1 ECE 316, Midterm Spring 2014, Solution

Q.1: a) $p(E\cap F) \overset{(a)}{=} p(\emptyset) = 0 \neq \underbrace{p(E)}_{>0}\cdot\underbrace{p(F)}_{>0}$,

where (a) is due to the fact that E & F are mutually exclusive

b) E & F are indep. $\longrightarrow p(E\cap F) = p(E)\cdot p(F)$

$p(E) = p(E\cap(F\cup F^c)) = p((E\cap F)\cup(E\cap F^c)) = p(E\cap F) + p(E\cap F^c) - \underbrace{p((E\cap F)\cap(E\cap F^c))}_{*}$

$* = p(E\cap(F\cap F^c)) = p(E\cap\phi) = 0$ "Tutorial note "4", Ex.1"

Therefore, $p(E) = p(E)\cdot p(F) + p(E\cap F^c) \longrightarrow p(E\cap F^c) = p(E)(1-p(F))$

$= p(E)\,p(F^c) \rightarrow F^c\,\&\,E$ are indep.

For E^c and F^c we ~~have~~ can use a similar approach or,

$p(E^c\cap F^c) = p((E\cup F)^c) = 1 - p(E\cup F) = 1 - [p(E)+p(F) - p(E)p(F)] = p(E^c)p(F^c)$

$\longrightarrow E^c$ and F^c are indep.

Q.2: a) $p(\text{two aces} / \text{ace of spades is chosen}) = \dfrac{p(\text{two aces, ace of spades})}{p(\text{ace of spades is chosen})}$

"Tutorial note "4", Q.2, part a"

$= \dfrac{\binom{3}{1}/\binom{52}{2}}{\binom{51}{1}/\binom{52}{2}} = \dfrac{3}{51} = \dfrac{1}{17}$

b) $P = \dfrac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$

"Tutorial note "3", Ex.2"

Q.3: a) $p(\text{Peter}) = 0.7$, $p(\text{James}) = 0.2$, $P(\text{Ford}) = 0.1$, $D = \text{Defective}$

$p(D|\text{Peter}) = 0.1$, $p(D|\text{James}) = 0.2$, $P(D|\text{Ford}) = 0.6$

$$p(D) = p(D|\text{peter})p(\text{Peter}) + p(D|\text{James})p(\text{James}) + p(D|\text{Ford})p(\text{Ford})$$

$$= (0.1)\times(0.7) + (0.2)\times(0.2) + (0.6)\times(0.1) = 0.17$$

b) $$p(\text{Peter}|D) = \frac{p(D|\text{peter})p(\text{peter})}{p(D)} = \frac{(0.1)\times(0.7)}{0.17} = \frac{7}{17}$$

"Tutorial note "4", Ex.1"

Q.4: a) $E(X) = \lambda$, $E(Y) = np$

$$E(3X+2Y) = 3.E(X) + 2.E(Y) = 3\lambda + 2np$$

b) $E(X! + e^X) = E(X!) + E(e^X)$

$$E(X!) = \sum_{k=0}^{\infty} \cancel{(k!)}\, \frac{e^{-\lambda}.\lambda^k}{\cancel{k!}} = e^{-\lambda}.\sum_{k=0}^{\infty} \lambda^k = e^{-\lambda}.\frac{1}{1-\lambda}, \text{ if } |\lambda| < 1.$$

$$E(e^X) = \sum_{k=0}^{\infty} (e^k).\frac{e^{-\lambda}.\lambda^k}{k!} = e^{-\lambda}.\sum_{k=0}^{\infty}\frac{(\lambda e)^k}{k!} = e^{-\lambda}.e^{(\lambda e)}$$

Q.5: a) $\smile Y \smile Y \smile Y \smile$ $\longrightarrow$ $P = \dfrac{\binom{4}{2}}{\binom{5}{2}} = \dfrac{6}{10} = \dfrac{3}{5}$

b) F = functional D = # of defective antennas

$$p(F) = p(F|D=0)p(D=0) + p(F|D=1)p(D=1) + \cdots + p(F|D=5)p(D=5)$$

$*$ $p(F|D=4) = 0$, $p(F|D=5) = 0$, $p(D=k) = \binom{5}{k}\left(\tfrac{1}{2}\right)^k\left(\tfrac{1}{2}\right)^{5-k}$

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$p(F|D=0)=1$, $p(D=0)=\binom{5}{0}(1/2)^0(1/2)^5=\frac{1}{32}$

$p(F|D=1)=1$, $p(D=1)=\binom{5}{1}(1/2)^1(1/2)^4=\frac{5}{32}$

$p(F|D=2)=\frac{3}{5}$ (part "a") , $p(D=2)=\binom{5}{2}(1/2)^2(1/2)^3=\frac{5}{16}$

$p(F|D=3)=\frac{1}{\binom{5}{3}}=\frac{1}{10}$, $p(D=3)=\binom{5}{3}(1/2)^3(1/2)^2=\frac{5}{16}$

$\longrightarrow p(F)=\frac{13}{32}$

"Textbook: exercise 4.12 of ch.4 and example 4.c of ch.1."