UNIVERSITY OF WATERLOO
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
ECE 316 Probability Theory and Random Processes
Midterm Examination
Wednesday February 13, 2008, 5:30 pm - 7:00 pm

Instructor: L.-L. Xie
Time of exam: 5:30 pm
Duration of exam: 90 minutes
Aids permitted: Hand calculators only
Answer all seven questions
Total marks = 35
Each question is of 5 marks

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1. How many different letter arrangements can be made from the following 11 letters:

M i s s i s s i p p i

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2. From a group of 9 women and 7 men a committee consisting of 4 women and 3 men is to be formed. How many different committees are possible if

(a) 2 of the men refuse to serve together;

(b) 1 man and 1 woman refuse to serve together?

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3. Forty percent of the people in a town watch both basketball games and baseball games regularly. 55 percent watch basketball games and 63 percent watch baseball games regularly. If one person is chosen randomly, what is the probability that this person watches

(a) neither basketball games nor baseball games regularly;

(b) basketball games or baseball games regularly?

4. Find the simplest expression for the following events:

(a) $((E^c \cup G)F^cG^c)^c$;

(b) $(E \cup F^c)(F \cup G^c)(G \cup E^c)$.

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5. Three cards are randomly chosen without replacement from an ordinary deck of 52 playing cards.

(a) Given that the ace of spades is chosen, what is the probability that all three cards are aces?

(b) Given that at least one ace is chosen, what is the probability that all three cards are aces?

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6. Suppose that n independent trials, each of which results in any of the outcomes 0, 1, or 2 with respective probabilities, p_0, p_1, and p_2, $\sum_{i=0}^{2} p_i = 1$, are performed. Find the probability that outcome 1 occurs at least once and outcome 2 occurs exactly twice.

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7. Three players A, B, C simultaneously toss coins. The coin tossed by player A turns up heads with probability P_1; the coin tossed by player B turns up heads with probability P_2; the coin tossed by player C turns up heads with probability P_3. If one person gets an outcome different from those of the other two, then he is the odd man out. If there is no odd man out, the players flip again and continue to do so until they get an odd man out. What is the probability that A will be the odd man?

USEFUL FACTS:

Basic combinations:

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}, \qquad \binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!}$$

The binomial theorem:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

DeMorgan's laws:

$$\left(\bigcup_{i=1}^{n} E_i \right)^c = \bigcap_{i=1}^{n} E_i^c, \qquad \left(\bigcap_{i=1}^{n} E_i \right)^c = \bigcup_{i=1}^{n} E_i^c$$

The inclusion-exclusion identity:

$$\begin{aligned} P(E_1 \cup E_2 \cup \cdots \cup E_n) = & \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} E_{i_2}) + \cdots \\ & + (-1)^{r+1} \sum_{i_1 < i_2 < \cdots < i_r} P(E_{i_1} E_{i_2} \cdots E_{i_r}) \\ & + \cdots + (-1)^{n+1} P(E_1 E_2 \cdots E_n) \end{aligned}$$

Conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)} \qquad \text{if} \quad P(F) > 0$$

The multiplication rule:

$$P(E_1 E_2 E_3 \cdots E_n) = P(E_1) P(E_2|E_1) P(E_3|E_1 E_2) \cdots P(E_n|E_1 \cdots E_{n-1})$$

Bayes' Formula:

$$P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)$$

and more generally, for mutually exclusive $F_1, F_2, \ldots, F_n$ with $\bigcup_{i=1}^{n} F_i = S$,

$$P(E) = \sum_{i=1}^{n} P(E|F_i)P(F_i), \qquad P(F_j|E) = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Independent events: The events $E_1, E_2, \ldots, E_n$ are said to be independent if, for every subset $\{i_1, i_2, \ldots, i_r\} \subset \{1, 2, \ldots, n\}$,

$$P(E_{i_1} E_{i_2} \cdots E_{i_r}) = P(E_{i_1}) P(E_{i_2}) \cdots P(E_{i_r})$$