P1.

i) $|S| = \binom{20}{4}$ , $|E| = \binom{5}{1} \cdot \binom{5}{1} \cdot \binom{5}{1} \cdot \binom{5}{1}$

E = distinct colors $\Rightarrow$ 1 blue, 1 black, 1 while, 1 green

$$\Rightarrow P = \frac{\binom{5}{1}^4}{\binom{20}{4}}$$

ii) $|S| = \binom{20}{4}$ , $|E| = \binom{5}{2} \cdot \binom{5}{2}$

$$\Rightarrow P = \frac{\binom{5}{2}^2}{\binom{20}{4}}$$

Q2) Let's define the following R.Vs:

$L$ = Liberal  $I$ = Independent  $C$ = Conservative

$V$ = person has voted

$p(L) = 0.2$  $p(I) = 0.5$  $p(C) = 0.3$

$p(V|I) = 0.6$  $p(V|L) = 0.3$  $p(V|C) = 0.4$

i) $p\{\text{person has voted}\} = p(V) = p(V|I)p(I) + p(V|C)p(C) + p(V|L)p(L)$

$= (0.6)(0.5) + (0.4)(0.3) + (0.3)(0.2)$

$\Rightarrow p(V) = \boxed{0.48}$

ii) $p(L|V) = \dfrac{p(L,V)}{p(V)} = \dfrac{p(V|L)p(L)}{p(V)} = \dfrac{(0.3)(0.2)}{0.48} = \dfrac{1}{8} = 0.125$

P.3 :

i) Prob $\{V < 18\}$ where $V \sim N(20, 2^2)$

let $Z \triangleq \frac{V - 20}{2} \Rightarrow Z \sim N(0, 1)$

Prob $\{V < 18\}$ = prob $\{Z < -1\}$ =

$\Phi(-1) = 0.16$

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ii) $E((\underbrace{V+2}_{\triangleq Y})^2) = E(Y^2)$

wher $Y \sim N(22, 2^2)$

$E(Y^2) = Var(Y) + (E(Y))^2 =$

$4 + (22)^2 = 488$

Q4) i) $W$ = the unit has the wrong weight (and may be the wrong color)

$C$ = " " " " " color (and maybe the wrong w)

$W \cap C$ = both wrong weight and wrong color

$P(W) = 0.02$ , $P(C) = 0.05$ , $P(W \cap C) = 0.01$

$P\{$the unit is defective in at least one respect$\}$

$= P\{W \cup C\} = P\{W\} + P\{C\} - P\{W \cap C\} = 0.02 + 0.05 - 0.01$

$= \boxed{0.06}$

ii) $X$ = # of units selected to find the first non-defective item

$P(X=k) \triangleq$ the $k^{th}$ item is non-defective and the previous $(k-1)$ items are defective

"$X$" follows Geometric distn. with $p = 1 - 0.06 = 0.94$

$\longrightarrow P(X=k) = (1-p)^{k-1} \cdot p$ , $k = 1, 2, 3, \cdots$

$= (0.06)^{k-1} \cdot (0.94)$ , $k = 1, 2, 3, \cdots$

$P_5$ :

i) both defective → $\frac{5 \times 4}{50 \times 49}$ (first choice, second choice)

ii) both non-defective → $\frac{45 \times 44}{50 \times 49}$