

Q) Another important type of questions in ordering problems are:

Some distinct items should be together.

This part is from that type.

We have 8 people and 4 couples

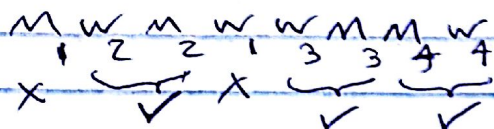
$M_1 W_1 : C_1$

$M_2 W_2 : C_2$

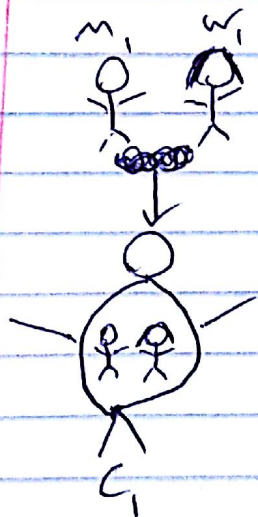
$M_3 W_3 : C_3$

$M_4 W_4 : C_4$

They want to sit together. ^{ONE} ~~Some~~ unacceptable examples:

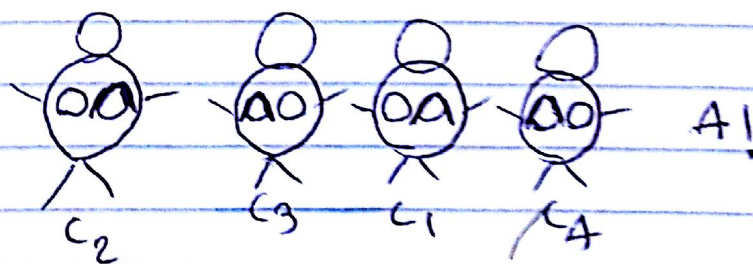


We try to coerce them into sitting together by bundling them together as new person.

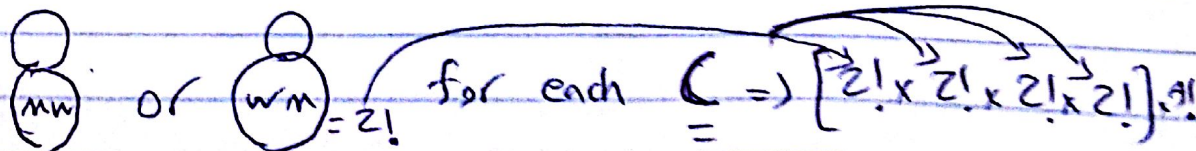


Now if we order new persons the condition is met:

One possible way:

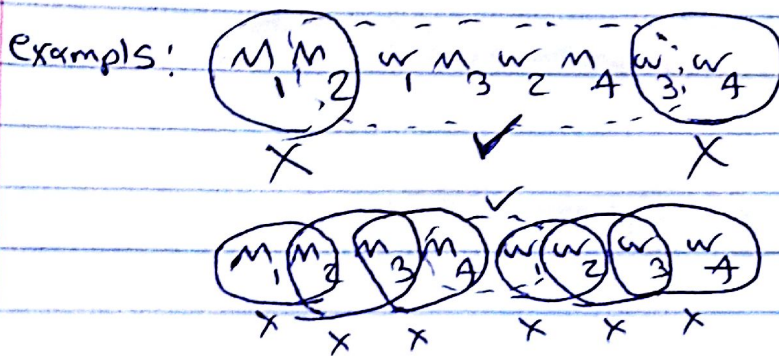


But ~~the~~ ^{inside} each new person we have two possible ordering



c) There is a special type of problems in which ~~some items can not~~ ^{Some items can not} be adjacent. For solving these problems we need walls between those items to prevent them from being adjacent.

In this part we can not have adjacent women or men. Some unacceptable



For solving this type first we look at problem to find out which items are restricted. In this question all men and women are restricted. So we can choose each one. Suppose I choose Men as walls. Some possible ordering for walls:

$M_1 \quad M_2 \quad M_3 \quad M_4$

$M_2 \quad M_3 \quad M_1 \quad M_4$

of orderings: $\underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1} = \underline{4!}$

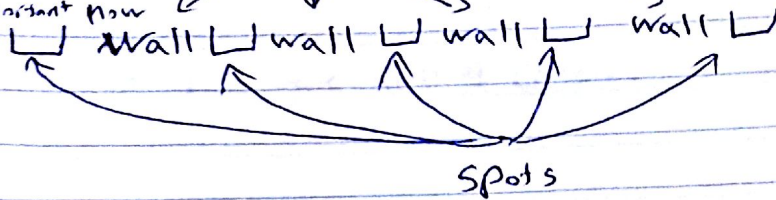
Now between ~~each~~ ^{two} walls I have an empty spot for a woman

(only one woman can be in each spot)

Since they can not be adjacent

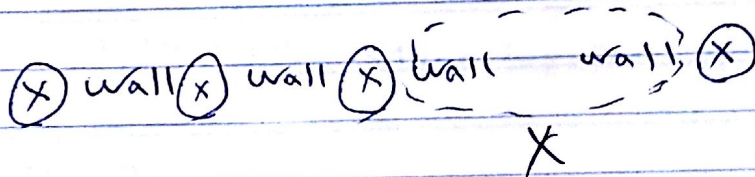
I have counted # of orderings of walls

So ordering is not important now



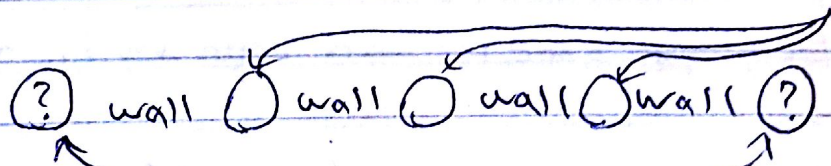
I have 5 spots for only 4 women. First I do not think about orderings of women and I only try to choose 4 spots from 5 possible for them, but $\binom{5}{4}$ is wrong because:

One possible way of choosing:



because walls are men and they should not be adjacent

In other words I have to choose all these three spots



and only one from these two = 2 ways

Now I have 4 spots for 4 different objects (women)

w_1
 w_2
 w_3
 w_4

$$4! = 1 \cdot 3 \cdot 2 \cdot 1 = 4!$$

$$\text{Final result} = 4! \times 2 \times 4!$$