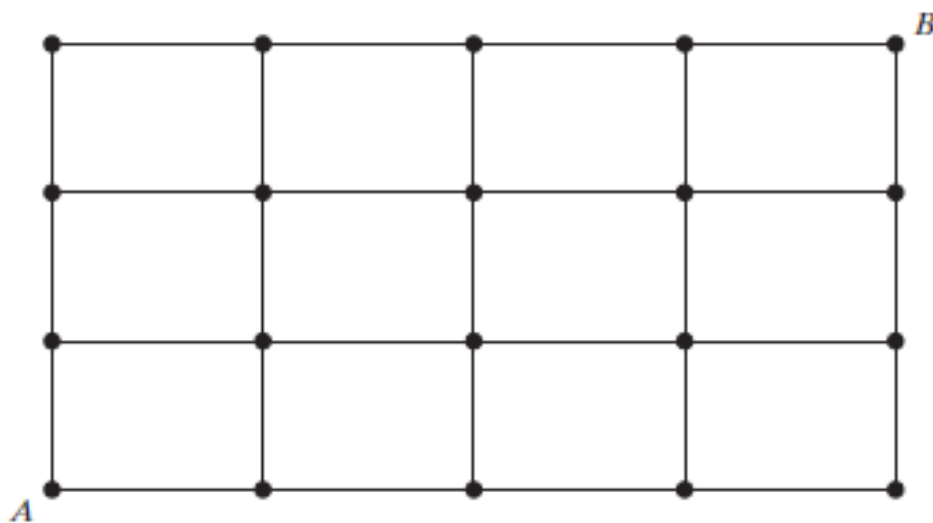


Hi,

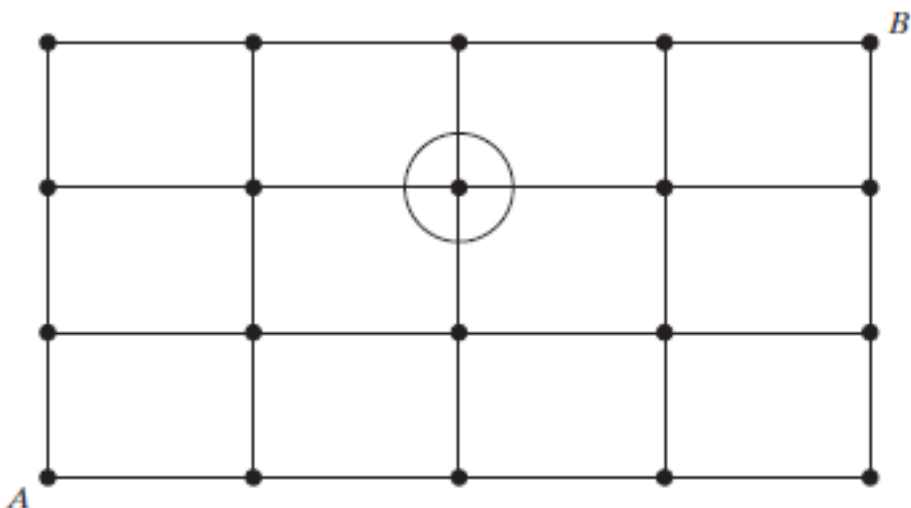
I chose some noteworthy problems from textbook. I will put solutions on Saturday night.

1. (a) How many different 7-place license plates are possible if the first 2 places are for letters and the other 5 for numbers?
(b) Repeat part (a) under the assumption that no letter or number can be repeated in a single license plate.
11. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if
 - (a) the books can be arranged in any order?
 - (b) the mathematics books must be together and the novels must be together?
 - (c) the novels must be together, but the other books can be arranged in any order?
13. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?
17. Seven different gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than one gift?

19. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
- (a) 2 of the men refuse to serve together?
 - (b) 2 of the women refuse to serve together?
 - (c) 1 man and 1 woman refuse to serve together?
20. A person has 8 friends, of whom 5 will be invited to a party.
- (a) How many choices are there if 2 of the friends are feuding and will not attend together?
 - (b) How many choices if 2 of the friends will only attend together?
21. Consider the grid of points shown here. Suppose that, starting at the point labeled A , you can go one step up or one step to the right at each move. This procedure is continued until the point labeled B is reached. How many different paths from A to B are possible?
- Hint:* Note that to reach B from A , you must take 4 steps to the right and 3 steps upward.



22. In Problem 21, how many different paths are there from A to B that go through the point circled in the following lattice?



28. If 8 new teachers are to be divided among 4 schools, how many divisions are possible? What if each school must receive 2 teachers?
29. Ten weight lifters are competing in a team weight-lifting contest. Of the lifters, 3 are from the United States, 4 are from Russia, 2 are from China, and 1 is from Canada. If the scoring takes account of the countries that the lifters represent, but not their individual identities, how many different outcomes are possible from the point of view of scores? How many different outcomes correspond to results in which the United States has 1 competitor in the top three and 2 in the bottom three?

- *32. An elevator starts at the basement with 8 people (not including the elevator operator) and discharges them all by the time it reaches the top floor, number 6. In how many ways could the operator have perceived the people leaving the elevator if all people look alike to him? What if the 8 people consisted of 5 men and 3 women and the operator could tell a man from a woman?
- *33. We have 20 thousand dollars that must be invested among 4 possible opportunities. Each investment must be integral in units of 1 thousand dollars, and there are minimal investments that need to be made if one is to invest in these opportunities. The minimal investments are 2, 2, 3, and 4 thousand dollars. How many different investment strategies are available if
- (a) an investment must be made in each opportunity?
 - (b) investments must be made in at least 3 of the

SELF-TEST PROBLEMS AND EXERCISES

1. How many different linear arrangements are there of the letters A, B, C, D, E, F for which
 - (a) A and B are next to each other?
 - (b) A is before B?
 - (c) A is before B and B is before C?
 - (d) A is before B and C is before D?
 - (e) A and B are next to each other and C and D are also next to each other?
 - (f) E is not last in line?

9. Consider three classes, each consisting of n students. From this group of $3n$ students, a group of 3 students is to be chosen.
- (a) How many choices are possible?
 - (b) How many choices are there in which all 3 students are in the same class?
 - (c) How many choices are there in which 2 of the 3 students are in the same class and the other student is in a different class?
 - (d) How many choices are there in which all 3 students are in different classes?
 - (e) Using the results of parts (a) through (d), write a combinatorial identity.
10. How many 5-digit numbers can be formed from the integers $1, 2, \dots, 9$ if no digit can appear more than twice? (For instance, 41434 is not allowed.)
17. Give an analytic verification of

$$\binom{n}{2} = \binom{k}{2} + k(n-k) + \binom{n-k}{2}, \quad 1 \leq k \leq n$$

Now, give a combinatorial argument for this identity.