session 10

- Sum of continuous random variables:

if X & Y be two continuous r.v's with density functions $f(x)$ and $g(x)$, respectively and also be independent,

$Z = X + Y$ has this distribution:

$$(f * g)(z) = \int_{-\infty}^{+\infty} f(z-y)\, g(y)\, dy$$

$$= \int_{-\infty}^{+\infty} g(z-x)\, f(x)\, dx$$

- As a good exercise you can check these facts:

① distribution of sum of two independent Uniform R.V's is:

$$f_Z(z) = \begin{cases} z & 0 \leq z \leq 1 \\ 2-z & 1 \leq z \leq 2 \\ 0 & \text{o.w} \end{cases}$$

② dist. of sum of two indep. exp. R.V's is:

$$f_Z(z) = \begin{cases} \lambda^2 z e^{-\lambda z} & z \geq 0 \\ 0 & \text{o.w} \end{cases}$$

③ Sum of two indep. standard normal R.V's:

$$f_Z(z) = \frac{1}{\sqrt{4\pi}} e^{-z^2/4}$$

hint 1: to prove this please note that: $\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-(y-\frac{z}{2})^2} dy = 1$

because $\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-(y-\mu)^2/2\sigma^2} dy = 1$, now fix $2\sigma^2 \stackrel{?}{=} 1$.

hint 2: Similarily we can prove that sum of $X \sim N(\mu_1, \sigma_1^2)$

and $Y \sim N(\mu_2, \sigma_2^2)$, $Z = X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

where X & Y should be indep.

Q1. If X and Y are independent Poisson random variables with respective parameters $\lambda 1$ and $\lambda 2$, compute the distribution of $X + Y$.

$Cov(X,Y) = E(XY) - E(X)E(Y)$

Correlation coeficient : $\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{var(X)\,var(Y)}}$

- if X & Y are uncorr., then $E(XY) = E(X)E(Y)$ and covariance is zero.
- if X & Y are indep., then they are uncorr.

but uncorr. $\not\Longrightarrow$ indep.

- if X & Y are completely corr. (e.g $Y = aX$) $\Rightarrow \rho(x,y) = 1$

Q2. Let X be the number of 1's and Y the number of 2's that occur in "n" rolls of a $fair\ die. Compute\ Cov(X,Y)$.

Q3. $If\ X1\ , X2\ , X3\ and\ X4\ are\ pairwise\ uncorrelated\ random\ variables, each\ having\ mean\ 0\ and\ variance\ 1, compute\ the\ correlation\ of$:

a) $X1 + X2\ and\ X2 + X3$
b) $X1 + X2\ and\ X3 + X4$

Q4. $Let\ X\ and\ Y\ be\ jointly\ Gaussian\ random\ variables\ with\ mean\ E(X) = E(Y) = \mu\ and\ Var(X) = Var(Y) = \sigma^2. Show\ that\ X + Y\ and\ X - Y\ are\ uncorrelated.$

Q5. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables having variance σ^2. Show that

$$\mathrm{Cov}(X_i - \overline{X}, \overline{X}) = 0$$