

University of Waterloo
Department of Electrical and Computer Engineering
ECE316 - Introduction to Probability Theory
Midterm Examination
March 2, 2012

Instructor: Amir Keyvan Khandani

Time allowed: 1.5 hours

Answer all questions.

Closed book. One sheet of $8\frac{1}{2}'' \times 11''$ formula (one side) is allowed.

1. (a) (5 points) How many vectors $(x_1, x_2, \dots, x_6)$ are there for which each x_i is either 0 or 1 and $\sum_{i=1}^6 x_i = 4$.
(b) (10 points) How many distinct non-negative integer-valued vectors $(x_1, x_2, \dots, x_5)$ are there for which
$$(x_1 + x_2)(x_3 + x_4 + x_5) = 11.$$

(c) (5 points) How many distinct vectors $(x_1, x_2, \dots, x_8)$ are there for which each x_i is a positive integer, $x_1 \geq 5$, and $\sum_{i=1}^8 x_i = 20$?
2. In a batch of manufactured units, 2% of units have the wrong weight (and perhaps also the wrong colour), 5% have the wrong colour (and perhaps also the wrong weight), and 1% have both the wrong weight and the wrong colour.
(a) (5 points) What is the probability that a randomly selected unit is not defective in any respect?
(b) (5 points) What is the probability that a randomly selected unit is defective in exactly one respect?
(c) (5 points) If we select 10 units at random, what is the probability that at least one unit is defective in all respects?
3. A student has asked his supervisor for a letter of recommendation for an award. He estimates that there is a 90% chance that he will get the award if he receives a strong recommendation, a 50% chance if he receives a moderately good recommendation, and a 10% chance if he receives a weak recommendation. He further estimates that the probabilities that the recommendation will be strong, moderate, and weak are 0.7, 0.2, and 0.1, respectively.
(a) (5 points) How certain is he that he will receive the award?
(b) (10 points) Given that he does receive the award, how likely should he feel that he received a strong recommendation?
(c) (10 points) Given that he does not receive the award, how likely should he feel that he received a weak recommendation?
4. (15 points) A newsboy purchases papers at 10 cents and sells them at 15 cents. However, he is not allowed to return unsold papers. If his daily demand is a binomial random variable with $n = 10$ and $p = 1/3$, approximately how many papers should he purchase so as to maximize his expected profit?
5. The probability mass function of a random variable X is given by:
$$P\{X = k\} = ak2^{-(k-1)}, \quad k = 1, 2, 3, 4.$$

(a) (5 points) For what value of a is this a probability mass function?
(b) (5 points) Find $P\{X > 2\}$.
(c) (5 points) Find the variance of the random variable X .
(d) (5 points) Find the expected value of the random variable $Y = X \log_2(X)$.
(e) (5 points) Compute the distribution function (CDF) of the random variable $Z = 2X + 3$.