$\boxed{\#1}$ a) $\binom{6}{4} = 15$

5

b) 5 + 5

$\begin{cases} x_1+x_2=11 \\ x_3+x_4+x_5=1 \end{cases}$ or, $\begin{cases} x_1+x_2=1 \\ x_3+x_4+x_5=11 \end{cases}$

$$\binom{11+2-1}{2-1}\binom{1+3-1}{3-1} + \binom{1+2-1}{2-1}\binom{11+3-1}{3-1}$$

$$= \binom{12}{1}\binom{3}{2} + \binom{2}{1}\binom{13}{2}$$

$$= 36 + 156 = \boxed{192}$$

c) 5

$\begin{cases} x_1+x_2+\cdots+x_8=20 \\ x_i>0 \rightarrow x_i \geq 1 \\ x_1 \geq 5 \end{cases}$

$\begin{cases} y_i = x_i - 1, \quad i = 2, 3, \ldots, 8 \\ y_1 = x_1 - 5 \end{cases}$

$\begin{cases} (y_1+5)+(y_2+1)+\cdots+(y_8+1)=20 \\ y_i \geq 0 \end{cases} \Rightarrow$

$\begin{cases} y_1+y_2+\cdots+y_8 = 20-12=8 \\ y_i \geq 0 \end{cases} \Rightarrow \binom{8+8-1}{8-1} = \binom{15}{7}$

$\boxed{\#2}$ $p\{\text{wrong weight}\} = .02$

$p\{\text{wrong colour}\} = .05$

$p\{\text{wrong weight \& wrong colour}\} = .01$

a) $p\{\text{not defective in any respect}\} = ?$

5

$= 1 - p\{\text{defective in weight } \underline{\text{or}} \text{ defective in colour}\}$

$= 1 - \left[p(\text{defective in weight}) + p(\text{defective in colour}) - p(\text{defective in both}) \right]$

$= 1 - [.02 + .05 - .01] = \underline{.94}$

b) $p\{\text{defective in exactly one respect}\} = ?$

5

$= p\{\text{defective in weight or colour}\} - p\{\text{defective in both}\}$

$= [.02 + .05 - .01] - .01 = \underline{.05}$

c) Let p denote the probability that a unit is

5 defective in all respects. According to the problem $p = .01$. Then, the number of units which are defective in both respects is a binomial random variable with $n = 10$ and $p = .01$. Therefore,

$p\left\{\begin{matrix}\text{at least one defective} \\ \text{unit in both respects}\end{matrix}\right\} = 1 - (1-p)^{10} = \underline{1 - (1 - .01)^{10}}$

#3 Let R be the event that he receives the award.

a) 5

$$P(R) = P(R \mid \text{Strong})\,P(\text{strong}) + P(R \mid \text{moderate})\,P(\text{moderate}) + P(R \mid \text{weak})\,P(\text{weak})$$

$$= (.9)(.7) + (.5)(.2) + (.1)(.1) = .74$$

b) 10 $P(\text{strong} / R) = ?$

Using Bayes's formula,

$$P(\text{strong} / R) = \frac{P(R \mid \text{strong})\,P(\text{strong})}{P(R)}$$

$$= \frac{(.9)(.7)}{.74} = \frac{63}{74}$$

c) 10

$$P(\text{weak} \mid R^c) = \frac{P(R^c \mid \text{weak})\,P(\text{weak})}{P(R^c)}$$

$$= \frac{(1-.1)(.1)}{1-.74} = \frac{9}{26}$$

#4

15

Let random variable D denote the daily demand and random variable X_m denote the profit when the newsboy purchases m papers ($m \leq 10$). We note that:

$$X_m = \begin{cases} m(15-10) & D \geq m \\ D(15-10)+(m-D)(-10) & D < m \end{cases}$$

The random variable X_m is a function of the random variable D. That is $X_m = g(D)$ where

$$g(x) = \begin{cases} 5m & x \geq m \\ 15x - 10m & x < m \end{cases}$$

Therefore,

$$E[X_m] = \sum_{d=0}^{10} g(d)\, P\{D=d\}$$

$$= \sum_{d=0}^{m-1} (15d-10m)\binom{10}{d}(1/3)^d(2/3)^{10-d}$$

$$+ \sum_{d=m}^{10} (5m)\binom{10}{d}(1/3)^d(2/3)^{10-d}$$

$\Rightarrow$ $E[X_0] = 0$, $E[X_1] = 4.74$, $E[X_2] = 8.18$, $E[X_3] = 8.69$

$E[X_4] = 5.3$, $E[X_5] = -1.5$, ... $\Rightarrow$ $\{m_{opt} = 3\}$

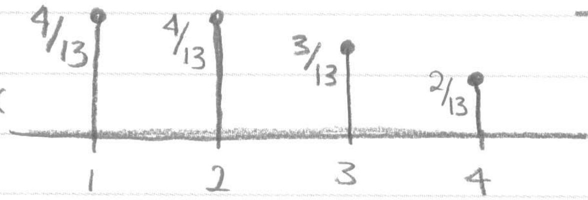
$\boxed{\#5}$ $P\{X=k\} = ak2^{-(k-1)}, \quad k=1,2,3,4$

a) 5

$$\sum_k P\{X=k\} = 1 \Rightarrow \sum_{k=1}^{4} ak2^{-(k-1)} = 1$$

$$\Rightarrow a + a + \tfrac{3}{4}a + \tfrac{a}{2} = 1 \Rightarrow \boxed{a = 4/13}$$

PMF of X



b) 5

$$P\{X>2\} = P\{X=3\} + P\{X=4\} = \frac{3}{13} + \frac{2}{13} = \boxed{\frac{5}{13}}$$

c) 5

$$Var(X) = E\left[(X-E[X])^2\right]$$

$$E[X] = \sum_k k\,P\{X=k\} = (1)\left(\frac{4}{13}\right) + (2)\left(\frac{4}{13}\right) + (3)\left(\frac{3}{13}\right) + (4)\left(\frac{2}{13}\right)$$

$$\Rightarrow \boxed{E[X] = \frac{29}{13}}$$

$$E[X^2] = \sum_k k^2 P\{X=k\} = (1)^2\left(\frac{4}{13}\right) + (2)^2\left(\frac{4}{13}\right) + (3)^2\left(\frac{3}{13}\right) + (4)^3\left(\frac{2}{13}\right)$$

$$\Rightarrow \boxed{E[X^2] = \frac{175}{13}}$$

$$Var(X) = E[X^2] - (E[X])^2 = \frac{175}{13} - \left(\frac{29}{13}\right)^2 = \boxed{8.485}$$

d) 5

$$E[Y] = E[X\log_2 X] = (1\log_2 1)(4/13) + (2\log_2 2)(4/13)$$

$$+ (3\log_2 3)(3/13) + (4\log_2 4)(2/13)$$

$$\Rightarrow E[Y] = \frac{8}{13} + (\log_2 3)(9/13) + 16/13$$

e) $F_Z(z) = P\{Z \leq z\} = P\{2X + 3 \leq z\}$

$$= P\left\{X \leq \frac{z-3}{2}\right\} = \begin{cases} 0, & \frac{z-3}{2} < 1 \\ 4/13, & 1 \leq \frac{z-3}{2} < 2 \\ 4/13 + 4/13 & 2 \leq \frac{z-3}{2} < 3 \\ 4/13 + 4/13 + 3/13 & 3 \leq \frac{z-3}{2} < 4 \\ 4/13 + 4/13 + \frac{3}{13} + \frac{2}{13} & \frac{z-3}{2} \geq 4 \end{cases}$$

$$= \begin{cases} 0, & z < 5 \\ 4/13, & 5 \leq z < 7 \\ 8/13, & 7 \leq z < 9 \\ 11/13, & 9 \leq z < 11 \\ 1, & z \geq 11 \end{cases}$$