Note: Students can add formulas on the second page, or use the first page and write additional formulas at the back. Calculators are allowed in the exam

The Binomial Theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

The Multinomial Theorem: $(x_1 + x_2 + \cdots + x_r)^n = \sum_{\substack{(n_1,n_2,\cdots,n_r): \\ n_1+\cdots+n_r=n}} \frac{n!}{n_1!\, n_2! \cdots n_r!} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$

There are $\binom{n+r-1}{r-1}$ distinct nonnegative integer-valued vectors $(x_1, \cdots, x_r)$ satisfying $x_1 + \cdots + x_r = n$.

$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c \qquad \left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i^c \qquad P(E^c) = 1 - P(E)$

$P(E_1 \bigcup E_2 \bigcup \cdots \bigcup E_n) = \sum_{i=1}^{n} P(E_i) + \sum_{i_1<i_2} P(E_{i_1}E_{i_2}) + \cdots + (-1)^{r+1} \sum_{i_1<i_2<\cdots<i_r} P(E_{i_1}E_{i_2}\cdots E_{i_r}) + \cdots + (-1)^{n+1} P(E_1 E_2 \cdots E_n)$,

where the summation $\sum_{i_1<i_2<\cdots<i_r} P(E_{i_1}E_{i_2}\cdots E_{i_r})$ is taken over all of the $\binom{n}{r}$ possible subsets of size r of the set $\{1, 2, \cdots, n\}$.

$P(E \bigcup F) = P(E) + P(F) - P(EF) \qquad P(E|F) = \frac{P(EF)}{P(F)}$

$P(E_1 E_2 \cdots E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1, E_2) \cdots P(E_n|E_1, E_2, \cdots, E_{n-1})$

$P(E) = P(EF) + P(EF^c) = P(F)P(E|F) + P(F^c)P(E|F^c) = P(F)P(E|F) + (1 - P(F))P(E|F^c)$

Bayes' formula: Suppose that $F_1, F_2, \ldots, F_n$ are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$, then $P(F_j|E) = \frac{P(EF_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$

Independent Events: Two events E and F are *independent* if $P(EF) = P(E)P(F)$ then $P(E|F) = P(E)$ and $P(F|E) = P(F)$. In general, the events $E_1, E_2, \cdots, E_n$ are said to be independent if for any subset $E_{i_1}, E_{i_2}, \cdots, E_{i_r}$ of them $P(E_{i_1}E_{i_2}\cdots E_{i_r}) = P(E_{i_1})P(E_{i_2})\cdots P(E_{i_r})$.

A single discrete random variable X:

Probability Mass Function (PMF) of X : $p_X(a) = P\{X = a\}$.

Cumulative Distribution Function (CDF) of X: $F_X(a) = P\{X \le a\} = \sum_{x \le a} p_X(a)$.

The expected value of random variable X : $E[X] = \sum_x x p_X(x)$

For any function $g(\cdot)$: $E[g(X)] = \sum_x g(x) p_X(x)$

The variance of the random variable X: $\mathrm{Var}(X) = E[(X - E[X])^2] \quad SD(X) = \sqrt{\mathrm{Var}(X)} \quad \mathrm{Var}(aX + b) = a^2 \mathrm{Var}(X)$

Name	PMF	Mean	Variance
Bernoulli with parameter p	$P\{X=1\} = p$, $P\{X=0\} = 1-p$	p	$p(1-p)$
Binomial with parameters n, p	$P\{X=k\} = \binom{n}{k} p^k (1-p)^{n-k}$, $k = 0, \cdots, n$	np	$np(1-p)$
Geometric with parameter p	$P\{X=n\} = p(1-p)^{n-1}$, $n = 1, 2, \cdots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson with parameter λ	$P\{X=n\} = e^{-\lambda}\frac{\lambda^n}{n!}$, $n = 0, 1, \cdots$	λ	λ

Two discrete random variables X and Y:

joint PMF: $p_{XY}(a, b) = P\{X = b, Y = b\}$ joint CDF: $F_{XY}(a, b) = P\{X \le a, Y \le b\}$.

Marginal PMF: $p_X(a) = \sum_b p_{XY}(a, b)$ and $p_Y(b) = \sum_a p_{XY}(a, b)$

Marginal CDF: $F_X(a) = F_{XY}(a, \infty)$ and $F_Y(b) = F_{XY}(\infty, b)$

Conditional PMF: $p_{X|Y}(x|y) = P\{X = x|Y = y\} = \frac{P\{X=x, Y=y\}}{P\{Y=y\}} = \frac{p_{XY}(x,y)}{p_Y(y)}$

Conditional CDF: $F_{X|Y}(x|y) = P\{X \le x|Y = y\} = \sum_{a \le x} p_{X|Y}(a|y)$

$E[g(X,Y)] = \sum_y \sum_x g(x, y) p_{XY}(x, y)$

Joint MGF: $M_{XY}(t, t') = E[e^{tX + t'Y}] = \sum_y \sum_x e^{tx + t'y} p_{XY}(x, y)$

Conditional Expectation: $E[X|Y = y] = \sum_x x p_{X|Y}(x|y) \qquad E[g(X)|Y = y] = \sum_x g(x) p_{X|Y}(x|y)$

$E[X] = \sum_y E[X|Y = y] P\{Y = y\}$

Independent Random Variables: Two discrete random variables X and Y are said to be independent if for every a and b we have $p_{XY}(a, b) = p_X(a)p_Y(b)$.

A single continuous random variable X:

Probability Density Function (PDF) of X : $f_X(a) = \frac{1}{\epsilon} P\{a - \frac{\epsilon}{2} \leq X \leq a + \frac{\epsilon}{2}\}$.

Cumulative Distribution Function (CDF) of X: $F_X(a) = P\{X \leq a\} = \int_{-\infty}^{a} f_X(x)dx$

$P\{X \in B\} = \int_B f_X(x)dx \qquad f_X(x) = \frac{d}{dx} F_X(x)$

The expected value of random variable X : $E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$

For any function $g(\cdot)$: $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)dx$

The variance of the random variable X: $\text{Var}(X) = E[(X - E[X])^2] \quad SD(X) = \sqrt{\text{Var}(X)} \quad \text{Var}(aX + b) = a^2 \text{Var}(X)$