University of Waterloo
Department of Electrical and Computer Engineering

E&CE-316 – Introduction to Probability Theory

Midterm Examination

June 9, 2006

Instructors: A. K. Khandani
Time allowed: 1.5 hours.
Closed book. One sheet of $8\frac{1}{2} \times 11$ review sheet (one side) allowed.
Answer all three questions. The parts with * sign are more difficult.
Questions and parts within questions are of equal value.

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1. Assume that 5 items are in an urn, numbered 1,2,3,4,5. Suppose that 2 items are white and 3 items are black.

1.1 (mark=2.5) Compute the probability that in a draw of 2 items (without replacement) we obtain 2 white items.

1.2 (mark=2.5) Compute the probability that in a draw of 2 items (without replacement) we obtain 2 black items.

1.3 (mark=2.5) Compute the probability that in a draw of 2 items (without replacement) we obtain one white and one black item.

1.4 (mark=2.5) Compute the probability that in a draw of 2 items (with replacement) we obtain 2 white items.

2. In a factory, units are manufactured by machines H_1,H_2,H_3 in the proportions $25 : 35 : 40$. The percentages $5\%, 4\%$ and 2%, respectively, of the manufactured units are defective. The units are mixed and sent to the customers.

2.1 (mark=2.5) Find the probability that a randomly chosen unit is defective.

2.2 (mark=2.5) Suppose that a customer discovers that a certain unit is defective. What is the probability that it has been manufactured by machine H_1?

2.3 (mark=2.5) Suppose that a customer buys units until he/she discovers a defective item and then stops buying that product. What is the probability that the customer buys more than 5 items before discovering a defective item.

2.4 (mark=2.5) *Suppose that a customer discovers that a certain unit is defective and we know that it is not produced by H_2, then what is the probability that it has been manufactured by machine H_1?

3. We know that there are $\binom{n+r-1}{r-1}$ distinct integer-valued vectors $(x_1, x_2, \ldots, x_r)$ satisfying:

$$x_1 + x_2 + \cdots + x_r = n \qquad x_i \geq 0, i = 1, \ldots, r$$

3.1 (mark=5) What is the number of distinct integer-valued vectors $(x_1, x_2, \ldots, x_5)$ satisfying:

$$x_1 + x_2 + \cdots + x_5 = 15 \qquad x_i \geq 2, i = 1, \ldots, 5$$

3.2 (mark=5) *What is the number of distinct integer-valued vectors $(x_1, x_2, \ldots, x_5)$ satisfying:

$$x_1 + x_2 + \cdots + x_5 = 13 \qquad 4 > x_i > 1, i = 1, \ldots, 5$$