M-user Gaussian Interference Channels:
To Decode the Interference or To Consider it as Noise

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***Abstract*— We address data transmission over the M-user Gaussian interference channel, where users send data using single Gaussian codebooks. We first present a polynomial-time algorithm for finding the maximum decodable subset among interfering users, provided the users' rates and powers are given. Given any ordering of users, we characterize an achievable rate vector in which users' rates are successively maximized based on the ordering. It is also shown that in a noncooperative scenario where users refuse to send below their conservative rates, there are achievable vectors that are feasible with respect to the conservative rates vector which can be obtained by using a simple iterative algorithm.**

I. INTRODUCTION

Treating interference as noise is not always an intelligent strategy for the Gaussian Interference Channel (GIC). For instance, in the case of strong and very strong two-user GICs, the capacity regions are achieved when both receivers decode both users' messages [1] [2]. However, when interference is caused by multiple users, the situation becomes more delicate. In this case, it may be possible to decode some parts of the interference and treat the rest as noise.

The capacity region of the GIC remains an open problem. The best inner bound for the case of a two-user GIC is the Han-Kobayashi achievable rate region characterized in [3]. In their scheme, each user splits its data into two parts and then uses different codebooks to transmit each part. The idea is to make some part of the data decodable by the unintended user. In [4], Chong et.al. have found a shorter description of the Han-Kobayashi achievable rate region. It is worth noting that even for the GIC the full characterization of the Han-kobayshi region is not known, due to the large number of variables involved in the problem. The situation is more complicated in the case of M-user GIC since generalizing the Han-Kobayashi method requires invoking an exponentially many dependent variables. The reason is that each user must split its data to 2^{M-1} parts.

We study the case where M transmitter-receiver pairs try to communicate reliably over a GIC. Furthermore, we make the following assumptions:

1) The users send at their maximum power.
2) They use single codebooks.
3) They use Gaussian codebooks for data transmission.

The problem here is to adjust the users' data rates such that each receiver can reliably decode its own data. However, the rate adjustment depends on the decoding strategies and there are exponentially many of them. For example, we can point out two extreme cases. In the first case, the strategy for each receiver is to consider the interference caused by other users as noise. In this case, we can find a point in the achievable rate region. In the second case, however, each receiver decodes all data transmitted from all transmitters. By this strategy, we obtain an achievable rate region corresponding to the capacity region of the M-user Compound Multiple Access Channel [5]. Therefore, by characterizing the achievable rate region for each strategy and taking the union over all regions, we obtain an achievable rate region.

We can enumerate all possible strategies. To this end, we see that there are 2^{M-1} possible subsets of users that can be considered as the decodable subset for a receiver. Since there are M receivers in the system, we have $2^{M(M-1)}$ possible strategies to choose from. Hence, finding the union of the achievable rate regions is computationally difficult.

In this paper, we take advantage of polynomial-time algorithms for discrete optimization problems to investigate different solutions for several problems in our system. The basic problem among them is finding the maximum decodable subset among interfering users provided users' rates and powers are given.

II. PRELIMINARIES

Throughout this paper, we use following notations. Vectors are represented by bold faced letters. Matrices and sets are denoted by capital letters where the difference is clear from the context. The transpose of a matrix or vector is denoted by apex T. For any set S and any vector $\mathbf{x}$, we use the compact notation $\mathbf{x}(S)$ to denote $\sum_{i\in S}\mathbf{x}_i$. The difference between two sets U and V is represented by $U-V$. The complement of a subset U is denoted by $\bar{U}$. The cardinality of a set E is denoted by $|E|$.

A. System Model

We focus on the M-user GIC where users use Gaussian codebooks for data transmission. The received symbol of user

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i can be modeled by

$$y_i = \sum_{i=1}^{M} h_{ij} x_j + z_i, \qquad (1)$$

where x_j is transmitted signal of user j and h_{ij} denotes the link's gain between jth transmitter and ith receiver. z_i is white Gaussian noise with zero mean and variance N_i. User i is also subject to an average power constraint P_i. It is more convenient to write the system model in matrix form as follows

$$\mathbf{y} = H\mathbf{x} + \mathbf{z}, \qquad (2)$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$ and $\mathbf{x} = [x_1, x_2, \ldots, x_M]^T$ denote the output and input vectors, respectively. $H = [h_{ij}]$ is the matrix of links' gains, and $\mathbf{z} = [z_1, z_2, \ldots, z_M]^T$ is the Gaussian noise vector. Even though, noise correlations do not affect the capacity region [12], we assume that the covariance matrix of the noise vector is a diagonal matrix. By scaling transformation, it is possible to write the channel model (2) in standard form which changes the noise variances and diagonal elements of H all to unity [6].

If users treat interference as noise, then the following rate is achievable for user i

$$R_i = \gamma\left(\frac{h_{ii}^2 P_i}{N_i + \sum_{j=1, j\neq i}^{M} h_{ij}^2 P_j}\right), \qquad (3)$$

where $\gamma(x) = \frac{1}{2}\log(1+x)$.

B. Submodular Functions

Definition 1: Let E be a finite nonempty set. A function $f : 2^E \to \mathbb{R}$ is called a submodular function if it satisfies

$$f(V \cup U) + f(V \cap U) \leq f(V) + f(U) \text{ for all } V, U \subseteq E, \qquad (4)$$

for any $V, U \subseteq E$. A function f is called suppermodular if $-f$ is submodular. A modular function is a function which is both submodular and suppermodular.

Submodular functions are the most important functions in discrete mathematics. They, in fact, play the same role in discrete mathematics as convex functions do in continuous mathematics [8]. Therefore, submodualr function minimization is an important issue in discrete mathematics. Beside, having a polynomial-time algorithm based on ellipsoid method [7], there are combinatorial algorithms for minimizing submodular functions in strongly polynomial time, c.f. [8] and [9]. In this paper, we make use of these algorithms to develop new algorithms for the problems we are interested in.

If a submodular function is nondecreasing, i.e. $f(U) \leq f(V)$ if $U \subseteq V$, and $f(\varnothing) = 0$ then the associated polyhedron

$$\mathcal{B}(f) = \{\mathbf{x} | \mathbf{x}(U) \leq f(U)\ \forall U \subseteq E, \mathbf{x} \geq 0\} \qquad (5)$$

is a polymatroid. Likewise, if a suppermodular function is nondecreasing and $f(\varnothing) = 0$ then the associated polyhedron

$$\mathcal{G}(f) = \{\mathbf{x} | \mathbf{x}(U) \geq f(U)\ \forall U \subseteq E\} \qquad (6)$$

is a contra-polymatroid.

C. Multiple Access Capacity Region

One of the most important results in Information Theory is the characterization of the capacity region of the Multiple Access Channel (MAC) [10], [11]. In fact, the MAC is the only multiuser channel that we know of the capacity region in general. In particular, the capacity region of M-user Gaussian Multiple access channel modeled by

$$y = x_1 + x_2 + \cdots + x_M + z, \qquad (7)$$

with noise variance N is characterized and can be stated as

$$\mathcal{C}_{MAC} = \{\mathbf{R} | \mathbf{R}(U) \leq \gamma\left(\frac{\mathbf{P}(U)}{N}\right)\ \forall V \subseteq E, \mathbf{R} \geq 0\}, \qquad (8)$$

where $\mathbf{R} = [R_1, R_2, \ldots, R_M]^T$, $\mathbf{P} = [P_1, P_2, \ldots, P_M]^T$ are the rates and the average powers vectors, respectively. $E = \{1, 2, \ldots, M\}$ is the set of users' indices. If we compare (5) and (8), we conclude that the capacity region of the M-user Gaussian MAC is a polymatroid provided that $\gamma\left(\frac{\mathbf{P}(V)}{N}\right)$ is a submodular function and nondecreasing. It is easy to check that γ is indeed a submodular function and nondecreasing [3].

III. Single Receiver Rate Maximization

In this section, we consider reliable data transmission over an interference channel where the interference is caused by $M-1$ interfering users with known powers, rates, and codebooks. The model of the channel can be written as

$$y = x + x_1 + x_2 + \cdots + x_{M-1} + z, \qquad (9)$$

where x and y denote transmit and received symbols, respectively. x_i is the input symbol of the ith interfering user with power P_i and rate R_i, and z is the additive white Gaussian noise with variance N. Consider the transmitter is subject to average power P and tries to send data at the maximum rate.

If the receiver treats the interference as noise, then the following rate is achievable

$$R = \gamma\left(\frac{P}{N + \sum_{i=1}^{M-1} P_i}\right). \qquad (10)$$

On the other extreme case, if it is possible for the receiver to decode all the interfering signals considering its own signal as noise then the following rate is achievable

$$R = \gamma\left(\frac{P}{N}\right). \qquad (11)$$

Even though, we first decode the interference in (11) and then we decode the intended signal, it is possible to decode jointly the intended signal with part of the interference. Therefore, in order to maximize the rate, it is needed to find the best decodable subset. To this end, the receiver first assumes that its own data is reliably decoded and its effect is removed. By this assumption, it looks for a decodable subset of users. Then it maximizes its own data rate in such a way that it can be decoded jointly with other decodable users.

A. Maximum Decodable Subset

Suppose the receiver has successfully decoded x in (9) and it can remove its effect. Then, we have

$$\tilde{y} = x_1 + x_2 + \cdots + x_{M-1} + z, \tag{12}$$

where users are transmitting at rates $\mathbf{R} = [R_1, \ldots, R_{M-1}]$ using Gaussian codebooks with average power constraints $\mathbf{P} = [P_1, \ldots, P_{M-1}]$. Let $E = \{1, 2, \ldots, M-1\}$ denote the set of users' indices. We are interested in decodable subsets of E. There are 2^{M-1} subsets and in order to check that a subset V with cardinality of k is decodable, $2^k - 1$ inequalities must be satisfied due to (8). Hence, in general the number of inequalities involved in the problem is

$$\sum_{k=0}^{M-1} \binom{M-1}{k} (2^k - 1) = 3^{M-1} - 2^{M-1}$$

which is exponential in the number of users. The problem is more complicated when we are searching for the best decodable subset in some sense. The following definition puts partial ordering on decodable subsets.

Definition 2 (Maximal decodable subset): A subset of users is a maximal decodable subset if all users in the subset are decodable by the receiver and it is not proper subset of any other decodable subset. If the maximal decodable subset is unique, we call it *maximum decodable subset*.

It is worth noting that even if a subset is jointly decodable it does not imply every subset of it is jointly decodable by considering the rest as noise.

Lemma 1: There is a maximum decodable subset for the receiver in (12).

Proof: Suppose the receiver is able to decode two subsets of users, namely, U and V such that none of them is subset of the other. U and V are proper subsets of their union $U \cup V$. Besides, their union is decodable by the receiver. This contradicts the fact that both subsets are maximal. □

Lemma 2: None of the signals in (12) is decodable iff users' rates satisfy

$$\mathbf{R}(U) > \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(E - U)}\right), \text{ for all } U \subseteq E. \tag{13}$$

Moreover, the region of those rates satisfying above inequalities form a contra-polymatroid region.

Proof: Suppose a rate vector $\mathbf{R}$ satisfies (13) and V is the maximum decodable subset. Form the characterization of the capacity region of the Gaussian MAC, we have

$$\mathbf{R}(V) \leq \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - V)}\right), \tag{14}$$

which is a contradiction and it completes the "if" part of the proof. Now, we need to prove that if the inequalities in (13) are not satisfied, there is at least a user which is decodable. Suppose there are some subsets that do not satisfy (13). Assume W has the minimum cardinality among all and satisfies

$$\mathbf{R}(W) \leq \gamma\left(\frac{\mathbf{P}(W)}{N + \mathbf{P}(E - W)}\right). \tag{15}$$

If $|W| = 1$, then the user in W is decodable by considering everything else as noise which is the desired result. Hence, we assume $|W| > 1$. If all members of W are jointly decodable, then we have found a subset which is decodable. Otherwise, there must be a subset of W, say V, which satisfies

$$\mathbf{R}(V) > \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - W)}\right). \tag{16}$$

By decomposing the γ function in (15), we obtain

$$\mathbf{R}(W) \leq \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - W)}\right) + \gamma\left(\frac{\mathbf{P}(X)}{N + \mathbf{P}(E - X)}\right), \tag{17}$$

where $X = W - V$. From the minimality of $|W|$, we have

$$\mathbf{R}(X) > \gamma\left(\frac{\mathbf{P}(X)}{N + \mathbf{P}(E - X)}\right). \tag{18}$$

By adding the two inequalities (16) and (18) and considering the fact that $\mathbf{R}(W) = \mathbf{R}(V) + \mathbf{R}(X)$, we conclude that

$$\mathbf{R}(W) > \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - W)}\right) + \gamma\left(\frac{\mathbf{P}(X)}{N + \mathbf{P}(E - X)}\right), \tag{19}$$

Which is a contradiction, and this completes the "only if" part of the proof. It is easy to see that the function on the right hand side of (13) is a suppermodular function and monotone, hence the region formed by rates satisfying (13) is a contra-polymatroid region. □

Theorem 1: A subset $S \subseteq E$ is maximum decodable subset iff the rates of the users satisfy the following inequalities

$$\mathbf{R}(V) \leq \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - S)}\right), \text{ for all } V \subseteq S, \tag{20}$$

$$\mathbf{R}(U) > \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(\bar{S} - U)}\right), \text{ for all } U \subseteq \bar{S}. \tag{21}$$

Proof: Inequality (20) corresponds to the capacity region of the MAC for members of S considering members of $\bar{S}$ as noise. Hence, the members of S are decodable iff the inequalities in (20) are satisfied. After decoding the users in S successfully, receiver can remove their effect. Now, by applying Lemma 2, we conclude that none of the users in $\bar{S}$ is decodable iff the inequalities in (21) are satisfied. This completes the proof. □

We first define function $f : 2^E \to \mathbb{R}$ as follows

$$f(V) = \gamma(V) - R(V), \tag{22}$$

for every $V \subseteq E$.

Lemma 3: The function f defined in (22) is a submodular function.

Proof: Since γ is a submodular function and R is a modular function, the function f is a submodular function. □

We now introduce the following submodular function minimization problem

$$f(W) = \min_{V \subseteq E} f(V) \tag{23}$$

If the minimum of f in (23) is zero, then all users are decodable by the receiver due to (8). Otherwise, there is at least a user of the set E which is not decodable. In the following theorem, we prove that indeed all members of the minimizer of f are not decodable, and they need to be considered as noise.

Theorem 2: None of the members of the subset W that minimizes f in (23) is decodable by the receiver provided the minimum is not zero and the minimum cardinal minimizer is used. In fact, all users in W must be considered as noise, i.e., if S is the maximal decodable subset then $W \cap S = \varnothing$.

Proof: Suppose $U = W \cap S$ and $\tilde{W} = W - U$. Since U is a subset of the maximum decodable subset S, we have

$$\mathbf{R}(U) \leq \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(\bar{S})}\right). \tag{24}$$

The inclusion $\tilde{W} \subseteq \bar{S}$ implies $\mathbf{P}(\tilde{W}) \leq \mathbf{P}(\bar{S})$. Hence

$$\mathbf{R}(U) \leq \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(\tilde{W})}\right). \tag{25}$$

From the definition of f in (22), we have

$$f(W) = \gamma\left(\frac{\mathbf{P}(W)}{N}\right) - \mathbf{R}(W). \tag{26}$$

The γ function in (26) can be decomposed as follows

$$\gamma\left(\frac{\mathbf{P}(U)}{N}\right) = \gamma\left(\frac{\mathbf{P}(\tilde{W})}{N}\right) + \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(\tilde{W})}\right). \tag{27}$$

Substituting into (26) and using $\mathbf{R}(W) = \mathbf{R}(\tilde{W}) + \mathbf{R}(U)$, we obtain

$$f(W) = f(\tilde{W}) + \gamma\left(\frac{\mathbf{P}(U)}{N + \mathbf{P}(\tilde{W})}\right) - \mathbf{R}(U). \tag{28}$$

Using the inequality in (25), we conclude that

$$f(\tilde{W}) \leq f(W), \tag{29}$$

which is a contradiction. This completes the proof. □

By applying Theorem 2 and using the well-known submodular function minimization algorithms as a subroutine, c.f. [9] and [8], we propose the following polynomial-time algorithm for finding the maximum decodable subset.

Algorithm 1 (Finding the maximum decodable subset):

1) Set $S = E$.
2) Find W such that
$$f(W) = \min_{V \subseteq S} f(V),$$
where f is the following function
$$f(V) = \gamma\left(\frac{\mathbf{P}(V)}{N + \mathbf{P}(E - S)}\right). \tag{30}$$
3) If $W = \varnothing$ Stop. S is the maximal decodable subset. Otherwise, $S - W \longrightarrow S$.
4) If $S = \varnothing$ Stop. None of the subsects of E is decodable. Otherwise, GO TO step 2.

Since in each iteration W is a nonempty set (otherwise, the algorithm stops), this algorithm converges at most in $|E|$ iterations. Hence, the total running time of the algorithm is polynomial in time.

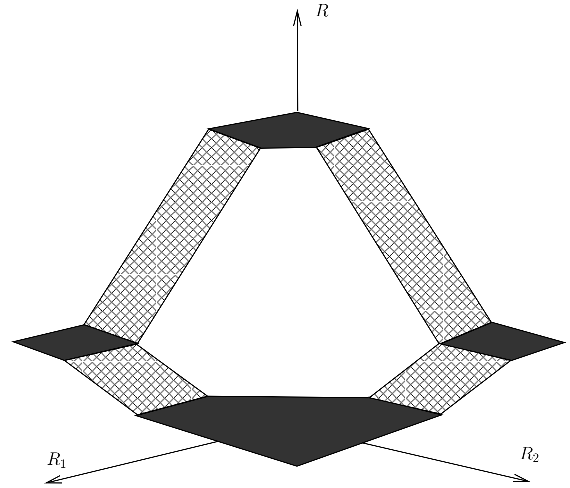

Fig. 1. The function $g(\mathbf{R})$ for a two-user GIC

B. Maximum Rate

We now return to the main problem which is rate maximization of user in (9). By making use of Algorithm 1, we first partition the set of interfering users into two subsets S and $\bar{S}$ where S is the maximum decodable subset and $\bar{S}$ is the noise part of the interference. By using (8), it is easy to prove that the maximum rate can be derived from the following optimization problem

$$R = g(\mathbf{R}) = \min_{U \subseteq S} \gamma\left(\frac{P + \mathbf{P}(U)}{N + \mathbf{P}(\bar{S})}\right) - \mathbf{R}(U). \tag{31}$$

The optimization problem in (31) is again a submodular function minimization and can be solved by polynomial-time algorithms.

In order to derive some properties of the function g defined in (31), we need the following definition.

Definition 3 (piecewise linear functions): A function $f : \mathbb{R}^M \rightarrow \mathbb{R}$ is piecewise linear if firstly its domain can be represented as the union of finitely many polyhedral sets, and secondly f is "affine" within each polyhedral set, i.e., $f(\mathbf{x}) = \mathbf{a}^T\mathbf{x} + b$ for some vector $\mathbf{a}$ and scalar b.
In the following theorem, we summarize some properties of g.

Theorem 3: The function g defined in (31) is continuous and piecewise linear. More precisely, g consists of 3^{M-1} hyperplanes.

Proof: We omit the proof here. □

In Fig. 1, an example of the function g for the two-user GIC is illustrated. As depicted in the figure, it is piecewise linear and has 9 faces.

IV. Users Ordering

In this section, we consider the M-user GIC modeled in (2). Given an ordering of users, we aim at maximizing users' rates in succession. In general, there are $M!$ orderings of users which result in $M!$ not necessarily distinct achievable rates in the capacity region; on the contrary, in the Gaussian MAC, every permutation leads to a distinct achievable rate. Without loss of generality, we may assume the order is the same as users' indices.

Setting the first user's rate to its maximum value, i.e., $\gamma\left(\frac{h_{11}^2 P_1}{N_1}\right)$ imposes some constraints on the other user's rates as they must be decoded by the first receiver. The reason is that the first receiver needs to decode first all other users to be able to decode its own signal at this rate. As we proceed, we maximize the rates of the users successively. For instance, for user i we treat users below the index i as interference with given rates and powers, and users above index i as they do not exist, i.e., we put constraints on their rates in such a way that the receiver can decode them first. In order to be decodable by receiver $j \in \{1, 2, \ldots, i\}$, the rate of user i must satisfy the following inequality which is a consequence of (31)

$$R_i \le R_{ij}(X_j) = \min_{W \subseteq X_j} \gamma\left(\frac{h_{ji}^2 P_i + \sum_{k \in W} h_{jk}^2 P_k}{N_j + \sum_{k \in (E - S_j)} h_{jk}^2 P_k}\right) - \mathbf{R}(W) \quad (32)$$

where X_j is the maximum decodable subset of users with indices less than i. The following algorithm maximizes the rates of the users successively.

Algorithm 2 (successive maximization of users' rates):

1) Set $R_1 = \gamma\left(\frac{h_{11}^2 P_1}{N_1}\right)$.
2) For $i = 1 : M$ set: $S_i = \{i, i+1, \ldots, M\}$ and $U_i = \bar{S}_i$.
3) For $i = 2 : M$ Do:
   a) Find the maximum decodable subset V in the set U_i for receiver i considering users in the set S_i are decoded first, and $S_i \cup V \to S_i$
   b) Solve the following optimization problem
   $$R_i = \min_{j \in \{1,2,\ldots,i\}} R_{ij}(S_j \cap U_i), \quad (33)$$
   where R_{ij} is defined in (32).

V. Fixed Point and Iterative Algorithm

We start this section with the following definition.

Definition 4 (conservative rate): The conservative rate of a user is the rate that is achievable while considering the interference caused by other users as noise.

In a noncooperative media, however, a user may reject any rate which is below its conservative rate. Therefore, some achievable rate vector characterized in the previous section may not be feasible in a noncooperative scenario. The reason is that a user to send at its maximum rate puts some constraints on the other users' rates which might make some users send below its conservative rate, whereas a user may reject to appease other users by sending at low rates. As a result, those strategies are feasible that every user has a rate no less that its conservative rate (3).

Suppose that users are transmitting at the rate vector $\mathbf{R}$. Now, by knowing the powers and rates of the other users, user i can maximize its own rate by running Algorithm 1 and solving the optimization problem in (31). Suppose the new rate of user i is $\tilde{R}_i = g_i(\mathbf{R}_{-i})$, where $\mathbf{R}_{-i} = [R_1, \ldots, R_{i-1}, R_{i+1}, \ldots, R_M]$. If all users do the same procedure simultaneously, then the new rate vector can be written as a function of $\mathbf{R}$, i.e.,

$$\tilde{\mathbf{R}} = \mathbf{G}(\mathbf{R}) = [g_1(\mathbf{R}_{-1}), g_2(\mathbf{R}_{-2}), \ldots, g_M(\mathbf{R}_{-M})]^T. \quad (34)$$

In the following theorem, we prove that the function $\mathbf{G}$ has a fixed point.

Theorem 4: The function $\mathbf{G}$ in (34) has a fixed point.

Proof: By applying Theorem 3 to each function g_i, we conclude that $\mathbf{G}$ is continuous. Since the rate of user i is bounded by $\gamma\left(\frac{h_{ii}^2 P_i}{N_i}\right)$, the domain of $\mathbf{G}$ is a compact subset of $\mathbb{R}^M$. Now, by applying Brouwer's fixed-point theorem [14], we conclude that $\mathbf{G}$ has a fixed point. □

In general, the fixed points of $\mathbf{G}$ are not unique. Moreover, if we start from a rate vector and update the vector based on (34), then in some cases, it does not converge to a fixed point. However, the following algorithm can find some fixed points of the function by updating just one rate at each iteration.

Algorithm 3 (Fixed Point Algorithm):

1) Set $\mathbf{R} = [\gamma\left(\frac{h_{11}^2 P_1}{N_1}\right), \gamma\left(\frac{h_{22}^2 P_2}{N_2}\right), \ldots, \gamma\left(\frac{h_{MM}^2 P_M}{N_M}\right)]^T$ and let π be a permutation on users.
2) Do the following until converges: For $i = 1 : M$. $R_{\pi(i)} = g_{\pi(i)}(\mathbf{R}_{-\pi(i)})$

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