Session 12

1) (10/60) A wide sense stationary process $X(t)$ with mean 0 and autocorrelation

$$R_X(\tau) = \delta(\tau)$$

is the input to an an LTI filter. The impulse response

$$h(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & otherwise \end{cases}$$

(a) What is mean value of the filter output process $Y(t)$?

(b) Find the output power spectral density $S_Y(f)$.

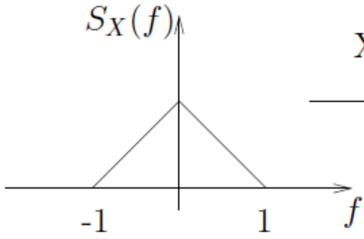
(c) Find the autocorrelation $R_Y(\tau)$.

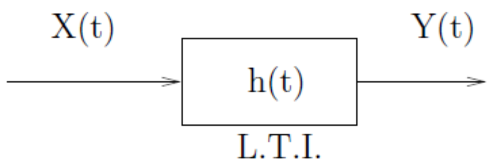
(d) Is $Y(t)$ wide sense stationary?

2) Define the random process $X(t) = A(t)\cos(wt + \theta)$, where the only random variable θ, is uniformly distributed over $[0, 2\pi]$.

a) Find $E[X(t)]$.
b) Find $R_X(t, \tau)$.
c) Is this a WSS process or not?
d) Find $S_X(f)$ for $A(t) = A$.
e) For $S_X(f)$ indicated in **Figure 1**, $E[X^2] = 1.5$, and the following $H(f)$, find $S_Y(f)$ and $R_Y(\tau)$.

$$H(f) = \begin{cases} 2, & |f| < 1/2 \\ 0, & \text{Otherwise} \end{cases}$$





3)

Poisson Random process $N(t)$ with parameter $\lambda = 0.1$ can be used to model the process of cars arriving at a gas station. Random variable T denotes the inter-arrival time between two consecutive cars.

(2) **5.1.** What is the probability that 5 cars arrive in the interval (t,t+10]?
(2) **5.2.** What is the probability that there is no car in the interval (t+10,t+12]?
(2) **5.3.** What is the mean and variance of T?
(2) **5.4.** What is the probability that $T < 3$?
(2) **5.5.** If there is no car for 5 time units, what is the probability that there will be no car for 3 more time units?

4)

(10/60) The joint pdf of X and Y is given by

$$f(x,y) = \begin{cases} c(x+y) & 0 \le x \le 2, 0 \le y \le 2 \\ 0 & otherwise \end{cases}$$

(a) Find c.

(b) Find the conditional distribution $f_{X|Y}(x|y)$.

(c) Compute $P\{X \ge 1/3 | Y = y\}$.

(d) Compute $E[X|Y = y]$.