Problem 1:

Suppose X, the lifetime of a certain type of electronic device (in hours), is a continuous random variable with probability density function $f(x) = \frac{10}{x^2}$ for $x > 10$ and $f(x) = 0$ for $x \leq 10$.

a. Find $P(X > 20)$.

b. Find the cumulative distribution function (cdf).

d. What is the probabilty that among 6 such types of devices at least 3 will function for at least 15 hours?

Problem 2:

A parachutist lands at a random point on a line AB.

a. Find the probability that he is closer to A than to B.

b. Find the probability that his distance to A is more thant 3 times his distance to B.

Problem 3:

Suppose the length of a phone call in minutes follows the exponential distribution with parameter $\lambda = 0.1$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait

a. more than 10 minutes.

b. between 10 and 20 minutes.

Problem 4:

Let X be an exponential random variable with $\lambda = 0.2$.

a. Find the mean of X.

b. Find the median of X.

c. Find the variance of X.

Problem 5:

The random variable X has probability density function

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E(X) = 0.6$ find

a. $P(X < \frac{1}{2})$.

b. $\text{Var}(X)$.

Problem 6:

For some constant c, the random variable X has probability density function

$$f(x) = \begin{cases} cx^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

a. $E(X)$.

b. $\text{Var}(X)$.

Problem 7:

To be a winner in the following game, you must be succesful in three succesive rounds. The game depends on the value of X, a uniform random variable on (0,1). If $X > 0.1$, then you are succesful in round 1; if $X > 0.2$, then you are succesful in round 2; if $X > 0.3$, then you are succesful in round 3.

a. Find the probability that you are succesful in round 1.

b. Find the conditional probability that you are succesful in round 2 given that you were succesful in round 1.

c. Find the conditional probability that you are succesful in round 3 given that you were succesful in round 2.

d. Find the probability that you are a winner.

Problem 8:

There are two types of batteries in a bin. The lifetime of type i battery is an exponential random variable with parameter λ_i, $i = 1, 2$. The probability that a type i battery is chosen from the bin is p_i. If a randomly chosen battery is still operating after t hours of use, what is the probability it willl still be operating after an additional s hours?

Problem 9:

You bet \$1 on a specified number at a roulette table. A roulette wheel has 38 slots, numbered 0, 00, and 1 through 36. Approximate the probability that

a. In 1000 bets you win more than 28 times.

b. In 10000 bets you win more than 270 times.