Solution.
Session 6.

Q1) $P(X>20)=\int_{20}^{\infty}\frac{10}{x^2}\,dx=\frac{1}{2}$

b) $F(x)=\int_{-\infty}^{x}\frac{10}{t^2}dt=\int_{10}^{x}\frac{10}{t^2}dt=1-\frac{10}{x} \quad 10\leqslant x$

c) The prob. of at least fifteen hours is $1-F(15)=\frac{2}{3}$

at least three of six devices should be functional.

so it has binomial distribution.

** hint: how did I understand I should use binomial?

Ans: since we have <u>two</u> possibilities for each devise. i.e: working more that 15 hours, working less than 15 hours.

$$\Rightarrow \text{answer}=\sum_{K=3}^{6}\binom{6}{K}\underbrace{\left(\frac{2}{3}\right)^{K}}\underbrace{\left(\frac{1}{3}\right)^{6-K}}$$

$p=\text{working}\geqslant 15^h$ $\qquad$ $1-p=\text{working}<15^h$

Q2) According to strategy:

$F_u(U)=P(U\leqslant u)=P(X^2+1\leqslant u)=P(-\sqrt{u-1}\leqslant X\leqslant\sqrt{u-1})=$

$F_X(\sqrt{u-1})-F_X(-\sqrt{u-1}) \qquad (u\geqslant 1)$

now get derivative from both sides

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$$f_u(u) = \frac{+1}{2\sqrt{u-1}} f_x(\sqrt{u-1}) - \frac{-1}{2\sqrt{u-1}} f_x(-\sqrt{u-1}) =$$

$$\frac{1}{2\sqrt{u-1}} \left(f_x(\sqrt{u-1}) + f_x(-\sqrt{u-1}) \right)$$

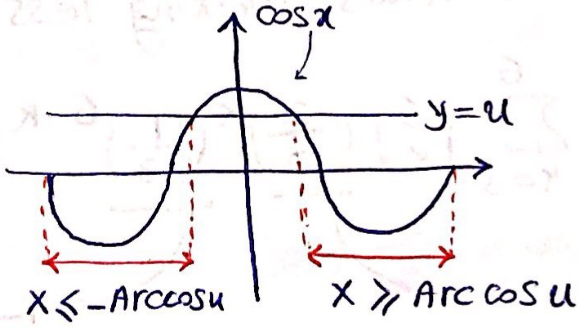
now we know that: $f_x(X) = X^2 - 2X$.

$$\Rightarrow f_u(u) = \frac{1}{2\sqrt{u-1}} \left((\sqrt{u-1})^2 - 2(\sqrt{u-1}) + (-\sqrt{u-1})^2 - 2(-\sqrt{u-1}) \right) = \dots$$

for $u \geq 1$

Q3) Again use the strategy:

$$F_u(U) = P(U \leq u) = P(\cos X \leq u) \longrightarrow -1 \leq u \leq 1$$



$$\Rightarrow P(\cos X \leq u) = \underbrace{P(X \geq \operatorname{Arccos} u)}_{1 - F_X(\operatorname{Arccos} u)} + \underbrace{P(X \leq -\operatorname{Arccos} u)}_{F_X(-\operatorname{Arccos} u)} =$$

derivative from both sides:

$$f_u(u) = \frac{1}{\sqrt{1-u^2}} \cdot f_x(\operatorname{Arccos} u) + \frac{1}{\sqrt{1-u^2}} f_x(-\operatorname{Arccos} u)$$

since f_x is uniform $\Rightarrow$ $f_x(\cdot) = \frac{1}{2\pi}$

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$$\Rightarrow f_u(u) = \frac{1}{\sqrt{1-u^2}}\cdot\frac{1}{2\pi} + \frac{1}{\sqrt{1-u^2}}\cdot\frac{1}{2\pi} = \frac{1}{\pi}\cdot\frac{1}{\sqrt{1-u^2}} \qquad -1 \leqslant u \leqslant 1$$

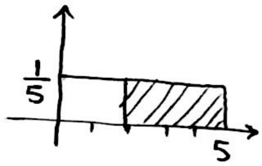
Q4) The roots of equation will be real when $\Delta \geqslant 0$,

$$Y^2 - Y - 2 \geqslant 0 \Rightarrow (Y-2)(Y+1) \geqslant 0$$

$\rightarrow Y \leqslant -1$ ✗

$\rightarrow Y \geqslant 2$ ✓

$$P(E) = P(Y \geqslant 2) = \int_2^5 f_y(y) = \int_2^5 \frac{1}{5}\,dy = \frac{3}{5}$$



Q5) According to session's 6 notes, we should convert this r.v to standard one.

a) $P\{X>5\} = P\{\frac{X-10}{6} > \frac{5-10}{6}\} = P(\overset{\text{standard normal r.v}}{Z} > -\frac{5}{6}) = 1-\Phi(-\frac{5}{6}) = 1-\Phi(-0.83)$

b) $P\{4<X<16\} = P\{\frac{4-10}{6} < \frac{X-10}{6} < \frac{16-10}{6}\} = P\{-1<Z<1\} = \Phi(1)-\Phi(-$

c) $P\{X<8\} = P\{\frac{X-10}{6} < \frac{8-10}{6}\} = P(Z<-\frac{1}{3}) = \Phi(-1/3)$.

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Q6. please change $\lambda = 12$ to $\boxed{\lambda = \frac{1}{2}}$. there was a typo.

a) $P\{T>2\} = \int_2^\infty \frac{1}{2} e^{-1/2 t} dt = e^{-1}$

b) we should compute $P\{T>10 \mid T>9\}$

because of memoryless property it is equal to :

$P\{T > 10-9\} = P\{T>1\}$

$P\{T>1\} = 1 - P\{T<1\} = 1 - (1 - e^{-1/2}) = e^{-1/2}$

Q7) $F_Y(y) = P(Y \leqslant y) = P(\log X \leqslant y) = P(X \leqslant e^y) = F_X(e^y)$

we know that for exponential random variable of X we have:

$F_X(x) = 1 - e^{-\lambda x}$ (prove it!)

$\Rightarrow F_Y(y) = 1 - e^{-e^y}$

$\Rightarrow f_Y(y) = (F_Y(y))' = e^{y + e^y}$

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Q 8) The number N of acceptable items is a binomial random variable so we can approximate it with a Gaussian with mean $\mu = pn = 150(0.95) = 142.5$, and a variance of $\sigma^2 = np(1-p) = 7.125$. From the variance we have a standard deviation of $\sigma \approx 2.669$. Thus the desired probability is given by

$$\begin{aligned} P\{N \geq 140\} &= P\{N \geq 139.5\} \\ &= P\left\{\frac{N - 142.5}{2.669} \geq \frac{139.5 - 142.5}{2.669}\right\} \\ &\approx P\{Z \geq -1.127\} \\ &= 1 - P\{Z \leq -1.127\} \\ &= 1 - \Phi(-1.127) \approx 0.8701\,. \end{aligned}$$

Where in the first line above we have used the continuity correction required when we approximate a discrete density by a continuous one, and in the third line above we use our Gaussian approximation to the binomial distribution. We note that we solved this problem in terms of the number of items that are *acceptable*. An equivalent formulation could easily be done in terms of the number that are *unacceptable* by using the complementary probability $q \equiv 1 - p = 1 - 0.95 = 0.05$.

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