf{H}\mathbf{x} + \mathbf{H}^\dagger\mathbf{z}_d\\ &= \Lambda\mathbf{x} + \tilde{\mathbf{z}}_d, && (10)\end{aligned}$$

where $\Lambda \triangleq |h_1^{'}|^2 + |h_2^{'}|^2$ and $\tilde{\mathbf{z}}_d \triangleq \mathbf{H}^\dagger\mathbf{z}_d$. In this case, the two-dimensional decision rule in the ML decoder will be as

$$\hat{\mathbf{x}} = \arg\min_{\hat{\mathbf{x}}\in\mathbb{S}^2}\|\tilde{\mathbf{r}}_d - \Lambda\hat{\mathbf{x}}\|^2. \tag{11}$$

where $\mathbb{S}^2$ denotes the corresponding signal constellation set.

[2]The required channel information for decoding at the destination can be acquired by transmitting pilot signals over different frequency subchannels.

III. Performance Analysis

In this section, we characterize the performance of the model described in Section II in terms of the outage probability. We derive an approximate formula for the outage probability based on the end-to-end SNR for the high SNR_{rd} regime. The optimization criterion is to minimize the end-to-end outage probability with respect to scaling factor α_i. To handle that, we first obtain the instantaneous end-to-end SNR of the proposed model. Using the fact that the destination node has perfect knowledge of the links S $\rightarrow$ RS_i $\rightarrow$ D, it is concluded that the matrix $\mathbf{H}|\mathbf{h}$ is deterministic. Noting that $\mathbb{E}\left[\mathbf{z}_d\mathbf{z}_d^\dagger|\mathbf{h}\right] = \sigma^2\mathbf{I}_2$ and $\mathbf{H}^\dagger\mathbf{H} = \Lambda\mathbf{I}_2$, it yields,

$$\mathbb{E}\left[\tilde{\mathbf{z}}_d\tilde{\mathbf{z}}_d^\dagger|\mathbf{h}\right] = \mathbb{E}\left[\mathbf{H}^\dagger\mathbf{z}_d\mathbf{z}_d^\dagger\mathbf{H}|\mathbf{h}\right] = \Lambda\sigma^2\mathbf{I}_2. \tag{12}$$

This implicitly indicates that the noise vector $\tilde{\mathbf{z}}_d|\mathbf{h}$ preserves the white Gaussian noise property. Since $\mathbb{E}\left[|x[i]|^2\right] = 1$, the instantaneous end-to-end SNR conditional upon the vector $\mathbf{h}$ is obtained as

$$\gamma_{\mathbf{h}} = \frac{\Lambda^2}{\Lambda\sigma^2} = \frac{|h_1^{'}|^2 + |h_2^{'}|^2}{\sigma^2}. \tag{13}$$

We assume that the QoS is provided when the end-to-end SNR exceeds a prescribed threshold γ_t. In the outage-based transmission framework, the outage event happens whenever $\gamma_{\mathbf{h}} < \gamma_t$. Hence, the outage probability is defined as

$$\text{P}_{out} \triangleq \Pr\{\gamma_{\mathbf{h}} < \gamma_t\}. \tag{14}$$

Clearly, the outage probability can be reduced by allocating the optimum power in each relay (through controlling the scaling factors α_i). Since, it is difficult to directly compute the exact expression for the outage probability, in the following, we investigate the outage probability of the underlying system in the high SNR_{rd} regime.

Lemma 1 *Let X_1 and X_2 be independent exponential random variables with parameters λ_1 and λ_2, respectively. Then, the probability density function (pdf) of $Z \triangleq \frac{X_1}{X_2}$ is obtained as*

$$f_Z(z) = \frac{\lambda_1\lambda_2}{(\lambda_1 z + \lambda_2)^2}U(z), \tag{15}$$

where $U(.)$ is the unit step function.

Proof: See [14] for the proof. ■

Theorem 1 *Under the high SNR_{rd} regime, the optimum scaling factor α_i which minimizes (14) is*

$$\hat{\alpha}_i = 0 \quad , \quad 0 < g_{sr_i} < \xi,$$

where $\xi \triangleq \sigma_r^2\gamma_t$.

Proof: Using the fact that g_{sr_1} is independent of g_{sr_2},

$$f_{g_{sr_1}g_{sr_2}}(v_1, v_2) = f_{g_{sr_1}}(v_1)f_{g_{sr_2}}(v_2) = e^{-v_1}e^{-v_2},$$

where $v_1,\ v_2 > 0$. Thus, the outage probability is given by

$$\text{P}_{out} = \mathbb{E}_{\mathbf{g}_s}[\Omega(\mathbf{g}_s)] = \int_0^\infty\int_0^\infty \Omega(\mathbf{g}_s)e^{-v_1}e^{-v_2}dv_1dv_2, \tag{16}$$

where $\mathbf{g}_s \triangleq [g_{sr_1} = v_1, g_{sr_2} = v_2]$ and $\Omega(\mathbf{g}_s) \triangleq \Pr\{\gamma_{\mathbf{h}} < \gamma_t | \mathbf{g}_s\}$. Using (7), (8) and (13), we have

$$\Omega(\mathbf{g}_s) = \Pr\left\{ \frac{X_1 v_1 + X_2 v_2}{(X_1 + X_2)\sigma_r^2 + \sigma_d^2} < \gamma_t \middle| \mathbf{g}_s \right\}, \quad (17)$$

where $X_i \triangleq g(v_i) g_{r_i d},\ i = 1, 2$ and

$$g(v_i) \triangleq \frac{\alpha_i}{v_i + \sigma_r^2}. \quad (18)$$

Under the Rayleigh fading channel model, $X_i | v_i$ is exponentially distributed with parameter $\frac{1}{g(v_i)}$ and with the following pdf

$$f_{X_i|v_i}(x_i|v_i) = \frac{1}{g(v_i)} e^{-\frac{x_i}{g(v_i)}} U(x_i).$$

In the high SNR$_{rd}$ regime, i.e., $\sigma_d^2 \ll \sigma_r^2$, (17) can be simplified as follows:

$$\begin{aligned} \Omega(\mathbf{g}_s) &= \Pr\{X_1 v_1 + X_2 v_2 < (X_1 + X_2)\xi | \mathbf{g}_s\} \\ &= \Pr\{(v_1 - \xi) X_1 < (\xi - v_2) X_2 | \mathbf{g}_s\}, \end{aligned} \quad (19)$$

where $\xi \triangleq \sigma_r^2 \gamma_t$. Depends on the values of v_1 and v_2 with respect to ξ, we have the following cases:

Case 1: $v_2 < \xi < v_1$

In this case, (19) can be written as

$$\Omega(\mathbf{g}_s) = \Pr\{Z < \phi | \mathbf{g}_s\}, \quad (20)$$

where $Z \triangleq \frac{X_1}{X_2}$ and $\phi \triangleq \frac{\xi - v_2}{v_1 - \xi}$. Using Lemma 1, the pdf of the random variable Z conditioned on $\mathbf{g}_s$ is obtained as

$$f_{Z|\mathbf{g}_s}(z|\mathbf{g}_s) = \frac{g(v_1) g(v_2)}{(g(v_2) z + g(v_1))^2} U(z). \quad (21)$$

Thus, (20) can be written as

$$\Omega(\mathbf{g}_s) = \int_0^{\phi} f_{Z|\mathbf{g}_s}(z|\mathbf{g}_s) dz = \frac{g(v_2)\phi}{g(v_2)\phi + g(v_1)}. \quad (22)$$

Case 2: $\xi < v_1$ and $\xi < v_2$

In this case, the quantity $\phi = \frac{\xi - v_2}{v_1 - \xi}$ is negative. Thus, since X_1 and X_2 are non-negative, it is concluded that

$$\Omega(\mathbf{g}_s) = \Pr\left\{ \frac{X_1}{X_2} < \phi \middle| \mathbf{g}_s \right\} = 0. \quad (23)$$

Case 3: $\xi > v_1$ and $\xi > v_2$

From (19), we have

$$\begin{aligned} \Omega(\mathbf{g}_s) &= \Pr\{Z > \phi | \mathbf{g}_s\} \\ &= 1 - \Pr\{Z \le \phi | \mathbf{g}_s\} \\ &\overset{(a)}{=} 1, \end{aligned} \quad (24)$$

where (a) follows from the fact that for $\xi > v_1$ and $\xi > v_2$, ϕ is a negative value, and this results in $\Pr\{Z \le \phi | \mathbf{g}_s\} = 0$.

Case 4: $v_1 < \xi < v_2$

In this case, (19) can be written as

$$\begin{aligned} \Omega(\mathbf{g}_s) &= \Pr\{Z > \phi | \mathbf{g}_s\} \\ &= 1 - \Pr\{Z \le \phi | \mathbf{g}_s\}, \end{aligned} \quad (25)$$

where ϕ is positive. Using (21) and (22), we have

$$\Omega(\mathbf{g}_s) = 1 - \frac{g(v_2)\phi}{g(v_2)\phi + g(v_1)} = \frac{g(v_1)}{g(v_2)\phi + g(v_1)}.$$

Now, we use the above results in order to compute the outage probability obtained in (16). To this end,

$$\begin{aligned} \mathrm{P}_{out} &= \int_0^{\infty}\int_0^{\infty} \Omega(\mathbf{g}_s) e^{-v_1} e^{-v_2} dv_1 dv_2 \\ &= (1 - e^{-\xi})^2 \\ &+ \int_0^{\xi} \left[\int_{\xi}^{\infty} \frac{g(v_1) e^{-v_2}}{g(v_2)\phi + g(v_1)} dv_2 \right] e^{-v_1} dv_1 \\ &+ \int_{\xi}^{\infty} \left[\int_0^{\xi} \frac{g(v_2)\phi e^{-v_2}}{g(v_2)\phi + g(v_1)} dv_2 \right] e^{-v_1} dv_1. \end{aligned} \quad (26)$$

It can be easily shown that the second and the third terms of (26) are symmetric. Thus, (26) can be simplified as

$$\begin{aligned} \mathrm{P}_{out} &= 2\int_0^{\xi} \left[\int_{\xi}^{\infty} \frac{e^{-v_2}}{g(v_1) + g(v_2)\phi} dv_2 \right] g(v_1) e^{-v_1} dv_1 \\ &+ (1 - e^{-\xi})^2. \end{aligned} \quad (27)$$

Noting that $\int_{\xi}^{\infty} \frac{e^{-v_2}}{g(v_1) + g(v_2)\phi} dv_2$ is positive, the optimum function $g(v_i)$ that minimizes (27) should satisfy $g(v_i) = 0$, when $0 < v_i < \xi$. In this case, the minimum P_{out} is equal to $(1 - e^{-\xi})^2$. Using $g(v_i) \triangleq \frac{\alpha_i}{v_i + \sigma_r^2}$, we therefore come up with the following result:

$$\hat{\alpha}_i = 0 \quad , \quad 0 < g_{sr_i} < \xi. \quad (28)$$

Note that we are not concerned with the computation of the exact values of the scaling factors at the relay nodes, i.e., $\hat{\alpha}_i$ for $g_{sr_i} \ge \xi$, as long as $\sigma_d^2 \to 0$. ∎

Remark 1: The case of high SNR$_{sr}$ is similar to the traditional STC problem. Therefore, the optimum power allocation policy for relay stations that minimizes (14) is the full power transmission.

IV. Numerical Results

In this section, we present some Monte-Carlo simulation results to evaluate the impact of the scaling factor α_i on the performance of the underlying system. We consider some forms of the scaling factors α_i in terms of g_{sr_i}. In the full power scheme and independent of the channel gain g_{sr_i}, each relay retransmits its received signal with full power, i.e., $\alpha_i = 1$. For the on-off power scheme, the relay i with g_{sr_i} above certain threshold transmits at full power, otherwise remains silent, i.e., $\alpha_i = 0$. We also consider the piecewise linear and the sigmoid function schemes[3].

Fig. 2 compares the outage probability of the full power and the on-off power schemes for different values of SNR$_{rd}$ and

[3]In the piecewise linear function, the scaling factor α_i is defined as

$$\alpha_i = \begin{cases} 0 & , \quad g_{sr_i} < T_1 \\ \frac{1}{T_2 - T_1} g_{sr_i} - \frac{T_1}{T_2 - T_1} & , \quad T_1 \le g_{sr_i} < T_2 \\ 1 & , \quad g_{sr_i} \ge T_2. \end{cases} \quad (29)$$

Also in the sigmoid function, the scaling factor α_i is defined as

$$\alpha_i = \frac{1}{1 + e^{-\epsilon_1 (g_{sr_i} - \epsilon_2)}}. \quad (30)$$

For these schemes, the optimum values of T_i and $\epsilon_i,\ i = 1, 2$, are obtained using exhaustive search.

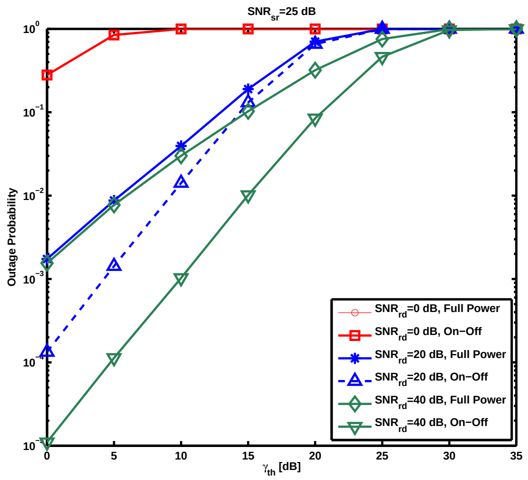


Fig. 2. P_{out} versus γ_{th} for full power and on-off power schemes.

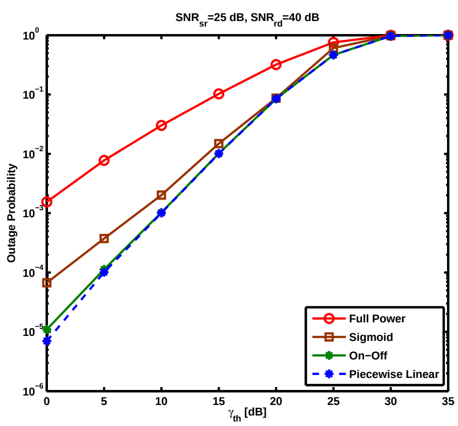


Fig. 3. P_{out} versus γ_{th} for high SNR_{rd}.

for BPSK constellation. The results are obtained by averaging over 10^6 channel realizations. To compare the results, we plot the outage probability of different schemes in Fig. 3 for high SNR_{rd}. It is observed that the performance of the underlying model is improved by using the on-off power scheme in comparison with the full power strategy. For example, we achieve 10 dB gain in using the on-off power strategy for $\mathrm{SNR}_{rd} = 40$ dB and $\mathrm{P}_{out} = 10^{-3}$ with respect to the full power scheme. Fig. 4 plots the uncoded average BER versus SNR_{rd} for different functions of α_i and for BPSK constellation. We observe an error floor in Fig. 4 for the high SNR_{rd}, which can be attributed to the amplified noise received at D through the relay-destination link. In fact, for sufficiently large SNR_{rd}, the performance is governed by SNR_{sr}.

Fig. 5-a illustrates the FER of the proposed model using the standard turbo code [15] with the rate of $1/3$ versus the SNR_{rd} for different functions of α_i, BPSK constellation and the frame length of 957 bits. Also, Fig. 5-b illustrates the FER of the

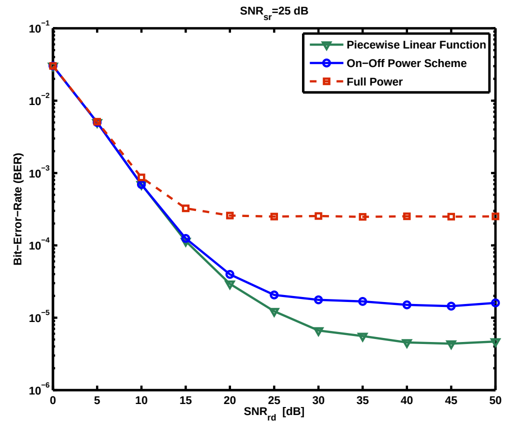


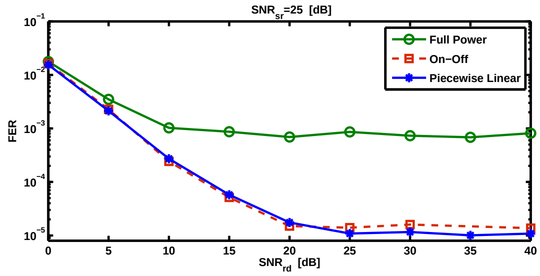
Fig. 4. Uncoded BER versus SNR_{rd} for different functions of α_i and for $\mathrm{SNR}_{sr} = 25$ dB,

proposed model using the convolutional code (introduced in section 8.4.9.2.1 of the IEEE 802.16.e standard [16]) with the rate of 0.5 versus the SNR_{rd} and the frame length of 576 bits. Compared to the full power scheme, the simulation results show a significant improvement in the system performance with the on-off power scheme in RSs. Also, it is observed from the simulation that the trend of the FER for higher order constellations is similar to the BPSK case [14].

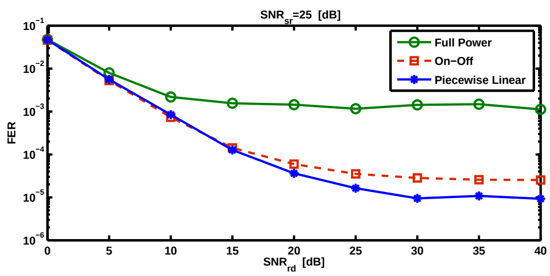
Up to now, we have investigated the power allocation in RSs in the AF mode. For this case, it is shown that the near-optimal power allocation in each relay is the threshold-based on-off power scheme that is a function of the channel gain of the link S→RS. To further improve the system performance, in particular when the quality of the link S→RS is good, the detect-and-forward (DF) scheme in each relay is suggested to eliminate the amplified noise at the RS by demodulating the received signal. According to Fig. 6, the *hybrid threshold-based AF/DF* scheme characterized by two threshold levels T_1 and T_2 at each RS is described as follows:

1- For $g_{sr_i} < T_2$, relay i performs the on-off power scheme developed in Section III.
2- For $g_{sr_i} > T_2$, relay i detects the transmitted symbol $x[k]$ from the received signal and forwards the estimated symbols $\hat{x}[k]$ to the destination.

In Fig. 7, we compare the FER of different transmission strategies in the RSs for convolutional code with the rate $1/2$, QPSK modulation and the frame length of 288 symbols. In the pure full power scheme, independent of the channel gain g_{sr_i}, each relay i transmits with full power all the time, i.e., $\alpha_i = 1$. For completeness, we also consider the threshold-based DF scheme, in which for g_{sr_i} less than a prescribed threshold, each relay i remains silent, otherwise switches to the DF mode. It is observed that the performance of the hybrid threshold-based AF/DF scheme is the same as the one threshold-based on-off power scheme and both of them better than the first two schemes. In fact, the gain of our approach comes from the combination of AF and thresholding schemes.



(a) Turbo Code



(b) Convolutional Code

Fig. 5. FER versus SNR$_{rd}$ for different functions of α_i and for a) turbo code with the rate $1/3$ and SNR$_{sr}$ $= 25$ dB, b) convolutional code with the rate $1/2$ and SNR$_{sr}$ $= 25$ dB.

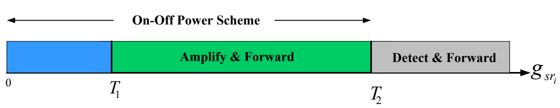


Fig. 6. Hybrid amplify-and-forward and detect-and-forward modes.

V. Conclusion

In this paper, a non-regenerative dual-hop wireless system based on a DSTC strategy has been considered. In the high SNR$_{rd}$, it has been shown that the optimum power allocation strategy in each relay which minimizes the outage probability is to remain silent, if its channel gain with the source is less than a prespecified threshold level. The simulations results show that the near-optimal power allocation scheme in each relay in order to minimize the outage probability or the FER is the threshold-based on-off power scheme. Also, the numerical results demonstrate a dramatic improvement in the system performance by using this scheme compared to the case that the RSs forward their received signals with full power. The advantage of using this protocol is that it does not alter the Alamouti decoder at the destination and it slightly modifies the relay operation which is practically appealing. Also, through using the on-off power allocation strategy at RSs, we significantly save the energy in the silent mode.

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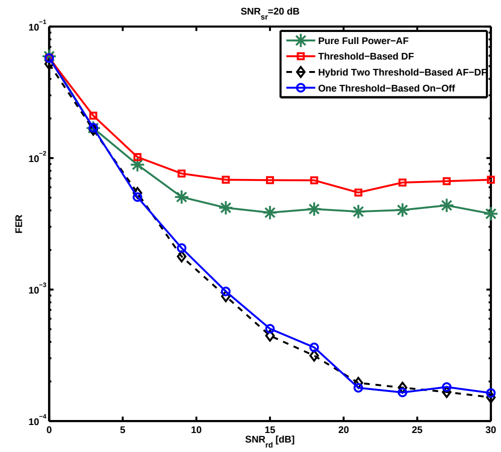

Fig. 7. FER versus SNR$_{rd}$ for different functions of α_i and for convolutional code with the rate $1/2$, SNR$_{sr}$ $= 20$ dB and for QPSK.

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