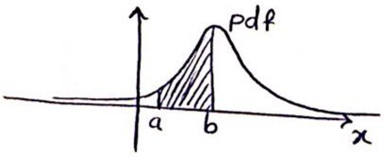
Continuous R.V

Discrete R.V	Continuous R.V
pmf: prob. mass function	pdf: prob. density function
Summation: $\sum_{x}$	Integration: $\int dx$

- CDF: cumulative distribution function $F(x) = \int_{-\infty}^{x} f(t)dt$
- Expectation: $E(x) = \int_{-\infty}^{+\infty} x f(x) dx$
- Variance : $Var(x) = E(x^2) - (E(x))^2$

* For continuous R.V's, the area under the curve of pdf is the same as probability.



$P(a \leq x \leq b) = \int_a^b f(x)dx$

1. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by:

$$f(x) = \begin{cases} \dfrac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

(a) Find $P\{X > 20\}$.

(b) What is the cumulative distribution function of X?

(c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?

Good strategy for finding pdf of $g(x)$ from pdf of "x"

① Try to find CDF of $g(X)$

for example : $U = g(x) = 2x+1$

$F_u(u) = P(U \leqslant u)$

② Then replace capital U with it's quantity

$P(U \leqslant u) = P(2X+1 \leqslant u) = P(X \leqslant \frac{u-1}{2})$

–hint : We should find a restriction respect to "X" at the end of this part.

③ Replace $P($ some restriction on $X)$ with $F($"sth"$)$

$P(X \leqslant \frac{u-1}{2}) = F_x(\frac{u-1}{2})$

④ Get derivative of both side of quality.

$(F_u(u))' = (F_x(\frac{u-1}{2}))' \Rightarrow f_u(u) = \frac{1}{2} f_x(\frac{u-1}{2})$

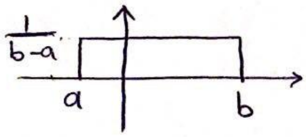
---

2. If pdf of 'X' is $f_x(x) = x^2-2x$, find the pdf of $U=X^2+1$.

---

uniform distribution:

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{O.W} \end{cases}$$



$$* \; E(X) = \frac{a+b}{2} \quad , \quad Var(X) = \frac{(b-a)^2}{12}$$

3. Let $X \sim U([-\pi, \pi])$. Find the distribution of the random variable $Y = \cos X$.

   The density of X is given by

$$f_X(x) = \begin{cases} \frac{1}{2\pi} & \text{if } x \in [-\pi, \pi] \\ 0 & \text{otherwise} \end{cases}$$

4. If Y is uniformly distributed over $(0, 5)$, what is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real?

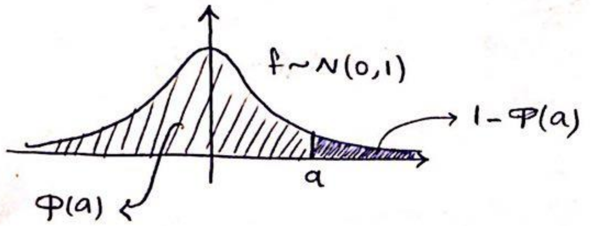
Normal (Gaussian) dist.

$$X \sim N(\mu, \sigma^2) \rightarrow \begin{cases} \text{Mean} : E(X) = \mu \\ \text{Variance} : Var(X) = \sigma^2 \end{cases}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

★ we define $\Phi(y) \triangleq P\{-\infty < Y \leqslant y\} = \int_{-\infty}^{y} f_Y(u)du$, where

$Y \sim N(0,1) \Rightarrow$ this called standard normal dis.



★ In other words, the CDF of a standard normal r.v is denoted by $\Phi(x)$, whose values at different points are given in a table. (see textbook)

★ To convert $X \sim N(\mu, \sigma^2)$ to standard normal dist., we define $Y = \frac{X-\mu}{\sigma} \sim N(0,1)$

★ we always try to convert Normal distribution to Standard one, because we Know $\Phi(x)$ at each point.

5. If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute:

(a) $P\{X > 5\}$;
(b) $P\{4 < X < 16\}$;
(c) $P\{X < 8\}$;

Exponential random variable:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$E(X) = \frac{1}{\lambda}$, $Var(X) = \frac{1}{\lambda^2}$, $F(a) = P(X \leq a) = 1 - e^{-\lambda a}$ $a \geq 0$

- Exponentially distributed random variables are memoryless.
  It means that:

$$P\{X > s + t | X > t\} = P\{X > s\} \quad \text{for all } s, t \geq 0$$

---

6. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 12$. What is
   (a) the probability that a repair time exceeds 2 hours?
   (b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?

7. If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable Y defined by $Y = \log X$.

8. *Each item produced by a certain manufacturer is independently of acceptable quality with probability.95. Approximate the probability that at most10 of the next 150 items produced are unacceptable.