Solutions to Midterm

#1. Since there are totally 11 letters, of which 4 s's are alike, 4 i's are alike, 2 p's are alike, the total number of different arrangements should be

$$\frac{11!}{4!\,4!\,2!} = 34,650$$

#2 (a). Without restriction, there are totally $\binom{9}{4}\binom{7}{3}$ different committees.

Now, we are required to compute the # of different committees with the restriction that 2 of the men refuse to serve together. Since there are $\binom{9}{4}\binom{5}{1}$ different committees if those 2 men serve together, we can subtract $\binom{9}{4}\binom{5}{1}$ from the total number to obtain the answer:

$$\binom{9}{4}\binom{7}{3} - \binom{9}{4}\binom{5}{1} = 3780$$

(b). Similarly, if that man and that women serve together, there are $\binom{8}{3}\binom{6}{2}$ possibilities. To get the answer, we can subtract $\binom{8}{3}\binom{6}{2}$ from the total number:

$$\binom{9}{4}\binom{7}{3} - \binom{8}{3}\binom{6}{2} = 3570.$$

#3. Let event $A = \{$this person watches basketball game$\}$.
$B = \{$this person watches baseball game$\}$,

Then, according to the question itself, we have known that:

$$P(AB) = 40\%.$$
$$P(A) = 55\%$$
$$P(B) = 63\%.$$

Therefore, for (a). $P(A^cB^c) = P(A\cup B)^c = 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(AB)]$

$$= 1 - [0.55 + 0.63 - 0.4] = 1 - 0.78 = 0.22$$

for (b), $P(A\cup B) = P(A)+P(B) - P(AB) = 0.55+0.63-0.4 = 0.78$.

#4. (a). $((E^c \cup G)F^cG^c)^c$

$=[(E^c \cup G) \cap (G^cF^c)]^c$

$=[(E^cG^cF^c) \cup (\underbrace{GG^cF^c}_{\emptyset})]^c$

$=(E^cG^cF^c)^c$

$= E\cup F \cup G$.

(b). $(E\cup F^c)(F\cup G^c)(G\cup E^c)$

$=\{[(E\cup F^c)F] \cup [(E\cup F^c)G^c]\}(G\cup E^c)$

$=[(EF)\cup(\underbrace{F^cF}_{\emptyset})\cup(EG^c)\cup(F^cG^c)](G\cup E^c)$

$=[(EF)\cup(EG^c)\cup(F^cG^c)](G\cup E^c)$

$=[(EFG)\cup(\underbrace{EG^cG}_{\emptyset})\cup(F^c\underbrace{G^cG}_{\emptyset})] \cup [(\underbrace{EFE^c}_{\emptyset})\cup(\underbrace{EG^cE^c}_{\emptyset})\cup(F^cG^cE^c)]$

$=(EFG)\cup(E^cF^cG^c)$

#5. (a) Let $A = \{$the ace of spades is chosen$\}$.

$B = \{$all three cards are aces$\}$.

then the desired probability is actually $P(B|A) = \frac{P(AB)}{P(A)}$.

Since $P(AB) = P\{$ace of spades is chosen and all three cards are aces$\}$

$$= \frac{\binom{3}{2}}{\binom{52}{3}}$$

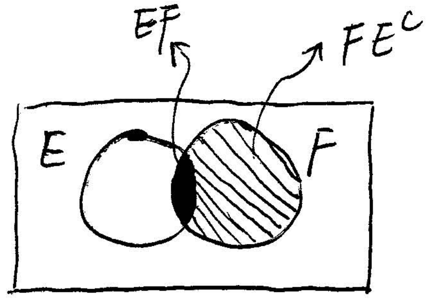
and $P(A) = \frac{\binom{51}{2}}{\binom{52}{3}}$,

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\binom{3}{2}}{\binom{51}{2}} = \frac{1}{425}.$$

(b). Let $C = \{$ at least one ace is chosen $\}$.
then the desired probability is $P(B|C) = \frac{P(BC)}{P(C)}$.
Since $P(BC) = P(B) = \binom{4}{3} / \binom{52}{3}$, $P(C) = 1 - \binom{48}{3} / \binom{52}{3}$

$$P(B|C) = \frac{P(BC)}{P(C)} = \frac{\binom{4}{3}/\binom{52}{3}}{1-\binom{48}{3}/\binom{52}{3}} = \frac{\binom{4}{3}}{\binom{52}{3}-\binom{48}{3}} = \frac{1}{1201}$$

#6. Let $E = \{$ outcome 1 occurs at least once $\}$
$F = \{$ outcome 2 occurs exactly twice $\}$.
then the desired probability is actually $P(EF)$.
Since $P(EF) = P(F) - P(FE^c)$, now it's required to Compute the probability $P(F)$ and $P(FE^c)$ respectively.



$P(F) = \binom{n}{2} P_2^2 (1-P_2)^{n-2}$,

$P(FE^c) = P\{$ outcome 2 occurs twice and outcome 1 never occurs $\}$ $\quad \therefore P(EF) = P(F) - P(FE^c)$

$= \binom{n}{2} P_2^2 P_0^{n-2}$ ($\because$ Except the twice outcome 2, all trials result in outcome 0)

Therefore, $P(EF) = P(F) - P(FE^c) = \binom{n}{2} P_2^2 (1-P_2)^{n-2} - \binom{n}{2} P_2^2 P_0^{n-2}$

#7. Let $P(A\ odd)$ denote the desired probability, then we have the following identity:

$$P(A\ odd) = \underbrace{P_1(1-P_2)(1-P_3)}_{\substack{\text{A heades, B,C tails} \\ \text{A odd}}} + \underbrace{(1-P_1)P_2P_3}_{\substack{\text{A tails, B,C heads} \\ \text{A out}}} + \underbrace{[(P_1P_2P_3) + (1-P_1)(1-P_2)(1-P_3)]\, P(A\ odd)}_{\text{Tie in this round, and then A out}}$$

Therefore, $$P(A\ odd) = \frac{P_1(1-P_2)(1-P_3) + (1-P_1)P_2P_3}{P_1+P_2+P_3-P_1P_2-P_1P_3-P_2P_3}$$