

## *Problems of fourth tutorial session:*

1. A gambling book recommends the following strategy for the game of roulette: Bet 1\$ on red, if red appears (which has probability of  $\frac{18}{38}$ ), take the 1\$ profit and quit. If red doesn't appear and you lose this bet (which has probability of  $\frac{20}{38}$ ), make additional 1\$ bets on red on each of the next two spins of the roulette wheel and then quit. Let "x" denote your winning when quit.
  - a) Find  $P\{x>0\}$ .
  - b) Find  $E(X)$ .
2. The PMF of a random variable "X" is given by  $P(X=k) = a \cdot k \cdot 2^{-(k-1)}$ ,  $k=1,2,3,4$ 
  - a) For what value of "a" is this a true PMF?
  - b) Find the variance of "x"
  - c) Find the expected value of random variable  $Y = x \cdot \log_2 x$
  - d) Compute CDF of x (  $F(x)=P(X \leq x)$  )
  - e) Compute the CDF of r.v  $Z=2x+3$ .
3. A box contains 5 red balls and 7 blue balls. Two balls are randomly withdrawn. If they are of the same color you win 1.10\$ otherwise you lose 1\$.
  - a) What is the expected value of the amount you win?
  - b) What is the variance of the amount you win?
4. Suppose that the probability that an item produced by a certain machine will be defective is .1. Find the probability that a sample of 10 items will contain at most 1 defective item.
5. Suppose that two teams play a series of games that ends when one of them has won  $i$  games. Suppose that each game played is, independently, won by team A with probability  $p$ . Find the expected number of games that are played when  $i=2$ . Also, show in both cases that this number is maximized when  $p = \frac{1}{2}$ .
6. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.
7. People enter a gambling casino at a rate of 1 every 2 minutes.
  - a) What is the probability that no one enters between 12:00 and 12:05?
  - b) What is the probability that at least 4 people enter the casino during that time