1

b) A, B must sit next to each other?

$\boxed{AB}$ + remaining persons = 7 persons $\xrightarrow{\#\text{ of orderings}}$ $\underline{7!}$ (Similar to part "a")

tie them together

$\underline{2!}$: orderings of two specific persons inside the box

– By multiplication rule in total : $\underline{2! \times 7!}$

C) 4 men and 4 women, no 2 men or 2 women can sit next to each other?

– Starting with a man, the next person should be a woman, the one after should then be a man and so on!

M: Man
W: Woman

$$\underset{M}{4} \times \underset{W}{4} \times \underset{M}{3} \times \underset{W}{3} \times \underset{M}{2} \times \underset{W}{2} \times \underset{M}{1} \times \underset{W}{1} = 4! \times 4!$$

$\boxed{\begin{matrix} W & W \\ W & W \end{matrix}}$ $\boxed{\begin{matrix} M & M \\ M & M \end{matrix}}$ → 4 possibilities

4 possibilities

– Starting with a woman would be similar : $4! \times 4!$

$\Rightarrow$ In total: $\underline{2 \times 4! \times 4!}$

d) 4 married couples, each couple must sit together? (Tie them together)

$\boxed{AB}$ $\boxed{CD}$ $\boxed{EF}$ $\boxed{GH}$: 4 persons → $4!$

2! 2! 2! 2!

orderings of persons inside the box

$\Longrightarrow$ In total: $4! \times [2! \times 2! \times 2! \times 2!]$

~~6 factorial is wrong.~~ ~~6 factorial/2fac*2fac*~~1fac

2

$n_1 + \dots + n_r = n \Rightarrow \dfrac{n!}{n_1! \dots n_r!} = \binom{n}{n_1}\binom{n-n_1}{n_2} \dots \binom{n-n_1-\dots-n_{r-1}}{n_r}$

Type 1 … Type r

$$= \frac{n!}{\cancel{(n-n_1)!}\, n_1!} \cdot \frac{\cancel{(n-n_1)!}}{n_2!\,(n-n_1-n_2)!} \cdots = \frac{n!}{n_1! \dots n_r!}$$

MAMMAL $\Rightarrow \dfrac{6!}{3!\,2!\,1!}$

1

#3 (a). Without restriction, there are totally $\binom{9}{4}\binom{7}{3}$ different committees.

Now, we are required to compute the # of different committees with the restriction that 2 of the men refuse to serve together. Since there are $\binom{9}{4}\binom{5}{1}$ different committees if those 2 men serve together, we can subtract $\binom{9}{4}\binom{5}{1}$ from the total number to obtain the answer:

$$\binom{9}{4}\binom{7}{3} - \binom{9}{4}\binom{5}{1} = 3780$$

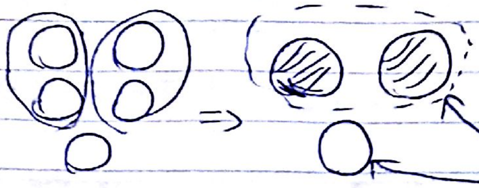
(b). Similarly, if that man and that women serve together, there are $\binom{8}{3}\binom{6}{2}$ possibilities. To get the answer, we can subtract $\binom{8}{3}\binom{6}{2}$ from the total number:

$$\binom{9}{4}\binom{7}{3} - \binom{8}{3}\binom{6}{2} = 3570.$$

3

4) Two-pair:

First way:



Ranks

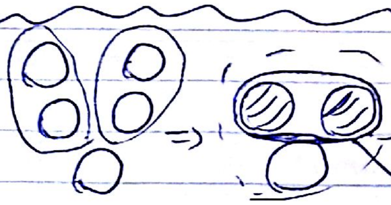
$$\binom{13}{2} \times \binom{13-2}{1} \times (\#\bullet)^2 \times (\#\circ)$$

$\#\bullet = \binom{4}{2}$ → Suits

$\#\circ = \binom{4}{1}$

or $\binom{13}{1} \times \binom{12}{2}$

Second way:



$$\binom{13}{3}\binom{3}{2} \times (\#\bullet)^2 \times (\#\circ)$$

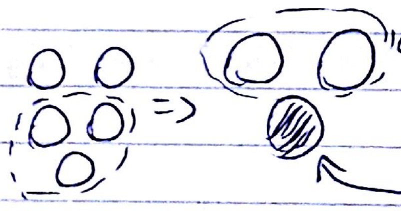
Note: $\binom{13}{3}\binom{3}{2} = \frac{13!}{3!\,10!}\,\frac{\cancel{3!}}{2!\,1!} = \frac{13!}{2!\,10!}$

$$||$$

$\binom{13}{2}\binom{11}{1} = \frac{13!}{2!\,\cancel{11!}}\,\frac{\cancel{11!}}{10!\,1!} = \frac{13!}{2!\,10!}$

Three-of-a-kind:

First way:



$$\binom{13}{1}\binom{13-1}{2} \times (\#\bullet)(\#\circ)^2$$

$\#\bullet = \binom{4}{3}$ $\quad \#\circ = \binom{4}{1}$

Second way: $\binom{13}{3}\binom{3}{1}(\#\bullet)(\#\circ)^2$

or $\binom{3}{2}$

Two following problems are important because in many different cases we will have some items and one or more restrictions on them and we are going to find different arrangements of them.

Because these type of problems are important, we are going to find an algorithmic way for solving them:

One restriction:

1) First we only consider items affected by restriction.

2) We find all possible arrangements of them under that specific restriction.

3) We forget that restricted ~~other~~ items are different and we ~~call~~ name all of them R and all non-restricted NR

4) We find all possible arrangements of R & NR items:

$$\frac{(\#R + \#NR)!}{(\#R)!\ (\#NR)!}$$ (If there ~~was~~ is not any restriction between R's & NR's) such as ~~adjacent~~ non adjacent

5) At the end we find all possible arrangements of NR items

More than one restriction: (Two restrictions)

1) First we only consider items affected by first restriction

2) We find number of all possible arrangements of items under first —

restriction

3) we forget that restricted items are different and we name all of them R_1

4) Now it is time for putting items under Second restriction among items with type R_1. In this step ~~we~~ first we suppose that we only have two types of items R_1 & R_2 and in next step we find all possible orderings under R_2 restriction.

named R_2

all possible arrangements of R_1 & R_2: $\dfrac{(\#R_1 + \#R_2)!}{(\#R_1)!\,(\#R_2)!}$

5) Now its time for considering only items under R_2 restriction and find all possible arrangements of them

6) If there are more than two restrictions we will continue else we put all previous restrictions under R_{total} and we suppose that all items under all restrictions are same and have type R_{total}

7) Now we first ~~find~~ find all arrangements of NR & R_{total} items

8) As the end we find all possible arrangements of items with type NR

Showing a Simple way!

One restriction: $\boxed{\text{Ordering or arrangements inside restriction}}$ × $\boxed{\text{\#Possible ways for arrangement of \#NR \& \#R items}}$ ×

$\boxed{\text{ordering of arrangements inside Non-restricted items}}$

Two restrictions!

$\boxed{\text{Arrangements of items under first restriction}}$ × $\boxed{\text{Arrangements of } \#R_1 \,\&\, \#R_2 \text{ items}}$ × $\boxed{\text{Arrangements of items under second restriction}}$

$\boxed{\text{Arrangements of } \underbrace{(\#R_1 + \#R_2)}_{\#R_3} \,\&\, \#NR \text{ items}}$ × $\boxed{\text{Arrangements of Non-restricted items}}$

Now we can look through problems 5 & 6:

5) This part is a little different to typical restriction which we consider with an algorithmic approach, And that is because restriction is non-adjacent

a) we want to have not adjacent vowels. ~~We can solve this problem~~ (I do not recommend this method)

If we first arrange vowels we will have:

① ② ③ ④ ⑤ ⑥ ⑦ ⑧

we will have nine spots (Boxes) for consonants. Under that condition

P7

we want to put 10 balls (consonants) inside 9 boxes with this condition that it should be at least one ball in each box (Except the first and last). So if we want to find possible arrangements of: 10 C & 8 V under that condition it will be = ? (we assume that all consonants are same)

Ordering of vowels × ? × (arrangement) of Consonants $= \dfrac{8!}{4!(1!)^4} \times \dfrac{10!}{4!\,2!\,2!\,(1!)^2} \times ?$

$4! \to$ E, $(1!)^4 \to$ A, I, O, U

Finding ? is not easy:

- first component: ⊗ V V V V V V V V V

  It is empty

we have 8 boxex and 10 same balls. we want to find all possible ways for arrangement with this condition: In each box there should be at least one ball. Finding this in direct way is hard so we use:

All possible − arrangements in which one or more is empty

A lot of work!

Second component: All boxes have ball inside => 10 balls 9 boxes

~~9 possible ways to have~~ 8 First 9 boxes contain 1 balls and one contains 2 balls => $\binom{9}{2}$ => All ways: 2×first component + second

Better way:

(1) First we assume that there are only Consonants:

$\#$ of arrangements $= \dfrac{10!}{4!\,2!\,2!}$ ★

(2) Now we suppose that all Consonants are Same:

O C O C .... C O

Spots for vowels

But we can not have a spot with two vowels. So number of ways to put vowels (first assume that all are same) $= \binom{11}{8}$ ★★

$\binom{11}{8}$: 11 = #Spots, 8 = #vowels

★: Using Combination assure that there is only one vowel inside each spot

(3) Now we should find arrangments of ~~Consonants~~ vowels: $\dfrac{8!}{4!}$ ★★★

Result: ★ × ★★ × ★★★

Note: You can change order of steps.

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b) (1) Only consider restricted items: A, E, I, O, U

$\quad$ 1, 4, 1, 1, 1

(2) Find number of arrangements with restriction as Condition.

(P.9) In this case because of restriction there is only one way AEEEEIOU

③ Now suppose that restricted items are called R and all are same and Non-restricted (consonants) are called NR

So: #R: 8 #NR: 10

$$\# \text{ Arrangements} = \frac{(8+10)!}{8!\,(10)!} = \binom{18}{10} = \binom{10+\overset{\text{Spots for C's or NR's}}{\overset{\downarrow}{9}}-1}{10}$$

④ Now we only consider non-restricted items

⑤ # Arrangements of NR items (consonants) $= \dfrac{\overset{\text{all consonants}}{10!}}{\underset{R}{4!}\,\underset{C}{2!}\,\underset{N}{2!}\,\underset{T}{1!}\,\underset{L}{1!}}$

Result: $1 \times \binom{18}{10} \times \dfrac{10!}{4!(2!)^2}$

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6) In this question we have two restrictions:

R_1: N preceding the first S

R_2: E " " " T

(1) Only consider items under restriction 1

3 N's 3 S's

(2) Possible arrangements (restriction should be meet)

$\underline{N}$ {(all possible arrangements of N's and S's)

$$\frac{(\overset{N's}{2} + \overset{S's}{3})!}{2!\,3!} = \binom{5}{3} \quad ★$$

put one of N's aside because we want to first N preced the first S

(3) We name all N's & S's (Items under restriction 1) R_1 and we suppose that those are same

(4) Now consider all items restricted by R_2: 5 E's & 1 T

We assume that all are same with name R_2 (we can first find arrangements of these items and after that distribute them between R_1's)

$$\frac{(\#R_2 + \#R_1)!}{(\#R_2)!\,(\#R_1)!} = \frac{12!}{6!\,6!} = \binom{12}{6} \quad ★★$$

P11

(5) Now we find arrangements between items restricted by R_2

R_2: first E preceding first T

put one aside: $1E$ (4 E's & 1 T) $\frac{(4+1)!}{4!\,1!} = 5$

6) Now we suppose that all restricted items are same and call R_3 and we want to find number of all possible arrangements of NR items and R_3 items (one example $\underbrace{NR\,NR \cdots NR\,R_3 \cdots R_3}$)

arrangements beneath them are not important in this case

$\#NR: \underset{L}{2}+\underset{A}{1}+\underset{H}{1}+\underset{I}{1}+\underset{V}{1} = 6$ / $\#R_3: \underset{N}{3}+\underset{S}{3}+\underset{E}{5}+\underset{T}{1} = 12$

$$\frac{(6+12)!}{6!\,(12)!} = \binom{18}{12} = \binom{18}{6}$$

(7) At the end we find # of arrangements between non restricted items

$$\frac{6!}{2!\,(1!)^4} = \frac{6!}{2!}$$

Result: $\binom{5}{3}\binom{12}{6}\,5\,\binom{18}{6}\frac{6!}{2!}$

P12