Solutions for the 2^{rd} tutorial session:

Q1) at first we specify the sample space.

first outcome → second outcome

$$6\left\{\begin{array}{cccc} (1,1) & (2,1) & \cdots & (6,1) \\ (1,2) & (2,2) & \cdots & (6,2) \\ \vdots & \vdots & & \vdots \\ (1,6) & (2,6) & \cdots & (6,6) \end{array}\right.$$

$\underbrace{\qquad\qquad\qquad\qquad}_{6}$

★ note that we have to consider (a,b) where $a \neq b$ different from (b,a)

$\Longrightarrow$ the size of the sample space is: $|S| = 6 \times 6 = 36$

$E = \{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

$\Longrightarrow |E| = 6$

$\Longrightarrow P(E) = \frac{|E|}{|S|} = \frac{6}{36} = \frac{1}{6}$

Q2) Sample space is choosing $(1+2)$ balls from $(6+5)$ balls.

So simply we have: $|S| = \binom{11}{3}$

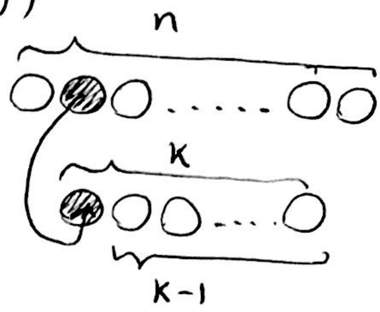
$|E| = \binom{6}{1}\binom{5}{2} \quad \Longrightarrow \quad P(E) = \frac{|E|}{|S|} = \frac{\binom{6}{1}\binom{5}{2}}{\binom{11}{3}}$

Q3) $|S| = \binom{n}{k}$

for event E, at first we choose the special one to make sure that we chose it. (with $\binom{1}{1}$ way)

after that we omit this special ball (red one) from the n balls. it remains $n-1$ balls

we have to choose $k-1$ balls from these balls.



So $|E| = \binom{1}{1}\binom{n-1}{k-1}$

$$\Longrightarrow P(E) = \frac{\binom{1}{1}\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}$$

Q4) ⓐ specify the players by A, B, C, D.

at first consider that A has all spades in his hands. So he has to choose $\binom{13}{13}$ spades. meanwhile $|S| = \binom{52}{13}$

so $P(E_a) = \frac{1}{\binom{52}{13}}$.

we have same expression for $B, C, \& D$. So: $P(E_b) = P(E_c) = P(E_D) = \frac{1}{\binom{52}{13}}$

$$\Longrightarrow P(E) = 4 \times \frac{1}{\binom{52}{13}}$$

* but what would happen if we consider $B, C, \& D$ in computing $P(E_a)$. in this way $|S|$ should be: $|S| = \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$

and for $|E| = \binom{13}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} \Longrightarrow P(E_a) = \frac{\binom{13}{13}\cancel{\binom{39}{13}}\cancel{\binom{26}{13}}\cancel{\binom{13}{13}}}{\binom{52}{13}\cancel{\binom{39}{13}}\cancel{\binom{26}{13}}\cancel{\binom{13}{13}}} = \frac{1}{\binom{52}{13}}$

As you see we get the same resault.

b) At first we distribute 4 Aces among them to make sure each of them has exactly one ace in his hands. we can do it in 4! ways. for the remaining 48 cards, we have to distribute them such that each player has 12 cards. (so totally each guy has 13 cards) this could be done in $\binom{48}{12,12,12,12}$ ways.

So $$P(E) = \frac{4! \times \binom{48}{12,12,12,12}}{\binom{52}{13,13,13,13}}$$

★ please note that $\binom{52}{13,13,13,13} = \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$.

Q5) from the course handout we know that:

(1) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(AB) + P(AC) + P(BC)] + P(ABC)$

and we know that for an arbitrary set, M we have:

$$P(M) = \frac{\text{number of members in } M}{N}$$

where N is the total number of club.

So simply, by multiplying both sides of equition (1) by N, we obtain:

$$|A \cup B \cup C| = |A| + |B| + |C| - [|AB| + |AC| + |BC|] + |ABC|$$

whe $|3|$ is the size of 3.

by using above formula we simply have:

$|A \cup B \cup C| = 43$.

Scanned by CamScanner

Q6) a) since they are mutually exclusive we have:

$$P(A \cup B) = P(A) + P(B)$$

(*) mutually exclusive means that:

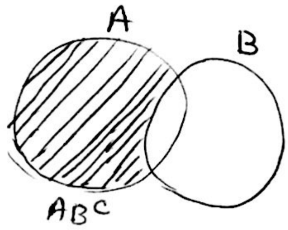
if A & B are mutually exclusive, they cannot be occurred simultaneously. means that if A occurred, B couldn't be occurred and vice versa.

(*) consept of mutually exclusive is defined for one experiment and one trial. eg: in rolling adice comming even and odd are mutally exclus

b) we know that $P(AB^c \cup B) = \underbrace{P(A \cup B)}$

$\Rightarrow P(AB^c \cup B) = P(AB^c) + P(B) = P(A \cup B) = P(A) + P(B)$

$\Longrightarrow P(AB^c) = P(A)$



AB^c & B are mutually exclusive.

c) $P(A \cup B) = P(A) + P(B) - P(AB)$

from a) $\longrightarrow$ $P(A \cup B) = P(A) + P(B)$ $\Big\} \longrightarrow P(AB) = 0$

Q7) if we have flush, if the cards have same suit. this is possible in 4 ways (spade/club/heart/diamond)

then we should select 5 of 13 suits, in $\binom{13}{5}$ ways.

So $|E| = 4 \times \binom{13}{5}$

$$\Longrightarrow P(E) = \frac{4 \times \binom{13}{5}}{\binom{52}{5}}$$

b) we have 2 same denomination (a,a). for a we have 13 possibilities. these 2 a's should select among 4 suits. So for (a,a) we have $13 \times \binom{4}{2}$ ways of selection.

for example, we choose queen for a, for choosing b, we have 12 possiblities for denomination and 4 for suits $\Rightarrow 12\times\binom{4}{1}$ for b.

and similarily for c, d we have $11\times\binom{4}{1}$ & $10\times\binom{4}{1}$ ways.

now for example our hand is $\underbrace{\overbrace{\text{queen}}^{a},\overbrace{\text{queen}}^{a}}_{a,a},\overbrace{2}^{b},\overbrace{10}^{c},\overbrace{\text{Jack}}^{d}$

but in our computation, we assume order for b, c, d. i,e: we suppose that at first we choose b, then c, then d. but you have to note that: the order doesn't matter in one hand. for example:

$$(\text{queen}, \text{queen}, 10, \text{jack}, 2) \equiv (\text{queen}, \text{queen}, 2, 10, \text{jack})$$

So we should devide the $|E|$ by $3!$. (which is permutation of b, c, d)

$$\text{So: } |E| = \frac{\overbrace{13\times\binom{4}{2}}^{a,a}\times\overbrace{12\times\binom{4}{1}}^{b}\times\overbrace{11\times\binom{4}{1}}^{c}\times\overbrace{10\times\binom{4}{1}}^{d}}{3!}$$

and $|S| = \binom{52}{5}$. $P(E) = \frac{|E|}{|S|}$.

Q8) $P(\text{if A wins at first withdraw}) = \frac{3}{7+3} = \frac{3}{10} = P_1$

the probability that A wins at the 3rd withdraw is that:

A at first picks up balls with prob. $= \frac{7}{10}$, now we have 3 red & 6 black balls. B should pick up black at second with prob. $= \frac{6}{9}$ & A should pick up red at 3'rd with prob. $= \frac{3}{8}$

So: $P_3 = \frac{7}{10}\times\frac{6}{9}\times\frac{3}{8}$

similarily $P_5 = \frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$

$P_7 = \frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{3}{4}$.

but if A cannot win the game till the 7'rd withdraw, cannot win the game never. Since all of 7 black balls were picked up, and at the 8^{rd} withdraw, B definitely picks up a red ball.

So $P(A_{win}) = P_1 + P_3 + P_5 + P_7$

Q9. we know $P(A^c) = 1 - P(A)$

$(\text{prob. of 6 comes up at least once})^c = (\text{prob. of never 6 comes up})$

and prob of never 6 comes up $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = (\frac{5}{6})^4$

So prob. of 6 comes up at least once $= 1 - (\frac{5}{6})^4$

Q10) each stranger may be born in 12 possible months.

So $|S| = \underbrace{12 \times 12 \times \cdots \times 12}_{12 \text{ times}} = 12^{12}$.

for our event, named E, we have to distribute these 12 months among these 12 guys with 12! ways.

$\Rightarrow P(E) = \frac{12!}{12^{12}} = \frac{11!}{12^{11}}$.